

Reanalysing a glacier mass balance measurement series – Ålfotbreen 2010-2019

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NVE Rapport nr. 6/2022

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Published by: Norwegian Water Resources and Energy Directorate
Author: Bjarne Kjøllmoen
Cover photo: Ålfotbreen in September 2019. Photo: Bjarne Kjøllmoen

ISBN: 978-82-410-2186-2
ISSN: 1501-2832
Case number: 200700140

Print: Norwegian Water Resources and Energy Directorate
Number printed: 20

Abstract: Reanalysis of mass balance measurements at Ålfotbreen and Hansebreen 2011-2019 including homogenization of both glaciological and geodetic observation series, uncertainty assessment and comparison of the glaciological and geodetic mass balance series.

Key words: Glaciological mass balance, Geodetic mass balance, Homogenization

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March, 2022

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Preface

This report documents the results from reanalysis of mass balance measurements at Ålfotbreen and Hansebreen. The time series is based on traditional glaciological observations using stakes and probings, as well as geodetic observations using laser scanning (LIDAR) and Digital Terrain Models (DTM).

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Summary

The glaciological and geodetic methods provide independent observations of glacier mass balance. The glaciological method is based on annual surface mass balance measurements, whereas the geodetic method includes surface measurements, and estimates of internal and basal mass balance over a period of years.

The glaciological mass balance series for Ålfotbreen and Hansebreen cover the periods from 1963 to 2020 and 1986 to 2020, respectively. In this report, a re-analysed time series for both glaciers over the period 2011-2019 is presented. Within this period, usable Digital Terrain Models (DTMs) from 2010 and 2019 were generated. The re-analysis includes homogenization of both glaciological and geodetic observation series, uncertainty assessment and comparison of the glaciological and geodetic mass balance.

The period of data set (2011-2019) was compared and the results did not show significant discrepancies between the glaciological and geodetic methods for the period 2011-2019. The mean annual difference was $0.10 \text{ m w.e. a}^{-1}$ for Ålfotbreen and $0.05 \text{ m w.e. a}^{-1}$ for Hansebreen. A hypothesis in Zemp et al. (2013) was tested and revealed that calibration was not required for any of the glaciers.

In the original report from 2010 the mapping company reported the aerial survey date to 2nd September. Later it has been shown that the correct date was 29th September. This change of date influenced the evaluation of the accuracy, the geodetic mass balance calculations, the comparison between geodetic and glaciological mass balance and hence, whether a calibration was necessary or not.

The consequence of the corrected date for the 2010 mapping was that calibration for Hansebreen was not necessary and the calibration for Ålfotbreen was slightly changed compared with the reanalysis in Kjøllmoen (2016). The re-calibrated glaciological cumulative mass balance for Ålfotbreen over 1998-2010 were -12.40 m w.e. The corresponding calibration in Kjøllmoen (2016) gave a cumulative mass balance of -12.85 m w.e.

1 Introduction

1.1 Background

The Norwegian Water Resources and Energy Directorate (NVE) operate the Norwegian mass balance observation programme. The observations are both traditional field measurements, referred to as the “glaciological method” (also called direct, conventional or traditional method) and geodetic surveys, referred to as the “geodetic method” (Cogley et al., 2011). This report describes reanalysis of the Ålfotbreen and Hansebreen mass balance time series 2011-2019. The mass balance time series 1963-2010 (Ålfotbreen) and 1986-2010 (Hansebreen) was reanalysed in Kjøllmoen (2016).

The glaciological mass balance method measures surface mass balance at point locations, and data are extrapolated over the entire glacier surface to obtain glacier-wide averages. The cumulative mass balance is the sum of the annual balances. In the geodetic method, cumulative balance is calculated from glacier surface elevations measured in different years by differencing Digital Terrain Models (DTMs) and by converting the volume change to mass change using a density conversion. The geodetic method is often used as a check on the accuracy of annual measurements by the glaciological method (e.g. Andreassen, 1999 and Zemp, 2010). If a comparison between the glaciological and the geodetic method of a time series show great discrepancies, a calibration of the glaciological mass balance series is required.

1.2 Ålfotbreen

Ålfotbreen ice cap (61°45'N, 5°40'E) has an area of 9.8 km² (Andreassen, 2022) and is, together with Blåbreen, the westernmost and the most maritime glacier in Norway (Fig. 1), except for some smaller ice patches to the west (Andreassen and Winsvold, 2012).

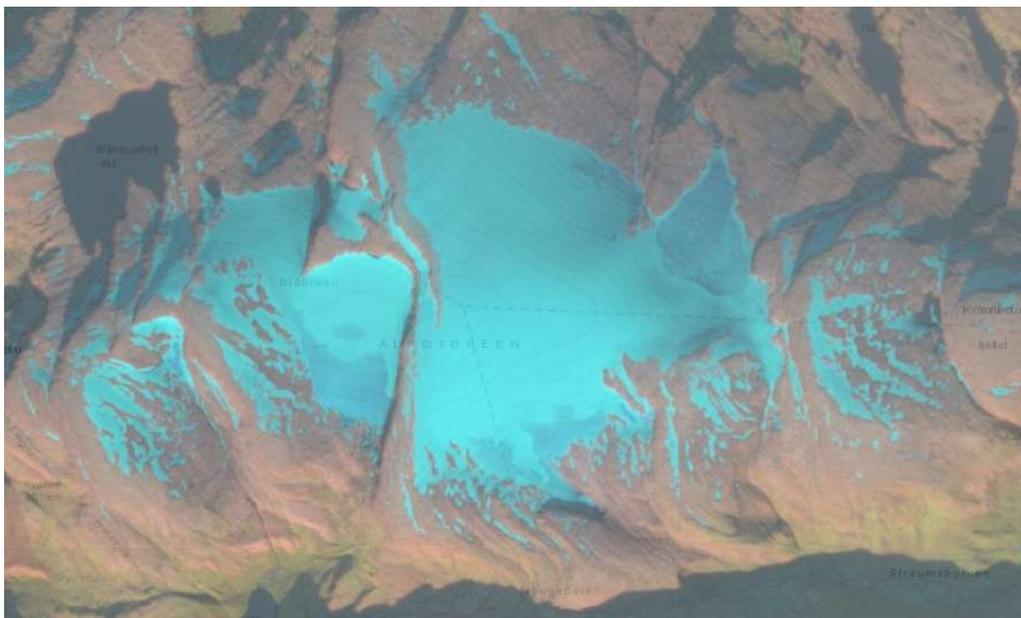


Figure 1
The ice cap Ålfotbreen photographed on 15th October 2020. Blåbreen to the left is separated from Ålfotbreen.
Source: Sentinel-2.

Ålfotbreen is a small ice cap resting on sandstones that have some characteristic staircase-like formations in the landscape (Fig. 2). The ice cap can be divided in three outlets, the two north-facing glaciers Ålfotbreen and Hansebreen, and the nameless south-facing outlet (Fig. 1). Ålfotbreen extends from 1360 down to 1000 m a.s.l., and Hansebreen from 1303 down to 927 m a.s.l. Both glaciers have a smooth and sloping surface with some crevasses.

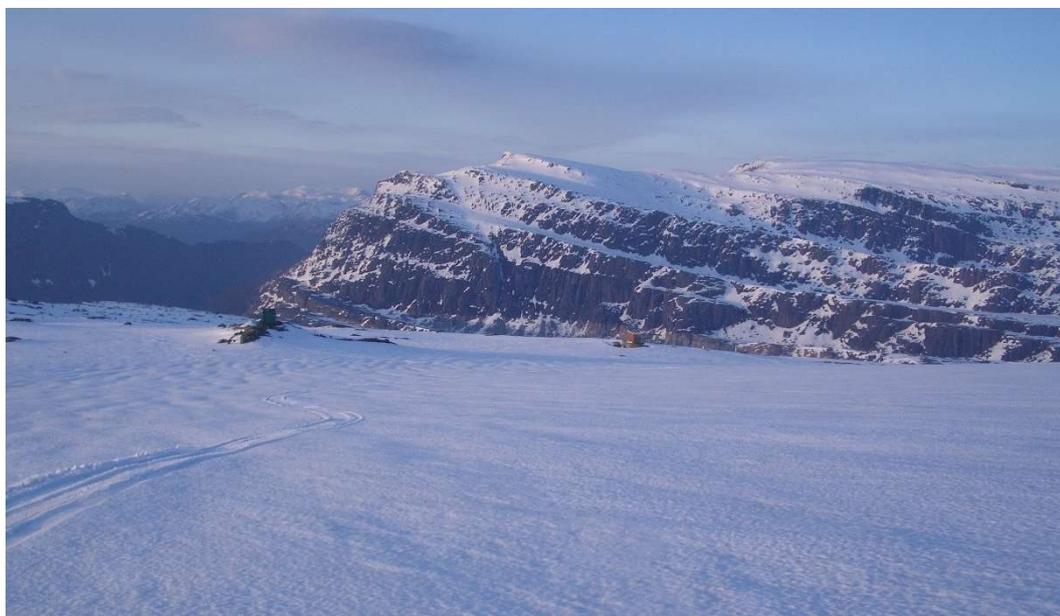


Figure 2
View from Ålfotbreen with the characteristic staircase-like sandstone formations surrounding the ice cap.
Photo: Laila Høivik.

1.3 Previous results

The two adjacent glaciers Ålfotbreen and Hansebreen have been subject for annual glaciological mass balance measurements since 1963 (Østrem and Liestøl, 1964) and 1986 (Laumann et al., 1988), respectively. The measurements at Ålfotbreen and Hansebreen are funded by Sogn og Fjordane Energi AS. The results for Ålfotbreen show a slight surplus from 1962 to 1988, a large surplus from 1988 to 1995 and a distinct deficit from 1995 to 2019. Hansebreen was in balance from 1986 to 2001. From 2001 to 2019 however, the measurements show a significant deficit.

Ålfotbreen and Hansebreen has been surveyed by aerial photography several years since 1945. Detailed glacier maps have been constructed from photographs taken in 1968, 1988 and 1997, and by laser scanning (LIDAR) in 2010 and 2019. Detailed glacier maps have been constructed from all these mappings.

Glaciological and geodetic mass balance for the periods 1968-1988 (only Ålfotbreen), 1988-1997 and 1997-2010 was compared in Kjøllmoen (2016). The discrepancies found between glaciological and geodetic balance for Ålfotbreen were significant for the period 1997-2010, but not significant for the two first periods (1968-1988 and 1988-1997). The discrepancies for Hansebreen were significant for both periods (1988-1997 and 1997-2010). Thus, the period 1997-2010 for Ålfotbreen and the periods 1988-1997 and 1997-2010 for Hansebreen were calibrated.

In this report the geodetic mass balance was calculated from LIDAR in 2010 and 2019. Thus, the glaciological and geodetic mass balances were compared for the 9-year period 2011-2019.

