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A hydraulics perspective on the power-law stage-discharge rating curve

Asgeir Petersen-Øverleir

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REPORT

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Norges vassdrags- og energidirektorat
Middelthunsgate 29
Postboks 5091 Majorstua
0301 OSLO

Telefon: 22 95 95 95
Telefaks: 22 95 90 00
Internett: www.nve.no

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Preface

Streamflow data are the most important input in hydrological studies. Discharges in natural rivers are almost exclusively obtained by continuously measuring the height of water level. The water level is subsequently converted to discharge by means of an estimated stage-discharge power-law rating curve. Thus the rating curve is of vital importance in surface hydrology. In this report the hydraulic appropriateness of the rating curve is developed. This treatment permits defining Bayesian priors to the rating curve parameters.

Oslo, November 2005

Morten Johnsrud
head of department

Svein Harsten
head of section

Summary

The results of this study are:

A. Given that the hydraulic channel control in which a gauging station is placed is power-law, i.e. the flow width can be expressed by a single power function of the flow depth, the classical power-law stage-discharge formula is identical to:
   a. the generalized equation for steady uniform flow in a wide channel
   b. the solution to the governing equations for critical flow from a reservoir
   c. the solution to the governing equations for normal uniform flow from a reservoir, given that the outlet is wide and the Chézy equation is applicable in the outlet channel

B. The shape of the hydraulic control cannot always be inferred from the corresponding estimated rating curve exponent.

C. The connections between the power-law rating curve formula and hydraulics provide material for defining hydraulically grounded informative prior densities for the power-law rating curve parameters. This paves the way for a Bayesian approach to rating curve estimation.

1 Introduction

The power-law stage-discharge rating curve

$$Q = ph^q \qquad (1)$$

where Q is discharge, h the depth of flow and (p, q) parameters, is the most common formula for describing the stage-discharge relationship at a gauging station. Eq. (1) is recommended in most of the standard hydrometric texts, e.g. [1-5]. Decades of experience has shown that Eq. (1) is appropriate in most cases, given that the stage-discharge relationship is not significantly affected by unsteadiness and/or backwater effects. The power-law formula has been successfully applied to measurement data from gauging stations in the world's largest rivers such as The Amazon and Rio Paraná [6] as well as small Norwegian mountain streams. Even so, the reasons for its success do not appear to have been adequately explained in the literature.

One simple explanation for the appropriateness of Eq. (1) is that experience demonstrates that the stage-discharge relationship at a well-located gauging station increases monotonically. Consequently, Eq. (1) yields an attractive formula for modelling stage-discharge relationships since a power-law model is both flexible and monotonically increasing (given that $q > 0$). However, by such an argument any monotonically increasing model could be applied.

The purpose of this study is to connect Eq. (1) to the hydraulic equations which govern the types of flow most often encountered in rivers in conjunction with hydrometric gauging stations. The reasons for achieving that are: (a) to demonstrate that a central formula in hydrometry is grounded in hydraulic theory; and (b) to provide a framework for applying at-site characteristics to Bayesian rating curve estimation.

2 Power-law channels

If the cross-sectional flow width w can be expressed by a single power function of the cross-sectional flow depth h, i.e.:

$$B = ah^b \qquad (2)$$

the channel in conjunction with the cross-section is said to be power-law [7]. The parameters a and b, which may be interpreted as scale and shape parameters, respectively, are decided by the characteristics of the river channel. A natural channel is typically very irregular and Eq. (2) will in most cases yield only an approximation. Fig. 1 displays an example from a levelled cross-section for a gauging station in the Chattahoochee River, United States. The soundings show that the channel is not of perfect regular geometric shape. Even so, fitting Eq. (2) to the true stage-width relationship calculated by a natural spline representation is reasonably accurate. In this case the parameters a and b were estimated to be 20.77 and 0.81, respectively.

For a given h the flow area becomes

$$A = \frac{a}{b+1} h^{b+1} \qquad (3)$$

Furthermore, the hydraulic depth D is given by

$$D = \frac{A}{B} = \frac{1}{b+1} h \qquad (4)$$

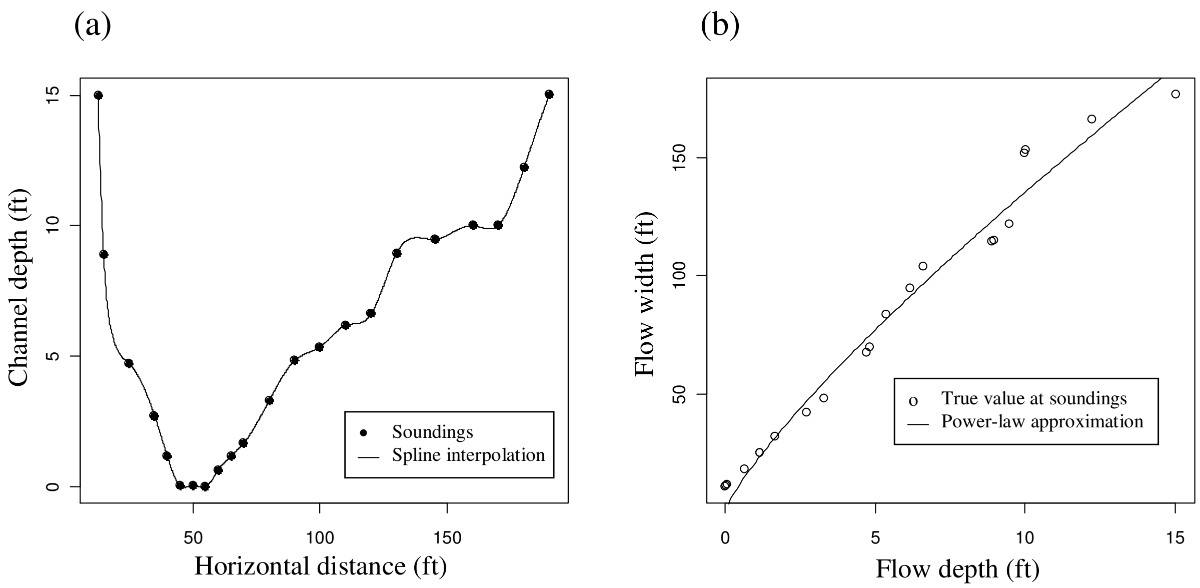

Figure 1: (a) The levelling soundings for the cross-sections of the Chattahoochee River at Littles Ferry Bridge, Georgia, United States, along with a natural spline representation. (b) The calculated depth-width relationship and the corresponding power-law approximation for the Chattahoochee River at Littles Ferry Bridge.

3 Steady uniform flow

Adequate gauging in larger rivers will typically involve siting the station in conjunction with a straight and long reach where the flow is sufficiently tranquil at all discharges. Similar gauging sites may also be found in smaller rivers if the corresponding catchments are of low gradient. In such situations the flow is often assumed uniform under steady conditions.

(a) (b)

Figure 2: Examples of flow in natural rivers that may be assumed uniform under steady conditions. (a) The river Tana at Polmak, Norway. (b) The river Altaelva at Harestrømmen, Norway.

Most practical uniform-flow formulae can be expressed in the generalised form [8]

$$V = FS_0^{1/2}R^x \quad (5)$$

where V is the mean velocity, F the channel resistance coefficient, R the hydraulic radius, S_0 the channel slope assumed to be equal to the friction slope, and x an exponent dependent on the friction law used. Since $Q = VA$, one gets

$$Q = FS_0^{1/2}R^x A \quad (6)$$

If the cross-section is wide, such that R can be approximated by the hydraulic depth, D, and the channel power-law, one obtains

$$Q \approx \frac{FS_0^{1/2} a}{(b+1)^{x+1}} h^{1+x+b} = ph^q \qquad (7)$$

Hence, assuming that the channel resistance coefficient is constant, the power-law rating curve model is applicable for wide power-law cross-sections in steady uniform flow situations.

4 Flow from a reservoir

Reservoirs such as lakes and larger pools are attractive sites for establishing gauging stations, mainly because (a) an outlet from a lake or larger pool is often very stable and provides a sensitive stage-discharge relation; and (b) a reservoir possesses notable heat storage. Outlets from such reservoirs are thus less vulnerable to significant ice build-up during cold periods due to the thermal energy of the water from the reservoir.

(a) (b)

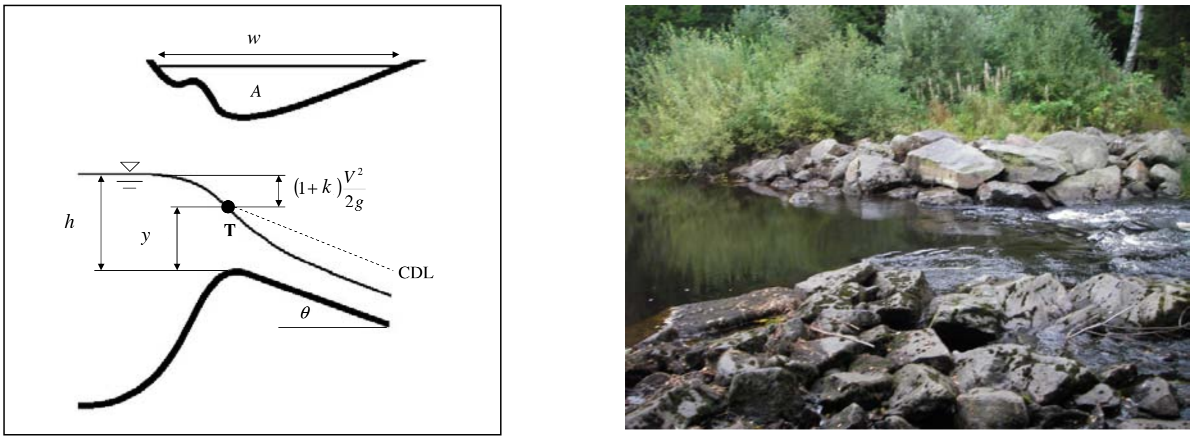

Figure 3: (a) Discharge from a reservoir into a channel wheere the outlet condition is critical. (b) Example of critical flow from a pool in the river Vrangselva at Magnor, Norway.