1.4 Outlook

The mass balance measurements at Ålfotbreen and Hansebreen was reanalysed following the reanalyses scheme proposed by Zemp et al. (2013). The major steps were:

1. Analysis and scrutiny of glaciological and geodetic measurements (ch. 2)
2. Homogenization of glaciological and geodetic measurements (ch. 3)
3. Uncertainty assessment (ch. 4)
4. Validation of glaciological measurements against geodetic measurements (ch. 4)

The output of the reanalysis is a *homogenized glaciological mass balance time series* with an uncertainty assessment, and if calibration is required, a *calibrated glaciological mass balance time series*.

A re-analysis for Ålfotbreen and Hansebreen over the period 1998-2010 was reported in Kjøllmoen (2016). Later it has been shown that the aerial survey date for 2010 was corrected from 2nd September to 29th September. The consequences of this correction are described in chapter 5.

2 Observations

2.1 Geodetic mass balance

Geodetic mass balance for the periods 1968-1988, 1988-1997 and 1997-2010 was reported in Kjølmoen (2016).

LIDAR from 29th September 2010 and 22nd September 2019 were used to produce detailed DTMs of the glacier surface of Ålfotbreen and Hansebreen.

The GIS-data processing of maps and DTMs by NVE was done using ArcGIS 9.3/10.2 software (©ESRI) and Surfer software version 15.

2.1.1 Mapping 2010

Vertical aerial photographs and LIDAR data were recorded simultaneously on 29th September 2010 by Terratec AS (Terratec AS, 2021).

The photographs were recorded by a Rollei metric ATC modular digital camera with a resolution of 60 megapixels. The mean flying height was 3000 m above ground level and the picture resolution was 25 cm GSD (Ground Sampling Distance). The resulting resolution of the orthophoto was given as 20 cm.

The LIDAR data was acquired using a Leica ALS50-II lidar instrument. The laser pulse rate was 81.100 Hz and the scan angle ± 20 degrees, resulting in a mean point density of 0.5 points per m². The expected accuracy of the LIDAR data was given as 10-20 cm by the mapping company.

The data delivery from Terratec was point clouds (las), glacier outlines and orthophoto (Fig. 3).

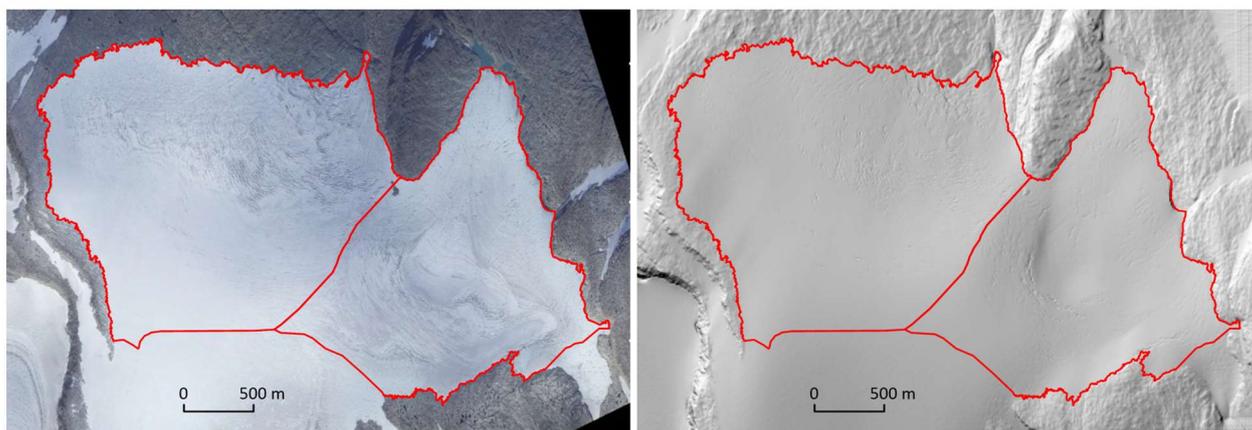


Figure 3
Orthophoto produced of aerial images from 29th September 2010 to the left and shaded relief based on the DTM 2010 to the right. The glacier boundary for Ålfotbreen and Hansebreen 2010 in red.

The gridding method used for generating a regular grid data set (10x10m) was “Kriging”.

The glacier outlines supplied from Terratec AS were too detailed, and hence, based on the orthophoto it was modified by NVE for the further calculations. The ice divide determined from the laser DTM₂₀₁₀ was used in the following calculations.

All data was referred to the UTM co-ordinate system zone 32, Euref 89 datum and the Norwegian height system NN1954.

2.1.2 Mapping 2019

LIDAR data were recorded on 22nd September 2019 by Terratec AS (Terratec AS, 2020) as a part of the national laser scanning program initiated by the Norwegian Mapping Authority.

The LIDAR data was acquired using a Riegl VQ1560i – L735 lidar instrument. The flying height was about 2500 m above ground level. The laser pulse rate was 350.000 Hz and the scan angle ± 40 degrees, resulting in a mean point density of 2.0 points per m².

The LIDAR data set was compared with measured control points in stable areas. The control revealed a systematic bias of 0.11 m. Thus the 2019 LIDAR data set was corrected. The control and calibration were done by Terratec AS. A control of the homogeneity did not reveal any systematic bias.

The data set produced by Terratec AS was point clouds (laz) (Fig. 4).

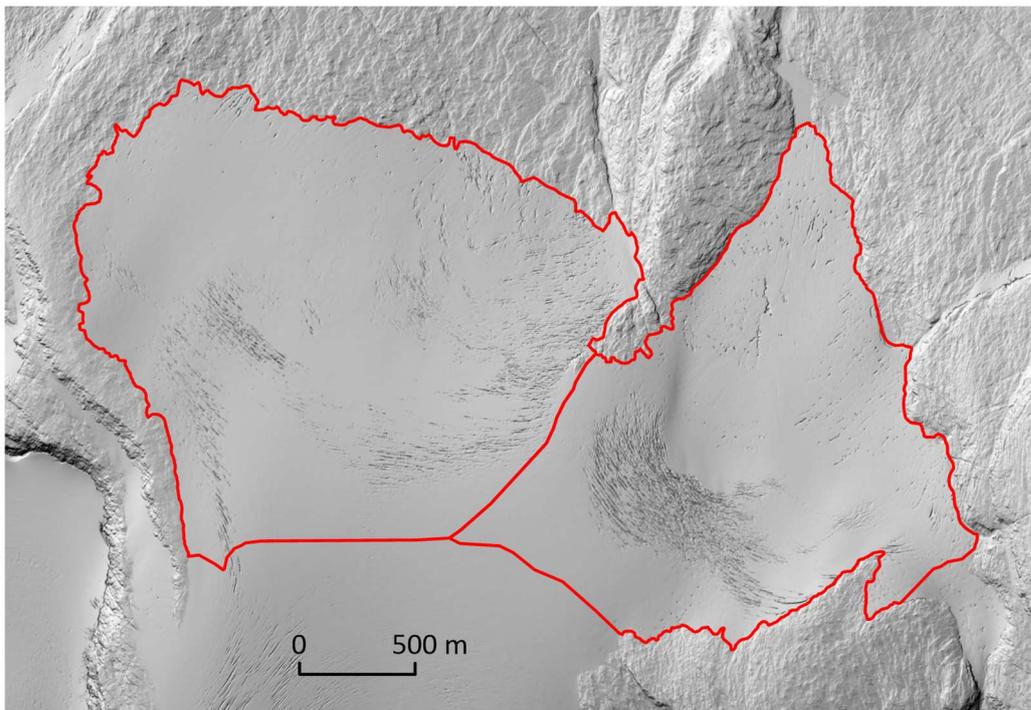


Figure 4
Shaded relief map based on the DTM 2019. The glacier boundaries for 2019 in red.

The gridding method used for converting point cloud to regular grid data set (10x10 m) was “Kriging”. The 10x10 m regular grid data set was used in the following calculations.

Neither orthophoto nor optical satellite imagery covering Ålfotbreen in 2019 were available. Thus, the glacier outlines were digitised using the shaded relief of the DTM₂₀₁₉ (Fig. 4). The ice divide determined from the laser DTM₂₀₁₀ was used in the following calculations.

All data was referred to the UTM co-ordinate system zone 32, Euref 89 datum and the Norwegian height system NN2000.

The height system NN2000 is some different from the NN1954 system and this difference is discussed in chapter 3.1.

2.1.3 Density

Determination of a density conversion factor was required in order to convert the volume change of snow, firn and ice to mass change. It is common to assume a constant density profile in the accumulation area, following Sorge's law (Bader, 1954). Hence, density of glacier ice, 900-917 kg m⁻³ (Cuffey and Paterson, 2010), is often used for the conversion (e.g. Haug et al., 2009 and Andreassen, 1999). This assumption, however, is valid only under steady-state conditions and was considered to be a maximum estimate in this study. Assuming a value of 850 ±60 kg m⁻³ to convert volume change to mass change is found to be appropriate for a wide range of conditions (Huss, 2013). Hence, this value was used for the conversion of the volumetric changes into water equivalent.

2.1.4 Adjustment for different dates

Comparison of glaciological and geodetic mass balance required an adjustment because the field measurements and aerial surveys were acquired at different dates. The related difference depends on the changes in mass balance between the field and aerial surveys. Accordingly, increasing time span will result in increasing difference. The season (summer/ autumn) and the general mass turn over will also influence the difference. Dates for field measurements and aerial surveys and corresponding adjustments are shown in table 1.

Table 1
Survey dates and adjustments for 2010 and 2019.

year	date			correction (m w.e.)		
	LIDAR	field survey _{summer}	field survey _{autumn}	category	ΔB _s Ålf.	ΔB _s Han.
2010	29 th September	12 th August	28 th September		0.00	0.00
2019	22 nd September	27 th August	25 th September	fresh snow	-0.02	-0.02

In 2010 the lidar data was acquired 29th September, and the ablation was measured on 28th September. In this context possible melting from 28th to 29th September is assumed to be insignificant. At the time of measurements, the fresh snow (0-10 cm) was estimated as <0.01 m w.e. Thus, adjustment due to different dates for 2010 was zero.

In 2019, the lidar survey date was 22nd September, and the ablation was measured on 25th September. At the time of ablation measurement 10 cm of solid fresh snow was measured at three stakes above 1230 m a.s.l. Below 1200 m elevation no fresh snow was observed. Data from nearby climate stations and study of satellite images indicate that the fresh snow had come between 16th and 19th September. Satellite images from 21st and 26th September show that some of the fresh snow melted over these five days (Fig. 5). Fresh snow at the lidar survey date (22nd September) and melting between the lidar survey date and the ablation measurement date (25th September) could not be measured directly. Hence, the correction due to fresh snow at the time of the lidar survey was assumed as -0.02 m w.e.