Assuming that the velocity head in the reservoir is negligible, the energy equation valid for the flow situation in Fig. 2 is [9]:

$$h = y + (1 + k)\frac{Q^2}{2gA_T^2} \qquad (8)$$

where k is the entrance loss coefficient, y the flow depth in the control section, h the stage related to the gauging site in the reservoir and A_T the flow area in the control section T.

4.1 Critical outlet conditions

If the downstream channel slope S_0 is greater than the water surface slope S_T , the flow condition at *T* is critical. The well-known governing equation for critical flow yields

$$\frac{Q^2}{gA_T^2} = \frac{A_T \cos\theta}{\alpha B_T} \qquad (9)$$

where α is the energy coefficient, θ is the angle of the outlet channel and B_T the flow-width in the control section *T*. Combining Eq. (8) and Eq. (9) gives

$$y = h - \frac{(1+k)\cos\theta}{2\alpha}\frac{A_T}{B_T} \qquad (10)$$

If the outlet is power-law, Eq. (10) can be written as

$$y = h\left[1 + \frac{(1+k)\cos\theta}{2\alpha(1+b)}\right]^{-1} \qquad (11)$$

Using Eq. (9) once again gives

$$Q = \sqrt{\frac{g\cos\theta a^2}{\alpha(1+b)^3}\left[1 + \frac{(1+k)\cos\theta}{2\alpha(1+b)}\right]^{-2b-3} h^{2b+3}} = ph^q \qquad (12)$$

which connects the power-law rating curve model to critical flow from a reservoir.

4.2 Normal outlet conditions

If the slope of the outlet channel is so-called mild, i.e. $S_0 < S_T$, then the flow through point *T* is normal and uniform, and Eq. (9) must be replaced by a uniform flow equation.

If the Chézy equation is applied, implying that $x = ½$ in Eq. (6), and assuming that the outlet is sufficiently wide so that W can replace R, one gets

$$Q^2 \approx C^2 S_0 \frac{A_T^3}{B_T} \qquad (13)$$

Combining Eq. (13) and Eq. (11), and assuming that the outlet is power-law, gives

$$Q \approx \sqrt{\frac{C^2 S_0 a^2}{(1+b)^3}\left[1+\frac{(1+k)\cos\theta}{2\alpha(1+b)}\right]^{-2b-3} h^{2b+3}} = ph^q \qquad (14)$$

which demonstrates that the power-law rating curve includes situations where; (1) the flow out of the reservoir is normal and uniform, (2) the Chézy equation is applicable and (3) the outlet and the downstream channel are wide and power-law.

5 Applications

5.1 Predicting channel shape by the rating curve exponent

In this section it will be suggested that the traditional hydrometric understanding of a strong relationship between channel shape and rating curve exponent is not always warranted. It is important that the practical implications of this fact are recognised as the credibility of the exponent of a fitted rating curve is often assessed in association with the shape of the corresponding channel control.

5.1.1 Critical flow from a reservoir

If the outlet is fairly symmetric around the stage of zero flow, and monotonically increasing, the shape of the cross-section can be inferred from the rating curve exponent. The traditional interpretation, based on Fig. 4, is that q = 3/2, 3/2 < q < 5/2, q = 5/2 and q >5/2 correspond to b = 0 (rectangular shape), 0 < b < 1 (parabolic shape), b = 1 (rectangular) and 1 < b (funnel-like shape), respectively.

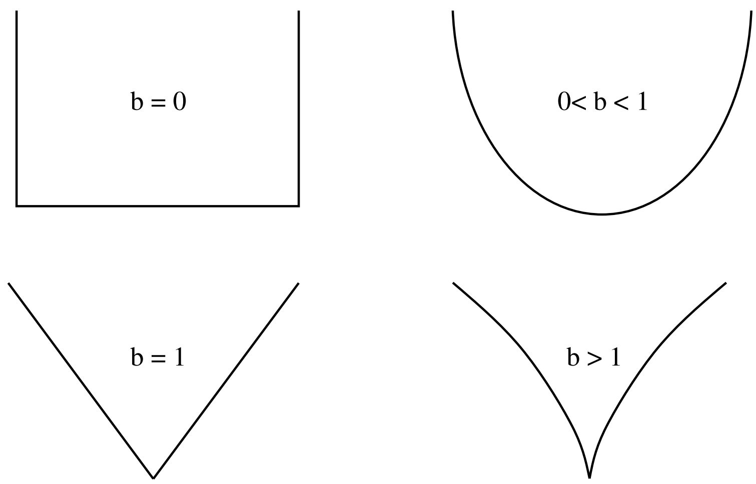

Figure 4: Theoretical cross-sections belonging to the power-law family.

However, natural channels and outlets are typically both asymmetrical and irregular. Due to this, the traditional interpretation can be quite misleading in some cases. An example of this, Fig. 5, shows a shallow and wide outlet from a Norwegian lake and the calculated depth-width relationship.

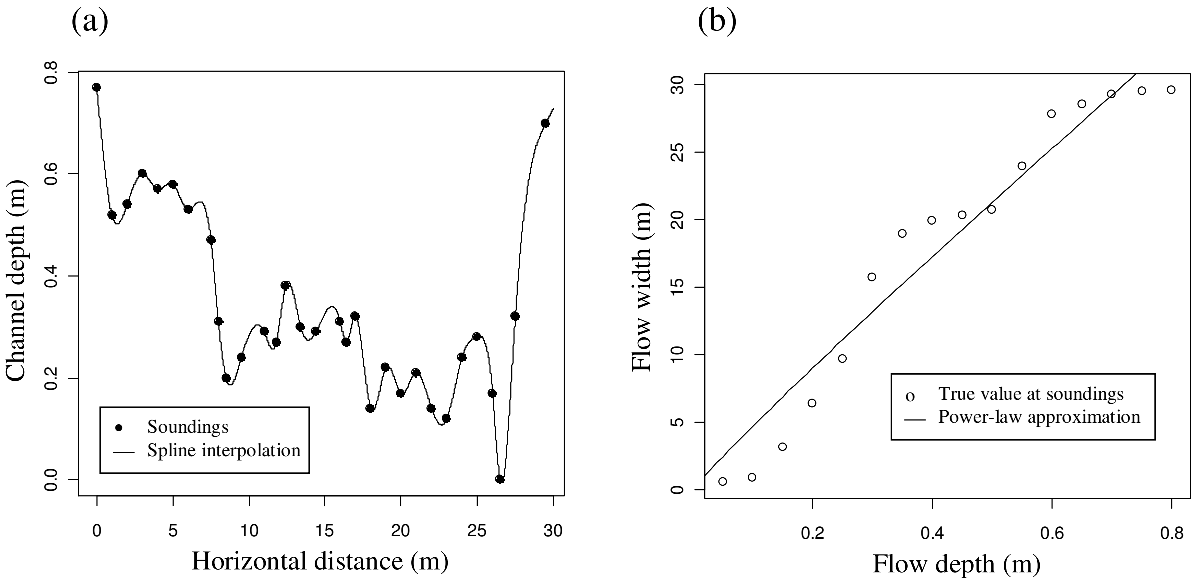

Figure 5: (a) The levelling soundings for the outlet of Byttevatn Lake, Norway, along with a natural spline representation. (b) The calculated depth-width relationship and the corresponding power-law approximation for the outlet of Byttevatn Lake, Norway.

The outlet of Byttevatn Lake has a fairly good power-law approximation, even though the wetted perimeter is very complex. In this case the shape parameter *b* was estimated to 0.94 which by the traditional interpretation should imply a parabolic cross-sectional shape. However, by looking at Fig. 5a one sees that this interpretation is questionable, as it is difficult to assign a main shape to the outlet. The same problem is encountered at Littles Ferry Bridge shown in Fig. 1a. It is unclear to the eye whether the cross-section is elliptical or triangular, although the corresponding *b* (0.84) states, by the traditional interpretation, that it is parabolic.

There is yet further potential for shape misclassification. It may be that the flow velocity in the reservoir is significant, so that ignoring velocity head in Eq. (10) can not be justified. In such situations the fitted model will be biased and may yield unrealistic parameter estimates which will have a reduced physical interpretation.

5.1.2 Steady uniform flow

Inferring cross-sectional shape from the rating curve parameter by means of the traditional interpretation can be very misleading in steady uniform flow situations. From Eq. (7) one sees that the exponent q is dependent on which friction law is used. If the Chézy friction law is used ($x = 1/2$), the traditional interpretation of channel shape versus rating curve exponent is applicable, given that the channel is wide and belongs to the group of shapes given in Fig. (6). However, as shown in section 4.1.1, even in these ideal situations the traditional interpretation may be flawed due to the complexity of natural channels. Moreover, if the true x is quite different from 1/2, for example unity which is widely accepted in open channel hydraulics [10], the traditional interpretation may be inaccurate.

A deep channel, markedly varied or unsteady flow, non-constant channel resistance are other factors that may weaken the validity of the traditional interpretation when steady uniform flow assumptions have been applied.

5.1.3 Compound width-depth relations

Some cross-sections are not amenable to a single valued power-law description. Strongly asymmetrical channels, e.g. -shaped, and compound channels, e.g. -shaped, belong to this class of cross-sections. In such cases the width-depth relationship must be represented by two or more power-law equations, which implies that the channel shape cannot be linked to a unique rating curve parameter.

5.2 Paving the way for Bayesian rating curve fitting: defining informative priors for the rating curve parameters

This section gives a brief coverage of how to utilise the hydraulic properties of the power-law rating curve in practical rating curve estimation. The objective here is solely to point out that results from classic hydraulics in combination with at-site characteristics may provide a basis for a particular informative Bayesian approach to rating curve fitting. The presented theory and methods are simple and are only intended to provide an insight into how to define hydraulics-grounded Bayesian priors. A comprehensive treatment of Bayesian rating curve fitting, which would require exploring elaborate statistical methods with many practical applications, is beyond the scope of this study.