According to the estimated melting from the lidar survey dates to the field survey dates, the geodetic mass balances were adjusted as:

$$- \Delta B_{s,2010} + \Delta B_{s,2019}$$



Figure 5
Satellite images covering Ålfotbreen from 21st (left) and 26th September (right) 2019. Both images show some fresh snow and the image from 26th September show that some of the fresh snow has melted.

2.1.5 Glacier basin

The hydrological and the glaciological basins were considered to be identical for both glaciers. The ice divide from 2010 and 2019 was quite similar and thus the ice divide from 2010 was used for both DTMs. The glacier basin areas are 3.97 km² (2010) and 3.49 km² (2019) for Ålfotbreen and 2.75 km² (2010) and 2.49 km² (2019) for Hansebreen. For the geodetic volume change calculations, a combination of the glacier boundaries was used so that the analyses mask will surround both glacier areas. Areas within the glacier basin defined as rock in both years were excluded.

2.2 Glaciological mass balance

Glacier surface mass balance at Ålfotbreen has been monitored annually since 1963 (Østrem and Liestøl, 1964). The adjacent glacier in east, Hansebreen, has been measured since 1986 (Laumann et al., 1988). The measurements have been carried out by NVE. The extent of measurements has varied over time, but the method of calculation has been homogenized for the whole period (Kjøllmoen, 2016). The measurements and calculations are in principle based on methods from Østrem and Brugman (1991) and as described in Andreassen et al. (2005) and Kjøllmoen et al. (2021).

The annual results are reported in “Glaciological investigations in Norway”, which are annual reports published by NVE. Reanalysed mass balance series for Ålfotbreen 1963-2010 and Hansebreen 1986-2010 was reported in Kjøllmoen (2016).

2.2.1 Monitoring program and field measurements

Normally, winter balance measurements were carried out between medio April and early May, while the annual balance measurements were carried out in October. Winter balance was measured using a number of stakes, as well as doing a number of snow depth soundings to the late-summer surface from previous year. In addition to snow depth, snow density was measured in one vertical profile usually centrally located on Ålfotbreen. The snow density measurements were done at the same time as the snow depth measurements. Annual balance was measured by stake readings.

A detailed description of the field measurements from 1963 to 2010 was given in Kjøllmoen (2016).

For the years 2011-2019 the number of stakes and snow depth measurements was quite stable. A network of 4-6 stakes on Ålfotbreen and 5-7 stakes on Hansebreen was maintained. The number of snow depth measurements varied between 75 and 88 on Ålfotbreen and between 55 and 58 on Hansebreen (Fig. 6 and Tab. 2). The snow density was determined in one location over this nine-year period.

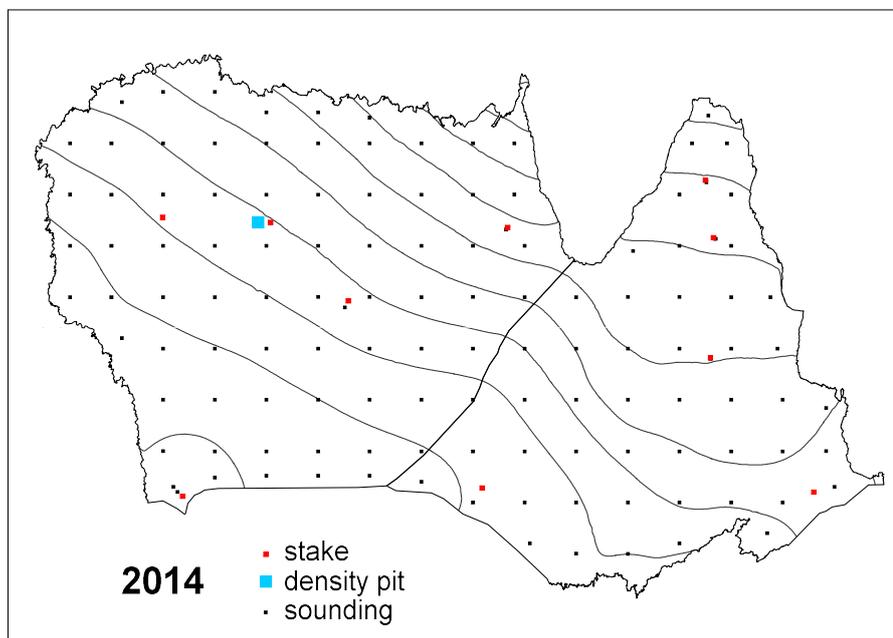


Figure 6
Typical stake network and snow depth soundings from 2014 representing the period 2011-2019 at Ålfotbreen and Hansebreen.

Table 2
A summary of the annual mass balance measurements at Ålfotbreen and Hansebreen over the period 2011-2019.

Year	Date		Stakes (number)			Density pit			Snow depth measurements				Data quality
	spring	autumn	b_w	b_s	b_a	position	depth (m)	ρ ($kg\ m^{-3}$)	number	\bar{x} (m)	min. (m)	max. (m)	
Ålfotbreen													
2011	27 th Apr.	13 th Oct.	5	6	6	37	5.54	558	88	6.2	4.0	7.9	Medium
2012	16 th Apr.	16 th Oct.	3	5	6	37	6.90	521	80	7.7	4.7	9.2	Medium
2013	23 rd May	25 th Sep.	4	4	5	37	4.90	510	78	5.9	2.6	7.9	Medium
2014	24 th Apr.	15 th Oct.	3	5	6	37	6.25	504	81	7.4	5.1	9.3	Medium
2015	21 st Apr.	16 th Oct.	3	4	5	28	6.24	501	80	8.4	6.2	10.3	Medium
2016	10 th May	5 th Oct.	5	6	7	28	5.95	533	78	7.7	4.9	9.0	Medium
2017	10 th May	19 th Oct.	6	6	7	28	6.40	533	83	6.1	3.1	8.2	Medium
2018	15 th May	11 th Oct.	6	6	7	28	5.44	572	84	5.0	2.5	6.4	Good
2019	20 th May	25 th Sep.	6	5	5	28	3.92	565	75	4.3	2.4	5.9	Medium
Hansebreen													
2011	27 th Apr.	13 th Oct.	4	5	5				56	6.0	4.2	8.2	Good
2012	16 th Apr.	16 th Oct.	3	5	5				57	7.0	5.0	9.9	Good
2013	23 rd May	25 th Sep.	5	5	5				56	5.5	3.4	8.2	Medium
2014	24 th Apr.	15 th Oct.	4	5	5				57	7.2	5.3	10.0	Medium
2015	21 st Apr.	16 th Oct.	4	5	5				58	8.1	6.2	10.0	Medium
2016	10 th May	5 th Oct.	5	5	6				55	7.1	4.6	10.0	Medium
2017	10 th May	19 th Oct.	7	7	7				58	6.4	4.5	10.0	Medium
2018	15 th May	11 th Oct.	4	7	7				55	4.6	3.2	5.9	Good
2019	20 th May	25 th Sep.	6	6	6				55	3.6	2.1	7.7	Good

2.2.2 Mass balance calculations

The mass balance was in principle calculated using a stratigraphic system, i.e. between two successive summer surfaces, as described in Cogley et al. (2011). The spatial interpolation of point measurements was done by estimating winter and summer balance in elevation intervals of 50 m vertical resolution. The altitudinal mass balance curves were made by plotting point measurements of winter, summer and annual balance versus altitude. Representative values for each 50-m elevation interval were then extracted from these scatter plots (Fig. 7). The method is called the profile method. The entire glacier area was well represented with measurements.

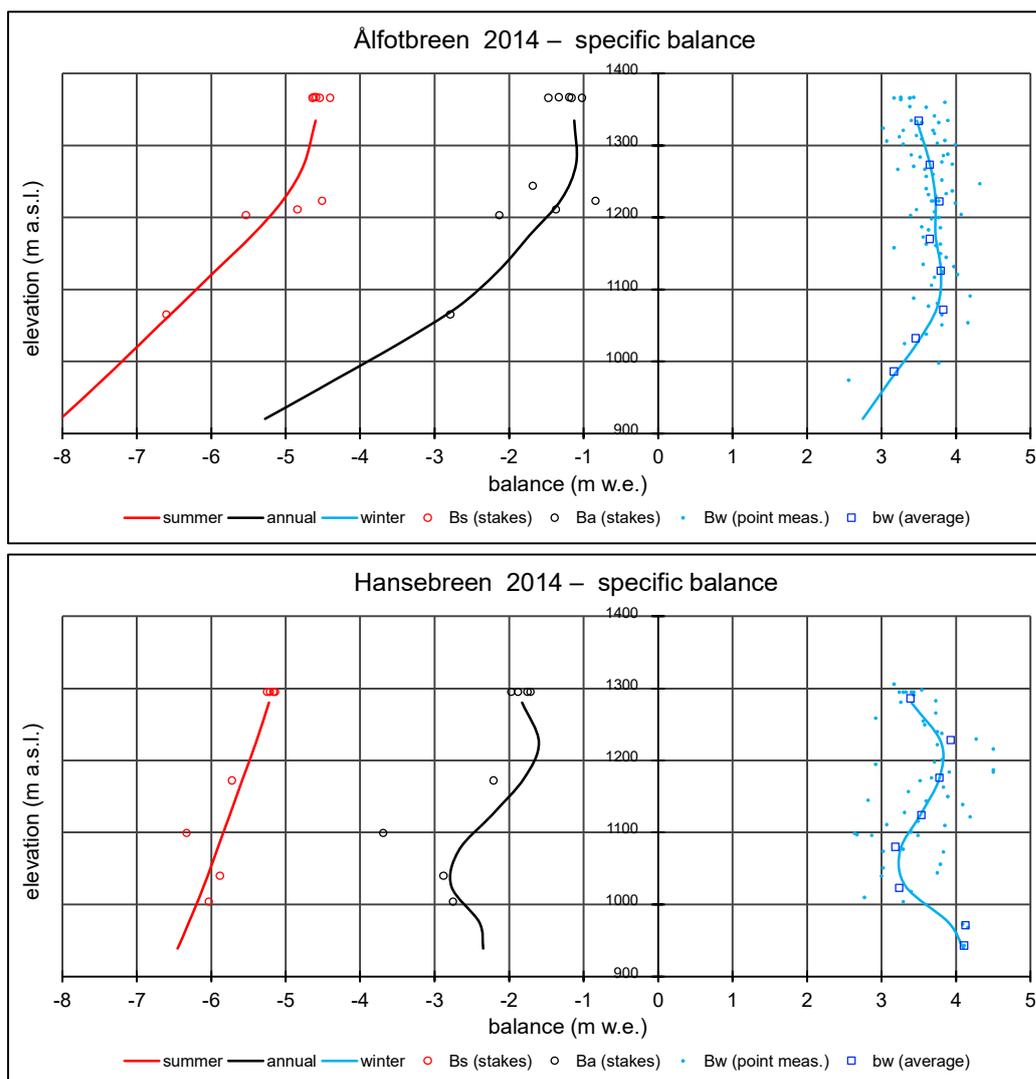


Figure 7
The altitudinal winter, summer and annual balance curves are plotted versus altitude. Point values for b_w (\square), b_s (\circ) and b_a (\circ), together with average b_w (\square) for each 50 m height interval are also plotted. This calculation method has been used for the whole period 1963–2019. The example diagrams above are from Ålfotbreen (upper) and Hansebreen (lower) in 2014.