Until now *h* is representing the effective depth of water on the channel control. Usually, the reference gauge is not zero when the effective gauge height is zero. Therefore, $\eta + c$, where -*c* is the unknown reference gauge height at zero flow and η the stage, or more precisely the water surface level in relation to the reference gauge, must usually replace *h* in the aforementioned equations. The parameter *c* can sometimes be ascertained in field surveys if the control reach is stable and well defined. However, as mentioned in the former section, most natural channels are not perfectly power-law, and fixing *c* might lessen the model's ability to emulate the main shape of the cross-section. In addition, the lowest point of a natural cross-section might not be easy to pinpoint with high accuracy, implying that a misclassification about *c* can easily be made. Hence the action of fixing *c* is not without limitations and when a fixed value is applied care should be taken.

Traditionally, rating curves have been fitted by the model

$$\log(Q_i) = \log(p) + q\log(\eta_i + c) + \varepsilon_i, \qquad \varepsilon_i \sim N(0, \sigma^2) \quad (15)$$

to measurement data using frequentistic regression methods [11, 12]. The main weakness of this approach is that it relies on the stage-discharge measurements to carry the sufficient information. This can be overcome by using Bayesian methods to solve the regression problem. Recently, Moyeed and Clarke [13] applied Bayesian methods for rating curve fitting. However, their study did not explore the full potential of a Bayesian approach as it neglected at-site characteristics and proper hydraulic theory to define the priors.

It is readily evident that Eq. (7), Eq. (12) and Eq. (14) provide possibilities to go beyond non-informative priors. Informative prior densities for *q* and log(*p*) can be based on either (a) a combination of known at-site characteristics and hydraulic theory; or (b) hydraulic theory exclusively. The first approach, which implies priors having smaller variability, is preferable if accurate at-site information is available, whereas the second approach yields a possibility for defining universally valid informative priors.

5.2.1 Steady uniform flow

A case study will be used to exemplify how informative priors using at-site information in combination with hydraulic theory can be defined in the case of steady uniform flow. The river Kautokeino at Masi gauging station, North Norway, drains an unregulated area of 5626 km^2. The undulating inland mountain plateau, known as Finnmarksvidda, accounts for most of the catchment. The flow in the channel section containing the station is assumed steady and uniform for all discharges. There are 35 stage-discharge measurements available at Masi. The measurements were made with a current meter or ADCP (Acoustic Doppler Current Profiler). The surface slope at Masi was determined by a field study to be about 0.003. Fig. 6 shows the relationship between the wetted perimeter and power-law approximated width-depth (a = 53.92 and b = 0.64).

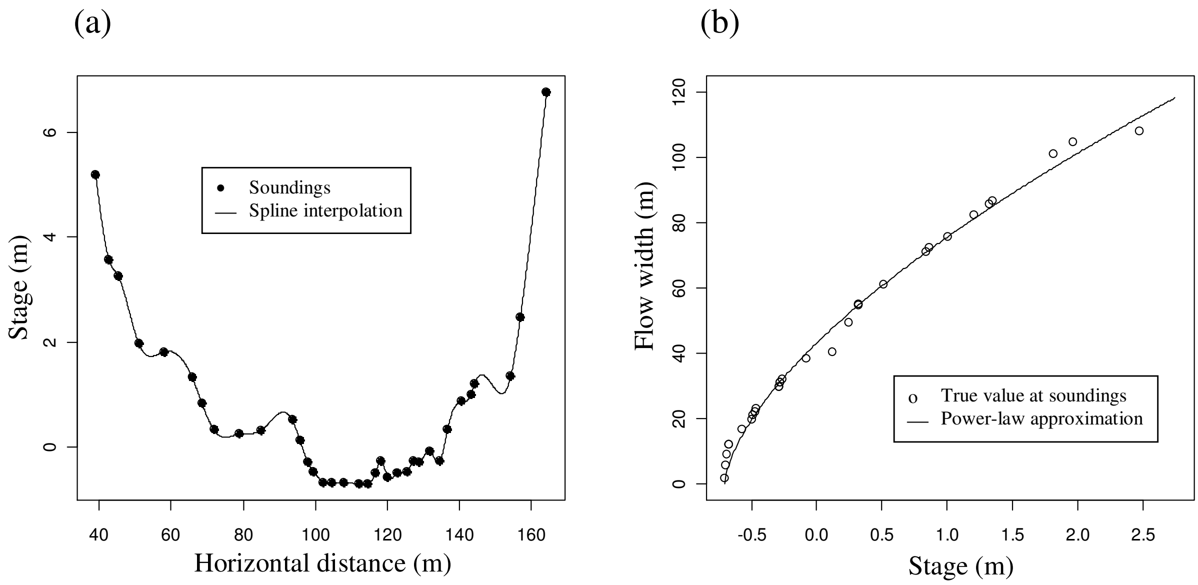

Figure 6: (a) The levelling soundings for the outlet of the Kautokeino River at Masi, Norway, along with a natural spline representation. (b) The calculated depth-width relationship and the corresponding power-law approximations for the Kautokeino River at Masi.

The friction law parameter x might be expected to vary between 1/2 and 2 [14], although experiments have yielded values as low as zero [15]. According to the Manning and Chézy formulae, which are the most widely used ones open-channel flow, M should be close to 2/3 and 1/2 respectively. Therefore x may be represented by a normal distribution with expectancy 0.67 and standard deviation 0.25. Chow [16] states that if the Manning friction law is used (x = 0.67), F should be between 7 and 40 in most natural streams. Hence taking into account that x may be different from 0.67, a log-normal distribution of expectancy 30 and standard deviation of 25 may be an appropriate prior for F.

The other model parameters also require priors in order to solve the regression problem using Bayesian methods. Experience at NVE (Norwegian Water and Energy Directorate) suggest the attainable accuracy of a discharge measurement is within 3 % of the true discharge, while poor measurements can have an uncertainty of about 15 %. Such values can be used to define a prior for the parameter σ (e.g. an inverse-chi-square distribution with expectancy 0.06 and variance 0.02). However, prior information may be less important for the measurement uncertainty since it is of less substantive interest than the rating curve parameters. Thus a non-informative prior may be preferred.

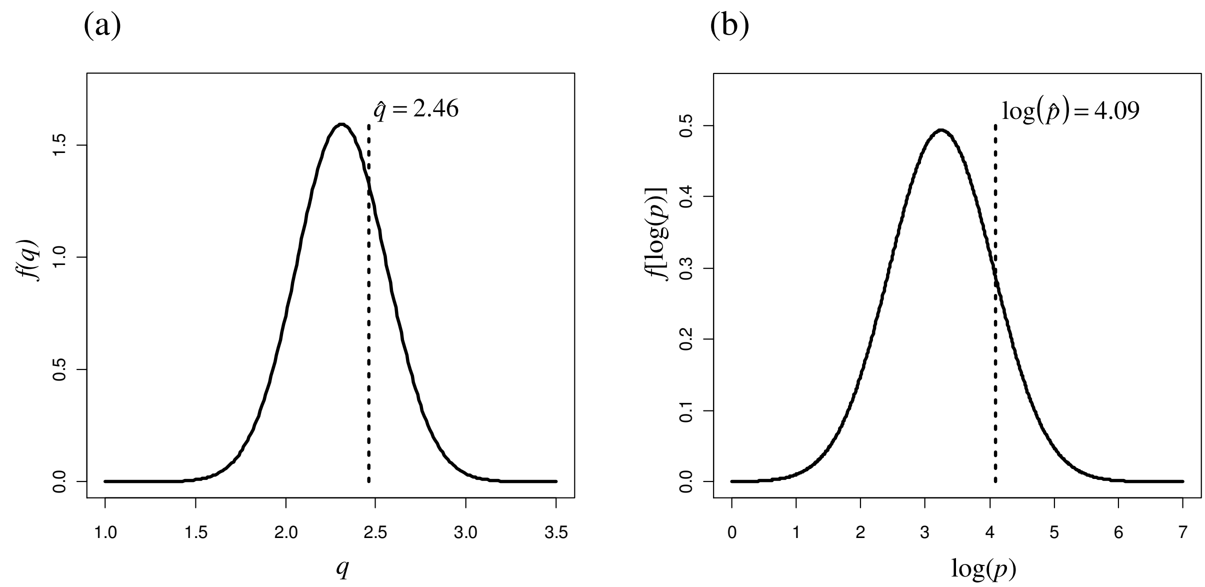

Figure 7: Informative prior density based on at-site characteristics and hydraulic theory for the rating curve exponent q (a) and the logarithm of the rating curve constant p (b) along with the frequentistic estimates for the Kautokeino River at Masi.

Theoretically, c is the negative point of zero flow. However, since most natural channels are seldom perfectly describable by a power-law, the parameter $-c$ should be interpreted as the approximate point of zero flow most representative for the actual cross-section. In any case, one would expect $-c$ to be in the vicinity of the lowest point on the cross-section. If the lowest point of the river is completely unknown, or the proposed model is applied in the upper segment of a compound rating curve where c has no clear physical meaning, the range covering plausible values of c should be wide, such that a prior for this parameter may be a distribution having support $\langle \infty, -\min(\eta_1, \cdots \eta_n) \rangle$.

Using the deterministically derived values of a, b and S_0 in combination with Eq. (7) and the hyperparameters of the aforementioned priors of x and F gives the marginal informative prior densities:

$$q \sim N\left(2.31, 0.25^2\right) \qquad (16a)$$

$$\log(p) \sim N\left(3.25, 0.81^2\right) \qquad (16b)$$

where

$$\mathrm{cov}(q, \log(p)) = -0.03 \leftrightarrow \mathrm{corr}(q, \log(p)) = -0.15 \qquad (16c)$$

The dependence between log(p) and q is due to the fact that they both are functions of the friction law parameter x. The densities are shown in Fig. 7.

The derived priors may also be used to assess the parameters estimated by frequentistic methods. Using non-linear least squares, the rating curve parameters at Masi were estimated to be: $\log(\hat{p})$ = 4.09 (1.63); $\hat{q}$ = 2.46 (1.38); and $\hat{c}$ = -0.14 (0.61), where the asymptotic standard deviations are given in the parentheses. From Fig. 7 one sees that these estimates seem reasonable according to the marginal prior densities. Furthermore, in Fig. 8 there is nothing to indicate that the frequentistic estimates do not provide good rating curve parameters.