2.2.3 Glacier boundaries

For both Ålfotbreen and Hansebreen the hydrological and glaciological basins were considered to be identical. Hence, the issue of which drainage basin used was ignored. The drainage divides, or rather the ice divides, were solely calculated from the mapped

glacier surface topography. The ice divide was calculated for both DTMs from 2010 and 2019. A comparison of the two ice divides showed only marginal differences, and hence, the ice divides from 2010 were continued in the further work. The glacier outlines from DTM₂₀₁₉ and ice divides from DTM₂₀₁₀ are shown in figure 4.

In the reported datasets from 2010 to 2019 (e.g. Kjølmoen et al., 2020), the mass balance calculations were based on the height-area distribution from the DTM₂₀₁₀.

2.2.4 Glaciological mass balance series

Ålfotbreen

The reanalysed (1963-2010) and original (2011-2019) glaciological mass balance series for Ålfotbreen gives a deficit of -7.7 m w.e. for the whole period 1963-2019. The results show a mass surplus from 1983 to 1995 ($+12.2$ m w.e.) and a mass loss from 1996 to 2019 (-21.5 m w.e.). The period 1963-1982 were nearly in balance ($+1.5$ m w.e.).

The mean winter, summer and annual mass balances for 1963-2019 were 3.58, -3.72 and -0.14 m w.e., respectively. The reanalysed (1963-2010) and original (2011-2019) annual winter, summer and annual mass balance results from 1963 to 2019 are shown in figure 8.

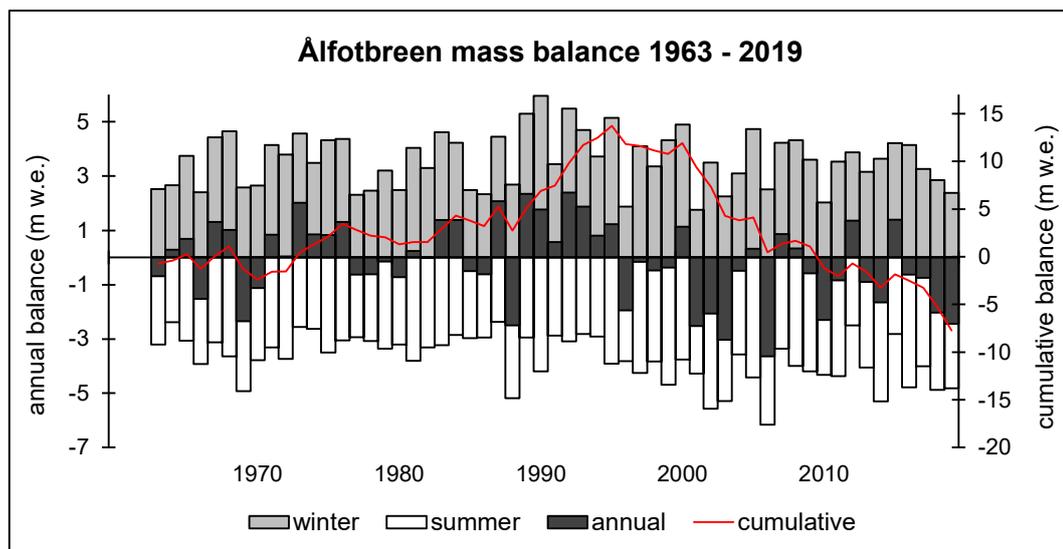


Figure 8
Reanalysed (1963-2010) and original (2011-2019) winter, summer and annual mass balance for Ålfotbreen over the period 1963-2019.

Hansebreen

The reanalysed (1986-2010) and original (2011-2019) glaciological mass balance series for Hansebreen gives a deficit of -25.0 m w.e. for the whole period 1986-2019. The results show a mass surplus from 1989 to 1995 ($+6.7$ m w.e.) and a distinct mass loss from 2001 to 2019 (-28.1 m w.e.). The period 1996-2000 were nearly in balance (-1.1 m w.e.).

The mean winter, summer and annual mass balances for 1986-2019 were 3.37, -4.10 and -0.74 m w.e., respectively. The reanalysed (1986-2010) and original (2011-2019) annual winter, summer and annual mass balance results from 1986 to 2019 are shown in figure 9.

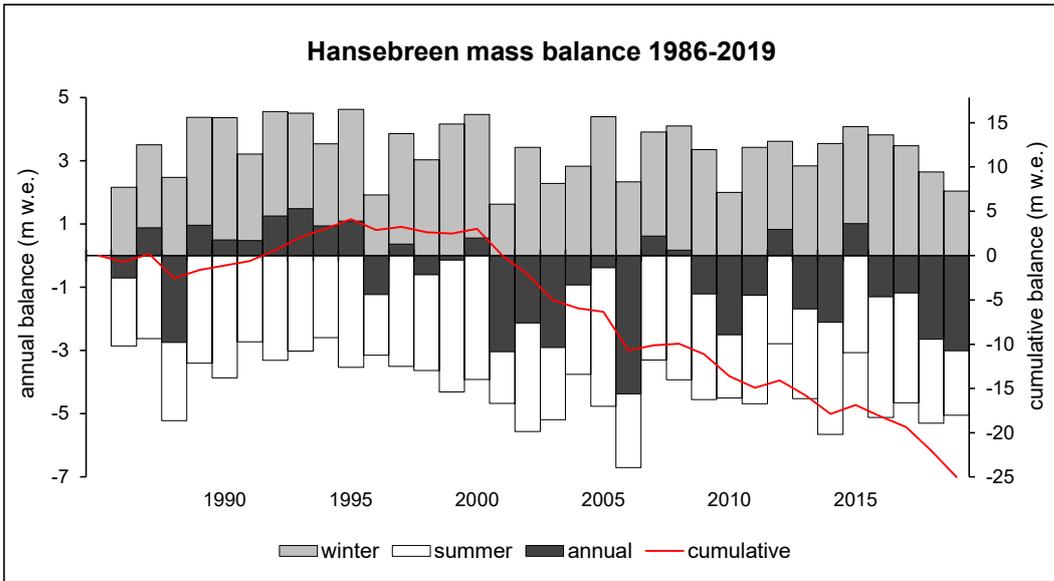


Figure 9
Reanalysed (1986-2010) and original (2011-2019) winter, summer and annual mass balance for Hansebreen over the period 1986-2019.

3 Homogenization

3.1 Geodetic mass balance

The accuracy of the final DTMs is principally influenced by the quality of the raw data and by the process from raw data to DTM. The raw data acquisition and the DTM processing were similar for 2010 and 2019.

The LIDAR data set from 2019 was referred to the Norwegian height system NN2000, while the data set from 2010 was referred to NN1954. The height difference between the two reference systems varies from -15 to + 35 cm, depending on where in Norway you are. Surveys from Ålfotbreen revealed height differences between the two systems between 4 and 5 cm. Hence, in order to ensure two comparable data sets the DTM from 2010 was converted to NN2000.

3.1.1 Mapping 2010

The 2010 DTM was based on data acquired by LIDAR (see chap. 2.1.1). Generally, the accuracy of data sets acquired by LIDAR is high and was estimated to be 10-20 cm (Terratec, 2021). The accuracy of the 2010 LIDAR data was documented by comparing the original LIDAR data set with nine control points measured with GNSS on the glacier surface. As the x and y co-ordinates of the LIDAR data set versus GNSS points/fixed points were not exact equal, interpolated values from a 0.5x0.5 m grid were extracted from the LIDAR data set.

GNSS on the glacier surface

The control points on the glacier surface were measured on 12th August, while the LIDAR data was acquired on 29th September. Based on stake readings on 12th August and 28th September, and air temperature from two climate stations, the elevation change for the control points were estimated for the period from 12th August to 29th September. The impact of a potential vertical ice motion was not considered. The results from the comparison are shown in table 3 and figure 10. The accuracy of the measured control points was assumed as ±0.1 m. The differences (Diff. = Height_{adj.} - Height_{DTM}) were between -0.36 and +0.38 m with an average of +0.01 m.

Table 3
Comparison of the glacier surface elevation between control points measured with GNSS and interpolated values from a 0.5 x 0.5 m grid extracted from the original LIDAR data set. The surface elevations measured on 12th August (Height_{GNSS}) were adjusted to elevations related to 29th September (Height_{adj.}).

Point No.	North	East	Height _{GNSS}	Height _{adj.}	Height _{DTM}	Diff. (m)
12-09	6 851 868.72	323 891.63	953.76	951.86	952.00	-0.14
13-05	6 851 302.80	324 053.01	1 064.57	1 062.82	1 063.18	-0.36
13-10	6 851 323.76	323 908.57	1 071.21	1 069.56	1 069.18	0.38
50-08	6 851 576.66	324 884.16	1 004.66	1 002.91	1 003.04	-0.13
37-10	6 851 359.32	322 769.37	1 205.78	1 204.13	1 203.82	0.31
80-03	6 850 706.44	324 911.14	1 100.60	1 098.80	1 099.00	-0.20
60-09	6 851 265.03	324 915.33	1 044.72	1 043.12	1 043.00	0.12
28-02	6 850 975.75	323 143.89	1 220.55	1 219.20	1 219.13	0.07

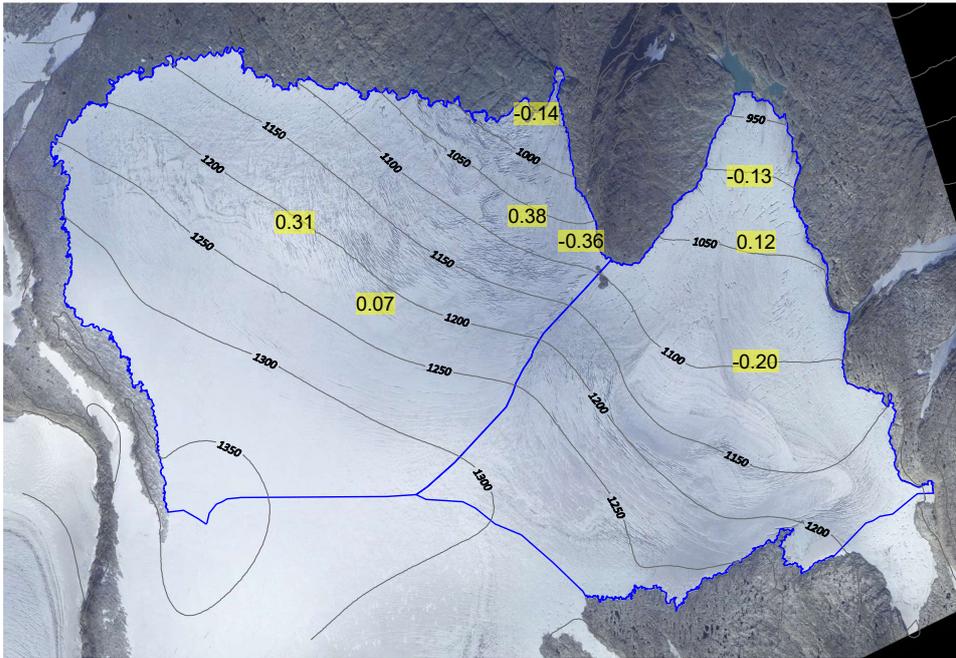


Figure 10
Spatial distribution of the eight control points measured on the glacier surface on 12th August.