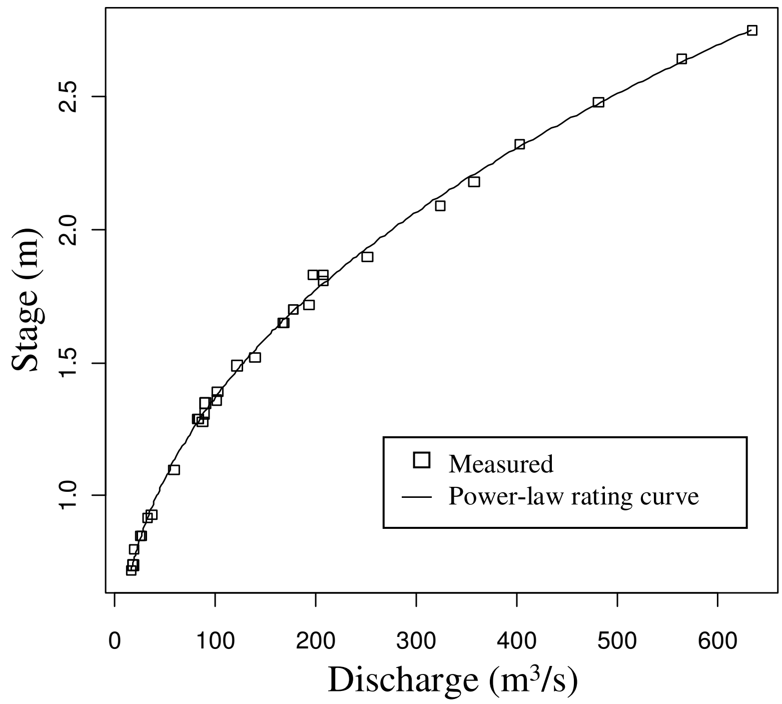

Figure 8: Results from frequentistic rating curve fitting for the Kautokeino River at Masi.

General informative priors that would remove the necessity for knowledge about at-site characteristics would be preferable to a computerised hydrometric database. Defining such priors for the parameters log(p) and q in the case of uniform flow, is fully possible by considering the geometric quantities a and b and the friction slope S_0 as statistical variables. It is readily evident that the two geometric parameters are correlated. Ideally one should therefore take into account the dependence when defining priors, though it is doubtful whether such a rigorous approach would yield more accurate estimates. A simple approach is to first consider the plausible range of a bank-full width of a river, given that the channel is rectangular. Most river channels are between 1 and, say, 300 metres. Hence, an “ad-hoc” prior density of a may be given a log-normal distribution with expectancy 50 and standard deviation 140, respectively. Furthermore, assuming that the shapes of the channels and outlets belong to the class given in Fig. 4, a normal distribution of expectancy and standard deviation of 0.5 and 0.25, respectively, may provide a good prior for b. Friction slopes outside the interval <0.01, 0.0001> are seldom encountered in hydraulic studies where uniform flow is assumed, therefore a log-normal distribution with expectancy 0.002 and standard deviation 0.0035 could be a defined as a plausible prior for this parameter. Using Eq. (7) and the aforementioned priors gives, by applying straightforward normal theory, one obtains the following universal priors for uniform flow situations:

$$q \sim N\left(2.17, 0.35^2\right) \quad (17a)$$

$$\log(p) \sim N\left(1.88, 1.75^2\right) \quad (17b)$$

where

$$\text{cov}(q, \log(p)) = -0.1 \leftrightarrow \text{corr}(q, \log(p)) = -0.16 \quad (17c)$$

The densities presented above are displayed in Fig. 9. Note how the absence of at-site information inflates the variance of the priors.