The evaluation based on the GNSS measurements on the glacier surface revealed differences within ± 0.4 m. Due to the time lag between the GNSS measurements on 12th August and the LIDAR acquisition on 29th September, the estimated surface elevation change in the intermediate period is an uncertain factor. The evaluation concluded that the quality of the LIDAR data was good, and correction of the 2010 DTM was not necessary.

3.1.2 Mapping 2019

As the 2010 DTM, the 2019 DTM was also based on data acquired by LIDAR.

The accuracy of the 2019 LIDAR data was documented by comparing the original LIDAR data set with control points measured with GNSS on the glacier surface. The 2019 LIDAR data was also compared with the 2010 LIDAR data in stable non-glacierized areas.

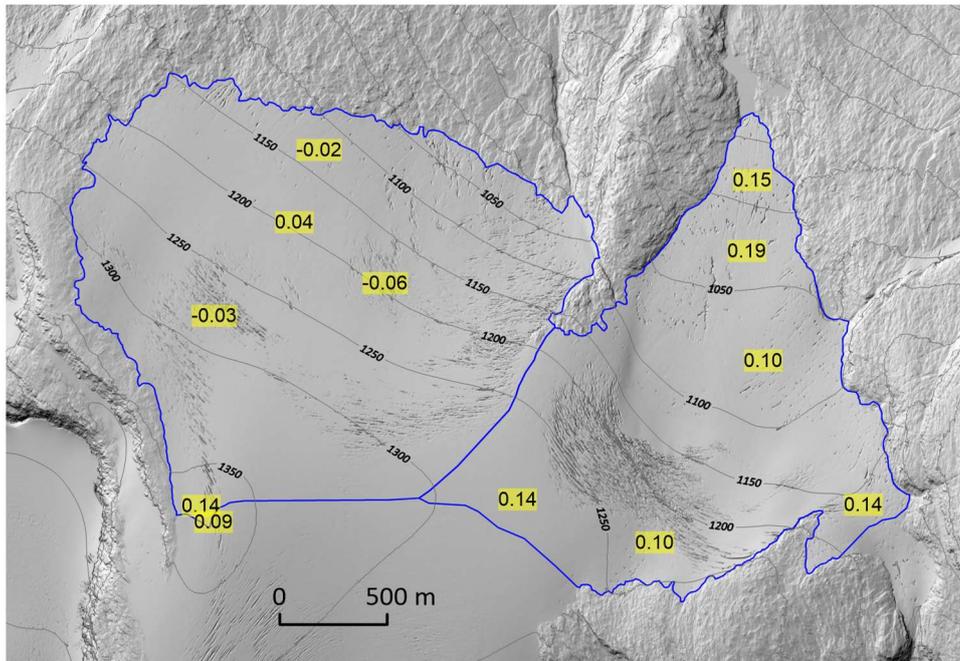
GNSS on the glacier surface

Twelve control points on the glacier were measured on 25th September, while the LIDAR data was acquired three days earlier, on 22nd September. Due to the short time lag (three days) between the LIDAR survey and the GNSS measurements, the impact of potential elevation changes and vertical ice motion was considered insignificant. The results from the comparison are shown in table 4 and figure 11. The height accuracy of the measured control points was assumed as ± 0.2 m. The differences ($\text{Diff.} = \text{Height}_{\text{GNSS}} - \text{Height}_{\text{DTM}}$) were between -0.06 and $+0.19$ m with an average of $+0.08$ m.

Table 4

Comparison of the glacier surface elevation between control points measured with GNSS and interpolated values from 0.2 a 0.2 m grid extracted from the original LIDAR data set.

Point No.	North	East	Height _{GNSS}	Height _{DTM}	Diff. (m)
49-19	6 850 044.49	322 319.08	1 358.43	1 358.29	0.14
49-07	6 849 968.75	322 376.67	1 360.20	1 360.11	0.09
88-18	6 849 872.79	324 430.70	1 226.53	1 226.43	0.10
85-18	6 850 050.01	325 402.42	1 155.57	1 155.43	0.14
80-19	6 850 724.00	324 936.16	1 079.07	1 078.97	0.10
60-18	6 851 236.81	324 859.73	1 028.67	1 028.48	0.19
50-19	6 851 568.25	324 885.21	983.30	983.15	0.15
30-19	6 850 933.94	322 379.71	1 280.35	1 280.38	-0.03
28-19	6 851 086.62	323 175.49	1 192.78	1 192.84	-0.06
37-18b	6 851 371.62	322 752.81	1 194.47	1 194.43	0.04
15-18	6 851 709.95	322 868.47	1 128.66	1 128.68	-0.02
90-03	6 850 074.35	323 792.60	1 286.50	1 286.35	0.14

**Figure 11**

Spatial distribution of twelve control points measured on the glacier surface on 25th September.

3.1.3 LIDAR 2019 vs LIDAR 2010

The 2019 LIDAR data was compared with the 2010 LIDAR data in non-glacierized areas. Ideally the non-glacierized terrain from two DTMs should correspond exactly. Due to all the inaccuracies, however, elevation differences will always occur when comparing two DTMs. Comparing elevation values in steep areas is considered to be very uncertain and should preferably be avoided. Thus, all areas steeper than 30° were removed. Accordingly, the results from 1 399 146 grid points (2x2 m) showed differences from +2.0 to -3.9 m with an average of -0.30 m. The standard deviation was 0.20 m. Generally, the results indicated that the 2019 DTM was 0.3 m above the 2010 DTM in non-glacierized areas (Fig. 12).

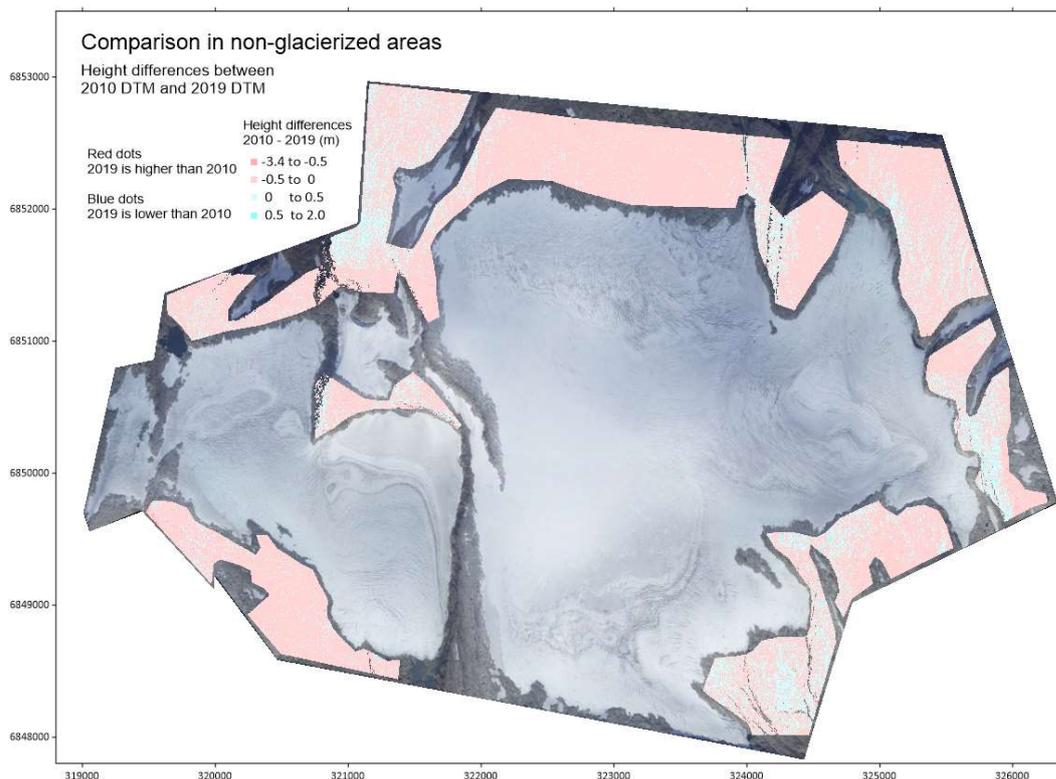


Figure 12
Aerial distribution of elevation differences in non-glacierized areas by comparing the 2019 DTM with the 2010 DTM. Thus, red dots indicate that the 2019 DTM is higher than the 2010 DTM and vice versa. Values in areas steeper than 30° were removed.

The maximum (+2.0 m) and minimum (-3.9 m) differences are rather great. The average difference is -0.30 m and 87 % of the control points had differences between 0 and -0.5 m. The reasons for the highest differences can be that the terrain is not stable, and reflection errors from sloping areas. Factors like material (gravel) from landslides and remaining snow will influence the surface elevation.

The reports from the mapping company (Terratec AS, 2021 and Terratec AS, 2020) do not indicate any systematic error in the two data sets. The average difference of -0.30 m and consistent differences between -0.5 and 0 however, suggest a significant height difference between the two DTMs. Hence the 2010 DTM was considered as the reference DTM and kept unchanged, while the 2019 DTM was lowered 0.30 m.

3.1.4 Mass change 2011-2019

The spatial distribution of thickness changes at Ålfotbreen and Hansebreen between 29th September 2010 and 22nd September 2019 is shown in figure 13. The geodetic mass balance over the period 2011-2019 was calculated within the hydrological basin using grid size of 10 x 10 m. The volume change was multiplied with the density conversion factor (850 kg m⁻³), divided with the mean area for 2010 and 2019, and adjusted for additional melting in 2010 and 2019. The results are given in table 5.

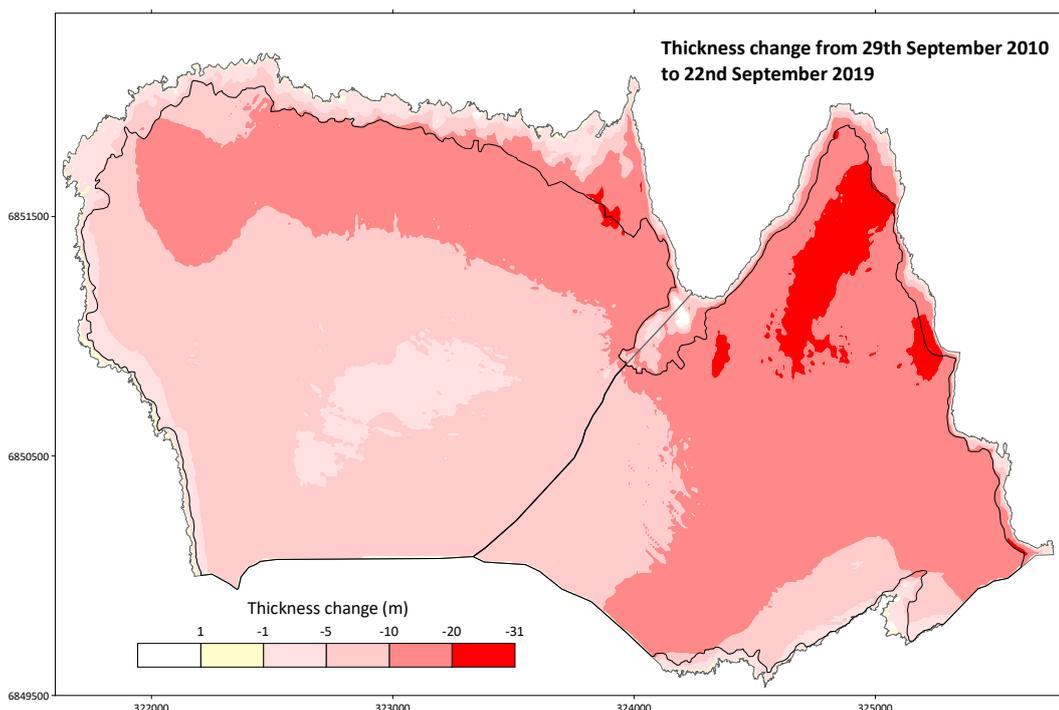


Figure 13
DTM differences within the hydrological basin of Ålfotbreen and Hansebreen from 29th September 2010 to 22nd September 2019. The glacier extents from 2010 (grey line) and 2019 (black line) are also shown.

Ice thickness change between 29th September 2010 and 22nd September 2019 varied from +1 to –31 meters (Fig. 13). Mean thickness change was –8.16 m for Ålfotbreen and –13.88 m for Hansebreen. Thus, geodetic mass balance over 2011-2019 was –7.45 m w.e. for Ålfotbreen and –12.48 m for Hansebreen.

Table 5
Volume change and geodetic mass balance for Hansebreen and Ålfotbreen between 29th September 2010 and 22nd September 2019.

period	area ₂₀₁₀	area ₂₀₁₉	vol. ch.	dens. fac.	date adj. (m w.e.)		geod. mb. (m w.e.)	
glacier	(km ²)	(km ²)	(mill. m ³)	(kg m ⁻³)	2010	2019	acc.	ann.
2011-2019								
Hansebreen	2.75	2.48	-38	850	0.00	-0.02	-12.48	-1.39
Ålfotbreen	3.98	3.48	-33	850	0.00	-0.02	-7.45	-0.83

area₂₀₁₀ is the glacier areas in September 2010

area₂₀₁₉ is the glacier areas in September 2019

vol. ch. is the volume change of ice, firm and snow over the given period

dens. fac. is the density used for converting from ice, firm and snow to water equivalent

date adj. is a correction for different dates for mapping and field survey for each year in the period

geod. mb. is accumulated (acc.) and annual (ann.) balance for the period

3.2 Glaciological mass balance

The methodology of the surface mass balance calculations was changed through the years from the beginning in 1963. Thus, a homogenization of the series 1963-2010 was implemented in Kjöllmoen (2016). Four major factors were considered and homogenized, 1) from contour-line method to profile method, 2) height-area distribution, 3) converting from snow depth to water equivalent and 4) ice-divide.

For the mass balance series 2011-2019 three of the factors (1, 3, and 4) were homogeneous with the period 1963-2010. As the new DTM from 2019 was available however, a homogenization of the series based on factor 2) height-area distribution, was required.

3.2.1 Height-area distribution

The original reported mass balance calculations 2011-2019 were based on height-area distribution from the DTM 2010. A period between two mappings is usually divided in two, where each map is applied to half of the period before the mapping year and half of the period after the mapping year (Fig. 14). Accordingly, the homogenization involved re-calculation of the reported years 2015, 2016, 2017, 2018 and 2019.

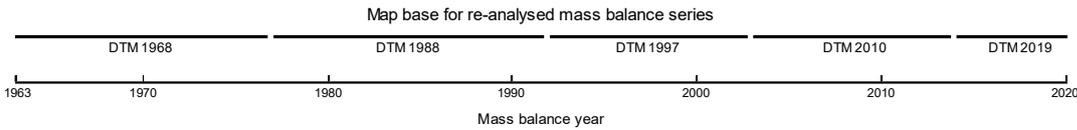


Figure 14
Upper line indicates map base for homogeneous mass balance series. Years denote year of validity period for each map.

3.2.2 Results

Homogenizing by re-calculation of the mass balance series from 2015 to 2019 ensure a uniform methodology. The re-calculation was based on the DTM 2019.

Original and homogenized mass balance values for the five years 2015, 2016, 2017, 2018 and 2019 are shown in table 6.

Table 6
Original and homogenized mass balance values for Ålfotbreen and Hansebreen for the years 2015, 2016, 2017, 2018 and 2019.

Year	Original mass balance series								Homogenized mass balance series							
	B _w	B _s	B _a	ΣB _a	ELA	AAR	DTM	Area	B _w	B _s	B _a	ΣB _a	ELA	AAR	DTM	Area
Ålfotbreen																
2015	4.21	-2.81	1.40	1.40	1020	96	2010	3.98	4.22	-2.75	1.47	1.47	<1000	100	2019	3.48
2016	4.15	-4.79	-0.64	0.76	>1368	0	2010	3.98	4.19	-4.70	-0.51	0.96	1320	14	2019	3.48
2017	3.26	-4.01	-0.75	0.01	1305	21	2010	3.98	3.26	-3.93	-0.66	0.30	1330	10	2019	3.48
2018	2.84	-4.88	-2.04	-2.02	>1368	0	2010	3.98	2.88	-4.82	-1.94	-1.64	>1360	0	2019	3.48
2019	2.38	-4.82	-2.44	-4.46	>1368	0	2010	3.98	2.46	-4.78	-2.32	-3.96	>1360	0	2019	3.48
Hansebreen																
2015	4.08	-3.07	1.01	1.01	<927	100	2010	2.75	4.05	-3.05	1.01	1.01	<927	100	2019	2.48
2016	3.82	-5.12	-1.30	-0.30	>1310	0	2010	2.75	3.80	-5.11	-1.31	-0.31	>1303	0	2019	2.48
2017	3.48	-4.66	-1.18	-1.48	>1310	0	2010	2.75	3.45	-4.64	-1.19	-1.50	>1303	0	2019	2.48
2018	2.65	-5.30	-2.65	-4.13	>1310	0	2010	2.75	2.63	-5.31	-2.68	-4.18	>1303	0	2019	2.48
2019	2.04	-5.05	-3.01	-7.14	>1310	0	2010	2.75	2.04	-5.03	-2.99	-7.17	>1303	0	2019	2.48

The differences between original and homogenized mass balance values for the five years were rather small for both glaciers. The homogenized winter balance values were slightly more positive than the original series for Ålfotbreen and slightly less positive for Hansebreen. The homogenized summer balance values were slightly less negative than the original series for both glaciers. The mean winter balance change was +0.033 m w.e. a⁻¹ for Ålfotbreen and -0.017 m w.e. a⁻¹ for Hansebreen. The mean summer balance change was -0.067 m w.e. a⁻¹ for Ålfotbreen and -0.012 m w.e. a⁻¹ for Hansebreen.

Accordingly, the cumulative annual balance for the five years 2015-2019 was changed from -4.46 to -3.96 m w.e. for Ålfotbreen and from -7.14 to -7.17 m w.e. for Hansebreen.

The homogenized mass balance series for Ålfotbreen over the period 1963-2019 shows a deficit of -7.22 m w.e., which gives a mean annual balance of -0.13 m w.e. a^{-1} . Over the 20-year period 2000-2019 however, the mean annual balance was -0.90 m w.e. a^{-1} . The homogenized series for Hansebreen over the period 1986-2019 shows a deficit of -25.05 m w.e. which gives a mean annual balance of -0.74 m w.e. a^{-1} . Over the 20-year period 2000-2019 however, the mean annual balance was -1.38 m w.e. a^{-1} .

4 Comparison

4.1 Comparison of glaciological and geodetic mass balances

Glaciological and geodetic mass balance for Ålfotbreen and Hansebreen are compared for the period 2011-2019 (autumn 2010 to autumn 2019). Glaciological mass balance is based on annual measurements of snow depth and snow density at the end of the winter, and of ablation measurements at the end of the summer. Geodetic mass balance is based on changes in elevation and area between two mappings.

In order to compare glaciological and geodetic mass balance, the errors for the different methods and the internal balance were estimated. Internal balance has been estimated earlier using the methods described in Oerlemans (2013) and Alexander et al. (2011) for ten glaciers in Norway (Andreassen et al., 2016). For this purpose, internal balance is expressed as melting inside and underneath the glacier due to heat of dissipation. Melting due to rain was considered negligible, as most of this melting affects snow, firn and ice on the surface, rather than the subglacial system.

Internal balance (B_{int}) was calculated for each elevation interval (100 meter) used in the surface mass balance by the formula

$$B_{int} = \frac{\sum_h g * ph * ah * (h - bL)}{A * Lm}$$

where g is the acceleration of gravity, h is mean elevation of elevation interval used in surface mass balance calculations, ph is precipitation at h , ah is glacier area of elevation interval h , bL is bed elevation at glacier snout, A is total glacier area og Lm is latent heat of fusion.

Precipitation was defined as a linear function of elevation. Daily precipitation was extracted from www.senorge.no, and the gradient was selected to give an annual precipitation 1.5 times the measured winter balance.

The internal balance was quantified as -0.06 m w.e. a^{-1} for Ålfotbreen and -0.04 m w.e. a^{-1} for Hansebreen (Andreassen et al., 2016). The uncertainty, $\sigma.B_{int}$, was assumed to be one third of the estimated internal melting, which amounts to ± 0.02 m w.e. a^{-1} for Ålfotbreen and ± 0.01 m w.e. a^{-1} for Hansebreen.