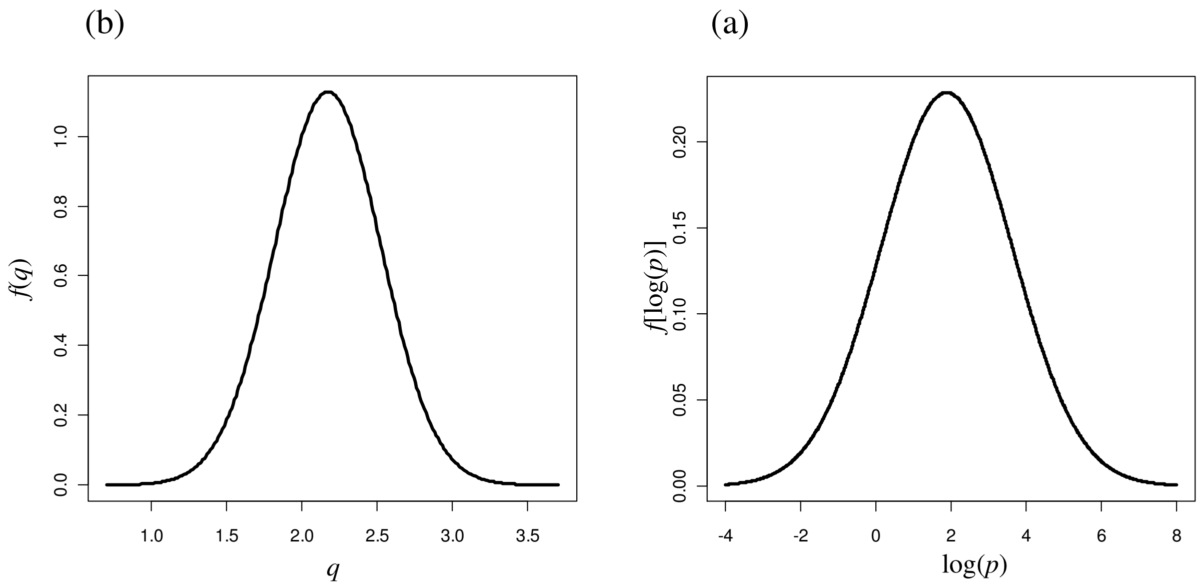

Figure 9: Universal informative prior density based on hydraulic theory for the rating curve exponent q (a) and the logarithm of the rating curve constant p (b) for steady uniform flow situations.

5.2.2 Critical flow from a reservoir

According to Chow [9] the typical value of the energy coefficient α in natural streams is between 1.15 and 1.50. Chow [9] also states that upstream weirs values of α greater than 2 have been observed, and that a value of $\alpha = 3.87$ was observed at the outlet section of a draft tube. Based on these figures one may assume that a log-normal distribution of expectancy and standard deviation of 1.3 and 0.15, respectively, is a plausible prior for this quantity in natural outlets. The value of cos(θ) can be taken as unity for most natural outlet channels. The coefficient of entrance loss k has, according to Chow [5], an average value of 0.25 for well-rounded outlets. It could then be plausible to assume that $0 \le k \le 0.5$ is a typical range of this quantity. Applying these values, one can assume that a log-normal distribution of expectancy and standard deviation of 0.25 and 0.1, respectively, is a reasonable prior density for this parameter in conjunction with natural outlets. The priors for b and a can be taken as in section 4.2.1.

The prior for q can be derived analytically. On the other hand, Eq. (12) does not provides a simple analytical expression for the marginal prior density of log(p). However, sampling from the above-mentioned priors of a, b, α and k one can obtain the informative priors for the log(p). One can see from Fig. 10b that the resulting density can be adequately fitted to a normal distribution. Thus, one obtains the universal priors in the case of critical flow from a reservoir:

$$q \sim N\left(2, 0.13^2\right) \qquad \text{(18a)}$$

$$\log(p) \sim N\left(2.70, 1.49^2\right) \qquad \text{(18b)}$$

where

$$\operatorname{cov}(q, \log(p)) = -0.03 \leftrightarrow \operatorname{corr}(q, \log(p)) = -0.16 \qquad \text{(18c)}$$

These priors have smaller variance than those defined for uniform flow situations. Consequently, should it be desirable to use priors designed to cover all flow situations, the prior densities corresponding to uniform flow should be used for high coverage certainty.

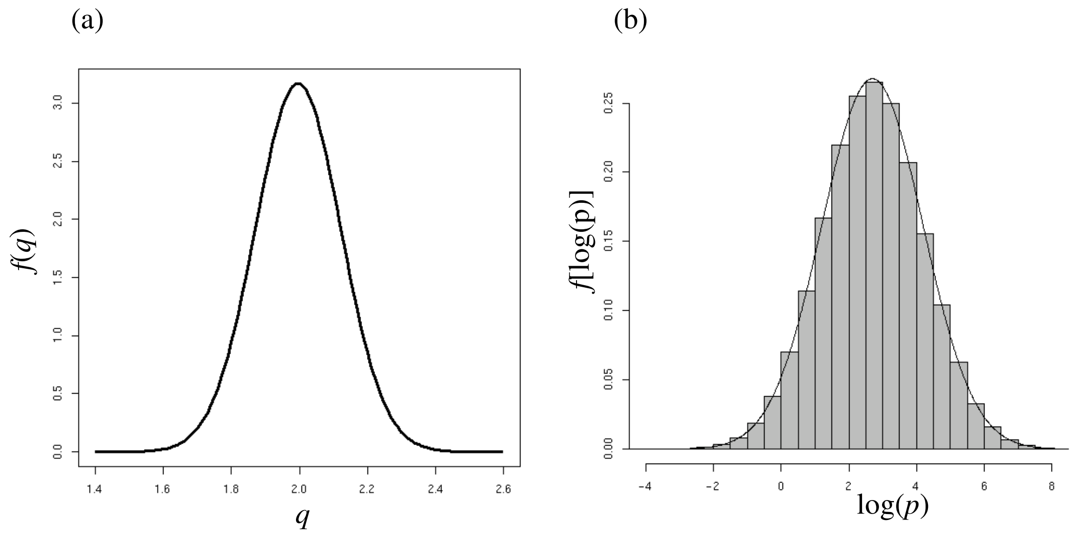

Figure 10: Universal informative prior density based on hydraulic theory for the rating curve exponent q (a) and the logarithm of the rating curve constant p (b) for critical flow from a reservoir.

The derivation of the priors may seem rather casual. However, as stated by Gelman et al. [16], although the prior distributions should include all plausible values of the rating curve parameters, the priors need not be realistically concentrated around the true values, because often the information about the parameters contained in the data will far outweigh any reasonable prior probability specifications.

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