In order to compare, the uncertainty of the measurements was estimated in accordance with Zemp et al. (2013) and Andreassen et al. (2016).

The results from glaciological, geodetic and internal mass balance are shown in table 7.

The results show a difference between glaciological and geodetic mass balance (Δ) over the period 2011-2019 as 0.10 m w.e. a^{-1} for Ålfotbreen and 0.08 m w.e. a^{-1} for Hansebreen.

Table 7

Comparison of glaciological and geodetic mass balances and results of the uncertainty analysis for Ålfotbreen and Hansebreen over the period 2011-2019. All mass balances and errors are in m w.e. a⁻¹.

glacier	years	B glac.	σ.glac. point	σ.glac. spatial	σ.glac. ref	B geod.	σ.geod. DTM	σ.dc	B int	σ.B. int	Δ
Ålfotbreen	9	-0.67	0.26	0.19	0.05	-0.83	0.05	0.05	-0.06	0.02	0.10
Hansebreen	9	-1.27	0.26	0.19	0.05	-1.39	0.05	0.08	-0.04	0.01	0.08

B glac. is mean annual glaciological mass balance

σ.glac. point is random error for each point value in the glaciological mass balance

σ.glac. spatial is spatial random error in the glaciological mass balance

σ.glac.ref is random error as a consequence of glacier area changes over time

B geod. is mean annual geodetic mass balance

σ.geod. DTM is random error for the DTMs

σ.dc is random error for the density conversion

B int is internal melting

σ.B. int is random error for the internal melting

Δ is the difference between glaciological and geodetic balance, corrected for internal melting

In order to check whether the annual discrepancy between glaciological and geodetic mass balance is significant different or not, a hypothesis where the uncertainties are taken into account, is tested (Zemp et al., 2013). If the answer of this hypothesis is «no», it is recommended to calibrate the glaciological mass balance series. If the answer is «yes», it means that the glaciological balance is not significant different from the geodetic balance. By checking this hypothesis for Ålfotbreen and Hansebreen, the answer was «yes», which suggest the geodetic and glaciological series are not significant different (Tab. 8). Hence, calibration of the series 2011-2019 was not required.

Table 8

Comparison and check of glaciological and geodetic mass balance including the uncertainties.

glacier	Δ	σ	H0	β	ε
Ålfotbreen	0.10	0.80	yes	87	0.47
Hansebreen	0.08	0.55	yes	91	0.52

Δ is the discrepancy (m w.e. a⁻¹) between glaciological and geodetic balance adjusted for internal melting

σ (dimensionless) is the reduced discrepancy, where uncertainties are accounted

H0 is the hypothesis whether the glaciological balance = the geodetic balance

β is the probability of accepting **H0** although the results of both methods are different at the 95 % confidence level

ε (m w.e. a⁻¹) is the limit for detection of bias

5 Reanalysing 1998-2010

In the reanalyses of Ålfotbreen and Hansebreen 1998-2010 (Kjøllmoen, 2016) the mapping in 2010 was essential. The DTM₂₀₁₀ was used in the mass balance homogenization process, in the geodetic mass balance calculations and for the comparison between geodetic and glaciological mass balance.

In the original report from 2010 the mapping company reported the aerial survey date to 2nd September. Later it has been shown that the correct date was 29th September. This change of date influences the evaluation of the accuracy, the geodetic mass balance calculations and the comparison between geodetic and glaciological mass balance.

In the following the consequences of the corrected data for the 2010 mapping are described.

5.1.1 Adjustment for different dates

Generally, comparisons of glaciological and geodetic mass balance require adjustments because field measurements and aerial surveys are usually acquired at different dates. Updated date for aerial survey in 2010 and corresponding mass balance adjustment is shown in table 9.

Table 9
Aerial survey dates and adjustments for 1968, 1988, 1997 and 2010.

year	date		adjustment (m w.e.)		
	aerial survey	field survey	category	Ålfotbreen	Hansebreen
1968	5 th Aug	30 th Sep	melting	-1.46	
1988	7 th Sep	22 nd Oct	melting	-0.52	-0.57
1997	14 th Aug	20 th Nov	melting	-1.14	-1.09
2010	29 th Sep	28 th Sep		0.00	0.00

For the first three years, the aerial survey was performed some weeks before the ablation measurements in fall. The melting for the intermediate periods was estimated using a simple equation based on air temperature from a nearby climate station. Fresh snow at the time of ablation measurements was included in the winter balance for the subsequent year and was, hence not taken into account in this adjustment.

In 2010 the lidar data was acquired 29th September, and the ablation was measured on 28th September. In this context possible melting from 28th to 29th September is assumed to be insignificant. At the time of measurements, the fresh snow (0-10 cm) was estimated as <0.01 m w.e. Thus, adjustment due to different dates for 2010 was zero.

According to the estimated melting from the aerial survey dates to the field survey dates, the geodetic mass balances were adjusted as:

$$-\Delta B_{s, \text{yearI}} + \Delta B_{s, \text{yearII}}$$

The adjustments calculated in Kjøllmoen (2016) was -0.46 m w.e. for Ålfotbreen and -0.51 m w.e. for Hansebreen.

5.2 Homogenization

5.2.1 Mapping 2010

See chapter 3.1.1.

5.2.2 Mass change 1998-2010

The geodetic mass balance over the period 1997(98)-2010 was calculated using grid size 10 x 10 m. Average volume change was multiplied with the density conversion factor (850 kg m^{-3}), divided with the mean area for 1997 and 2010 and adjusted for additional melting in 1997. The results are given in table 10.

Geodetic mass balance over 1997-2010 was -13.14 m w.e. for Ålfotbreen and -16.92 m w.e. for Hansebreen.

The corresponding results from Kjøllmoen (2016) was -13.59 m w.e. and -17.42 m w.e. , respectively.

5.2.3 Mass change Ålfotbreen 1969-2010 and Hansebreen 1989-2010

In order to include as much as possible of the glaciological mass balance time series the mass change was also calculated for the periods 1968-2010 for Ålfotbreen and 1988-2010 for Hansebreen. The calculation method including grid size, density conversion factor, area and adjustment for additional melting is similar to the 1998-2010 mass change. The results are given in table 10.

Geodetic mass balance for Ålfotbreen over 1969-2010 was -7.63 m w.e. The result from Kjøllmoen (2016) was -8.08 m w.e.

Geodetic mass balance for Hansebreen over 1989-2010 was -11.33 m w.e. The result from Kjøllmoen (2016) was -11.83 m w.e.

Table 10
Volume change and geodetic mass balance for Ålfotbreen (1998-2010 and 1969-2010) and Hansebreen (1998-2010 and 1989-2010).

glacier	area _{yearI}	area _{yearII}	vol.ch.	dens. fac.	date adj. (m w.e.)		geod. mb. (m w.e.)	
	(km ²)	(km ²)			(mill. m ³)	(kg m ³)	yearI	yearII
Ålfotbreen								
1998-2010	4.48	3.97	-71	850	-1.14	0.00	-13.14	-1.01
1969-2010	4.49	3.97	-45	850	-1.46	0.00	-7.63	-0.18
Hansebreen								
1998-2010	3.18	2.75	-63	850	-1.09	0.00	-16.92	-1.30
1989-2010	3.07	2.75	-41	850	-0.57	0.00	-11.33	-0.51

5.3 Comparison and calibration

5.3.1 Comparison of glaciological and geodetic mass balances

Results from the glaciological, geodetic and internal mass balance calculations for the period 1998-2010 as well as the uncertainties are shown in table 11.

Table 11

Results of the uncertainty analysis. B is (glaciological [B glac.], geodetic [B geod.] and internal [B int]) mass balance and σ is the estimated random error. All balances and errors are in m w.e. a⁻¹. Δ is the difference between geodetic and glaciological balance, corrected for internal balance.

glacier	years	B glac.	σ .glac. point	σ .glac. spatial	σ .glac. ref	B geod.	σ .geod. DTM	σ .dc	B int	σ .B. int	Δ
Ålfotbreen	13	-0.53	0.26	0.19	0.05	-1.01	0.04	0.06	-0.06	0.02	0.42
Hansebreen	13	-1.01	0.26	0.19	0.05	-1.30	0.06	0.08	-0.04	0.01	0.25

B glac. is mean annual glaciological mass balance

σ .glac. point is random error for each point value in the glaciological mass balance

σ .glac. spatial is spatial random error in the glaciological mass balance

σ .glac.ref is random error as a consequence of glacier area changes over time

B geod. is mean annual geodetic mass balance

σ .geod. DTM is random error for the DTMs

σ .dc is random error for the density conversion

B int is internal melting

σ .B. int is random error for the internal melting

Δ is the difference between glaciological and geodetic balance, corrected for internal melting

The results show annual discrepancies as 0.42 m w.e. for Ålfotbreen 1998-2010 and 0.25 m w.e. a⁻¹ for Hansebreen 1998-2010.

The corresponding values in Kjøllmoen (2016) was 0.46 m w.e. and 0.29 m w.e, respectively.

In order to check whether the annual discrepancy between glaciological and geodetic mass balance is significant different or not, the hypothesis in Zemp et al. (2013) was tested. The answer from this check was «no», for Ålfotbreen and «yes» for Hansebreen, which suggest the geodetic and glaciological series are significant different for Ålfotbreen 1998-2010, but not significant different for Hansebreen 1998-2010 (Tab. 12). Hence, calibration of the glaciological mass balance series 1998-2010 was required for Ålfotbreen, but not required for Hansebreen.

The corresponding answer from this check in Kjøllmoen (2016) was «no» for both glaciers and consequently, both Ålfotbreen and Hansebreen 1998-2010 were calibrated. Due to the change of mapping date in 2010 the calibration of Hansebreen 1998-2010 is reversed.

Table 12

Comparison and check of glaciological and geodetic mass balance including the uncertainties over the period 1998-2010 for Ålfotbreen and Hansebreen.

glacier	Δ	σ	H0	β	ϵ
Ålfotbreen	0.42	3.60	no	5	0.42
Hansebreen	0.25	1.87	yes	54	0.48

Δ is the discrepancy (m w.e. a⁻¹) between glaciological and geodetic balance adjusted for internal melting

σ (dimensionless) is the reduced discrepancy, where uncertainties are accounted

H0 is the hypothesis whether the glaciological balance = the geodetic balance

β is the probability of accepting H0 although the results of both methods are different at the 95 % confidence level

ϵ (m w.e. a⁻¹) is the limit for detection of bias

5.3.2 Calibration of glaciological mass balance

Based on the comparison and hypothesis in chapter 5.3.1 the mass balance period 1998-2010 was calibrated for Ålfotbreen, but not calibrated for Hansebreen. The similar comparison and hypothesis in Kjøllmoen (2016) suggested calibration for both glaciers.

The annual periodic glaciological mass balance for Ålfotbreen 1998-2010 needed to be corrected with $0.42 \text{ m w.e. a}^{-1}$. The corresponding correction value in Kjølmoen (2016) was $0.46 \text{ m w.e. a}^{-1}$. Whether the discrepancy is a result of a bias in winter or summer balance was not proved. Thus, corrections of both winter and summer balances were applied. The percentual distribution (winter vs. summer) of the annual corrections can be done in several ways. In this calibration, the winter and summer corrections were assessed according to the size of the balance values; the greater balance value, the greater part of the correction. For instance, for the year 2004 the homogenized B_w and B_s were 3.32 and -3.35 m w.e. , respectively. The annual correction for the period 1998-2010 (-0.42 m w.e.) was then distributed as 50 % ($(3.32/(3.32+3.35))*100$) to B_w , and 50 % ($(3.35/(3.35+3.32))*100$) to B_s , resulting in calibrated B_w as 3.11 m w.e. ($3.32+(-0.42*50 \%)$), and B_s as -3.35 m w.e. ($3.35+(-0.42*50 \%)$). Winter, summer and annual balance curves for 2004 before and after the calibration are shown in figure 15.

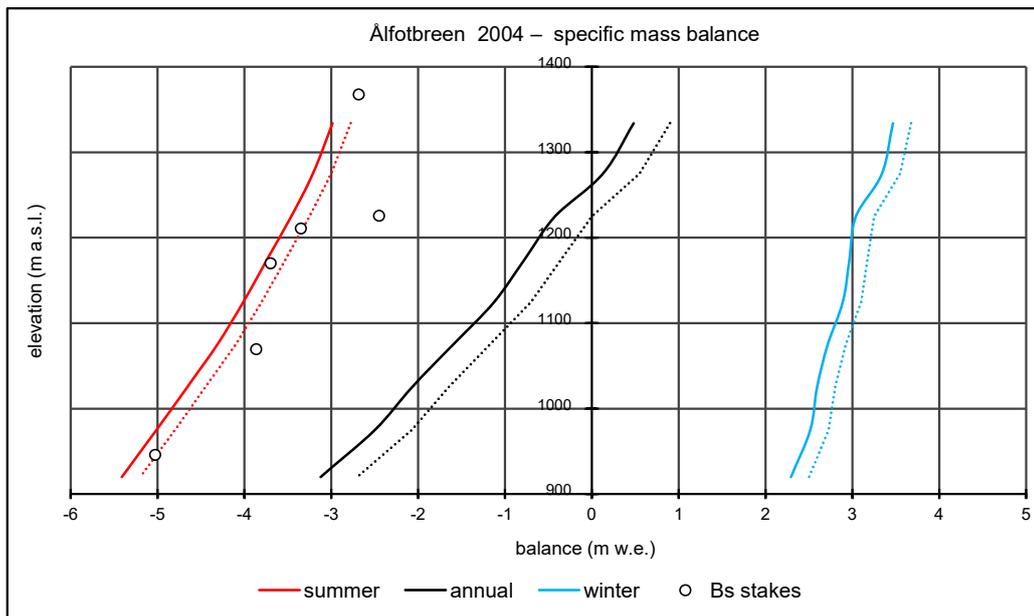


Figure 15
Winter, summer and annual balance curves in 2004 before (dotted) and after (solid) the calibration. Summer balance at each stake is also shown (○).

The calibrated cumulative mass balance for Ålfotbreen over 1998-2010 was -12.4 m w.e. and the mean annual balance values for the period were $3.45 (B_w)$, $-4.40 (B_s)$ and $-0.95 \text{ m w.e.} (B_a)$, respectively.

The calibrated results from Kjølmoen (2016) were cumulative mass balance as -12.8 m w.e. and mean annual balance values as $3.43 (B_w)$, $-4.42 (B_s)$ and $-0.99 \text{ m w.e.} (B_a)$, respectively.

The re-calibrated mass balance series for the years 1998-2010 together with the homogenized series 1998-2010 from Kjølmoen (2016) are shown in table 13, and the updated mass balance series for the period 1963-2019 is shown in figure 16.

Table 13

Homogenized and re-calibrated mass balance series for Ålfotbreen over 1998-2010.

Year	Homogenized mass balance series								Re-calibrated mass balance series					
	B _w	B _s	B _a	ΣB _a	ELA	AAR	DTM	Area	B _w	B _s	B _a	ΣB _a	ELA	AAR
1998	3.58	-3.60	-0.03	-0.03	1225	53	1997	4.48	3.37	-3.81	-0.45	-0.45	1250	45
1999	4.55	-4.47	0.08	0.05	1220	55	1997	4.48	4.33	-4.67	-0.34	-0.79	1260	42
2000	5.17	-3.58	1.59	1.64	1055	91	1997	4.48	4.92	-3.75	1.17	0.38	1100	85
2001	1.90	-3.97	-2.07	-0.42	>1383	0	1997	4.48	1.77	-4.26	-2.49	-2.11	>1383	0
2002	3.69	-5.30	-1.62	-2.04	>1383	0	1997	4.48	3.51	-5.55	-2.04	-4.15	>1383	0
2003	2.41	-4.98	-2.57	-4.62	>1383	0	1997	4.48	2.27	-5.27	-2.99	-7.14	>1383	0
2004	3.32	-3.35	-0.03	-4.64	1225	51	2010	3.97	3.11	-3.56	-0.45	-7.59	1265	37
2005	4.99	-4.21	0.77	-3.87	1050	93	2010	3.97	4.76	-4.40	0.35	-7.24	1180	66
2006	2.65	-5.85	-3.19	-7.06	>1368	0	2010	3.97	2.52	-6.14	-3.61	-10.85	>1368	0
2007	4.49	-3.17	1.32	-5.74	990	98	2010	3.97	4.25	-3.35	0.90	-9.95	1050	93
2008	4.57	-3.78	0.79	-4.96	1120	82	2010	3.97	4.34	-3.98	0.37	-9.59	1155	73
2009	3.83	-3.95	-0.13	-5.09	1235	48	2010	3.97	3.62	-4.17	-0.55	-10.14	1305	21
2010	2.19	-4.03	-1.84	-6.93	>1368	0	2010	3.97	2.04	-4.30	-2.27	-12.40	>1368	0

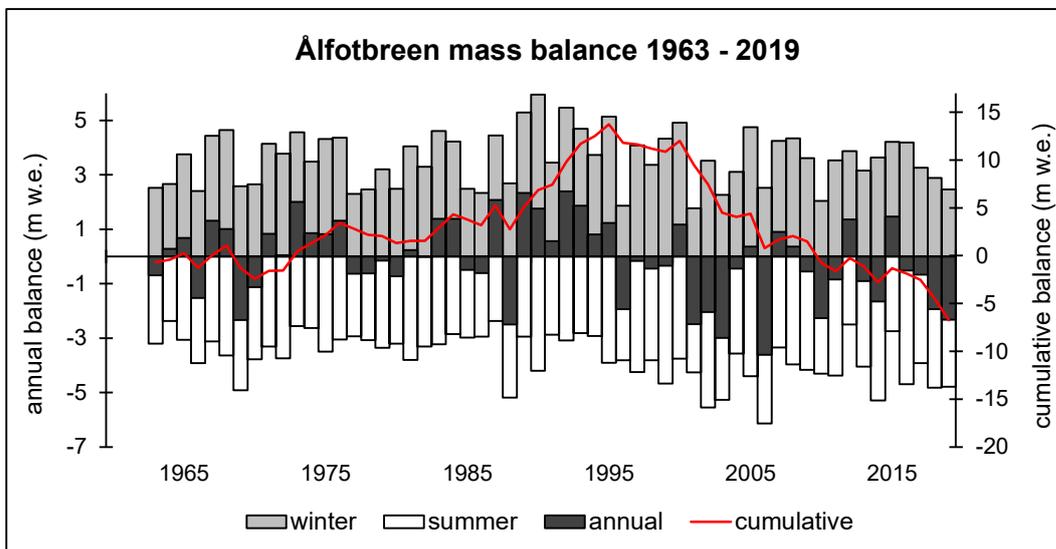


Figure 16

Calibrated mass balance series for Ålfotbreen over 1963-2019.

Conclusions

The aim of this report was to homogenize the glaciological mass balance series 2011-2019 for Ålfotbreen and Hansebreen, compare the series with the corresponding geodetic mass balance, and hence, reveal a possibly significant discrepancy followed by a calibration of the glaciological series. Within this period, Digital Terrain Models (DTMs) for 2010 and 2019 were produced.

In order to obtain comparable values, the glaciological and the geodetic mass balances were first homogenized. The homogenized cumulative glaciological mass balance over the years 2011-2019 were -6.00 m w.e. for Ålfotbreen and -11.40 m w.e. for Hansebreen. The corresponding geodetic mass balance was -7.17 m w.e. for Ålfotbreen and -12.21 m w.e. for Hansebreen. The internal mass balance was quantified as -0.51 m w.e. for Ålfotbreen and -0.37 m w.e. for Hansebreen. Accordingly, the mean annual difference ($\Delta_a = B_{a\text{glac.}} - B_{a\text{geod.}} + B_{a\text{int.}}$) over 2011-2019 was 0.10 m w.e. a^{-1} for Ålfotbreen and 0.05 m w.e. a^{-1} for Hansebreen. A hypothesis in Zemp et al. (2013) was tested and revealed that calibration was not required for any of the glaciers.

In the reanalyses of Ålfotbreen and Hansebreen 1998-2010 (Kjøllmoen, 2016) the mapping in 2010 was essential. The DTM_{2010} was used in the mass balance homogenization process, in the geodetic mass balance calculations and for the comparison between geodetic and glaciological mass balance.

In the original report from 2010 the mapping company reported the aerial survey date to 2nd September. Later it has been shown that the correct date was 29th September. This change of date influenced the evaluation of the accuracy, the geodetic mass balance calculations, the comparison between geodetic and glaciological mass balance and hence, whether a calibration was necessary or not.

The consequence of the corrected date for the 2010 mapping was that calibration for Hansebreen was not necessary and the calibration for Ålfotbreen was slightly changed compared with the reanalysis in Kjøllmoen (2016). The re-calibrated glaciological cumulative mass balance for Ålfotbreen over 1998-2010 were -12.40 m w.e. The corresponding calibration in Kjøllmoen (2016) gave a cumulative mass balance of -12.85 m w.e.

The reanalysed mass balance values were updated in NVE's databases by flagging the series as homogenized. The annual mass balance data are available for download from NVE's glacier application <http://glacier.nve.no/glacier/viewer/ci/en/>. The reanalysed data will be submitted to WGMS.

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