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SoilFlow : The Soil Water and Energy Balance Model

Technical Manual

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Abstract: This document is the technical manual of the SoilFlow model (Soil Water Balance and Energy model). The model is developed at the Hydrology department of the Norwegian Water Resources and Energy Directorate (NVE), primarily to be used as a tool in landslide and flood forecasting, and seasonal drought evaluation.

Key words: Soil moisture, hydraulic conductivity, pore pressure, snow water equivalent, modelling

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List of notations and symbols

Symbol	Description	Unit
α	Albedo	-
α_{sp}	Albedo of snow	-
α_g	Van Genuchten parameter (related to inverse of air entry potential)	-
θ	Volumetric water content of soil, including ice	cm^3/cm^3
$\theta_{hypstart}$	θ above which we have the hyperbolic-linear part of the SWRC	-
θ_{ice}	Liquid-equivalent volumetric ice content of soil	-
θ_r	Van Genuchten parameter, residual water content	-
θ_s	Water content at saturation	-
ρ_a	Density of air	g/cm^3
ρ_l	Density of liquid water	g/cm^3
$\rho_{spi(top)}$	Density of the topmost layer of snow	g/cm^3
ρ_{spi}	Density of snowpack, not including liquids	g/cm^3
c_{pa}	Heat capacity of air	J/g.k
c_{pi}	Heat capacity of ice (per equivalent water volume)	$J/K.cm^3$
c_{pw}	Heat capacity of water	$J/K.cm^3$
$clouds$	Fraction of sky covered by clouds (between 0 and 1)	-
$d_{s(top)}$	Diameter of snow crystals in the topmost layer of snow	mm
d_{sp}	Depth of snowpack	cm
e_a	Actual water pressure	kPa
e_s	Saturated water pressure	kPa
h	Total water potential (as height of water column)	cm
h_m	Matric potential (as height of water column)	cm
h_{pond}	Depth of pond above soil column	cm
h_{surf}	Height of surface water	cm
k_h	Heat conductivity	J/(cm day K)
k_{hmid}	Heat conductivity between two layers	J/(cm day K)
k_{pond}	First order coefficient for pond surface runoff	1/day
k_{sat}	Saturated hydraulic conductivity	cm/day
k_{sp}	Snow thermal conductivity	J/cm/K/day
k_w	Hydraulic conductivity	cm/day
m_{sp}	Mass of snowpack per area	g/cm^2
m_{spi}	Mass of ice in snowpack per area	g/cm^2
m_{spl}	Mass liquid in snowpack per area	g/cm^2
m_{splh}	Snowpack liquid holding capacity	g/cm^2
m	Van Genuchten empirical parameter	-
n	Van Genuchten empirical parameter (pore size distribution parameter)	-
nlay	Number of discretization layers	
q_h	Flux of heat upwards	$J/cm^2/day$

q_{drain}	Water drainage in cm3/cm3	-
q_{lsp}	Liquid flow from snowpack	cm/day
$q_{shortcut}$	Flow rate from pond through shortcut to bottom of profile	cm/day
q_{root}	Root uptake of water	1/day
q_w	Upward water flux	cm/day
r_{hsnow}	Aerodynamic resistance of snow	s/m
r_a	Aerodynamic resistance	s/m
r_s	Surface resistance	s/m
z_m	Height of wind measurement	m
$z[i]$	Depth to midpoint of layer i	cm
$\Delta z[i]$	Thickness of layer i	cm
C_b	Volumetric heat capacity of dry soil	$J/cm^3.K$
E_0	Potential evapotranspiration	cm/day
F_{pond}	Factor for continuous transition from pond to no-pond	-
G	Ground heat flux	$J/cm^2/day$
H	Thermal energy content of soil.	J/cm3
LAI	Leaf area index	m^2/m^2
L_m	Latent heat of fusion (melting)	J/cm^3
L_v	Latent heat of vaporization	J/cm^3
Q_r	Rain	cm/day
Q_s	Snowfall	cm/day
R_n	Net radiation	$J/cm^2/day$
R_{ns}	Net shortwave radiation	$J/cm^2/day$
R_s	Incoming shortwave radiation	$J/cm^2/day$
R_{so}	Extraterrestrial radiation	$J/cm^2/day$
T	Temperature, Celsius	˚C
T_{Kair}	Air temperature in Kelvin	K
T_{sp}	Temperature of snowpack	˚C

Preface

This manual gives a general description of the water and energy balance model called the SoilFlow model. It describes the model objectives and gives an overview of model components as well as a detailed mathematical description of model components. It explores the model's mathematical formulation, the numerical solution selected and the method that is coded in a MATLAB Software environment.

The manual is primarily intended for professionals engaged in hydrological model development, but also for other users who need to learn more about the technical part of the SoilFlow model, as well as input data and programming environment. The reader is assumed to have a good knowledge of mathematical modeling and a background in MATLAB programming.

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Oslo, April 2021

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Summary

This document is the technical manual of the SoilFlow model (Soil Water Balance and Energy model). The model is developed at the Hydrology department of the Norwegian Water Resources and Energy Directorate (NVE), primarily to be used as a tool in landslide and flood forecasting, and seasonal drought evaluation.

The SoilFlow model is a one-dimensional physical water and energy balance model, that simulates water and heat transport in snow and soil simultaneously. The model simulates both saturated and unsaturated flow, freezing and thawing, evapotranspiration, surface runoff, and drainage.

The SoilFlow model runs in a MATLAB environment, where the main evaluating functions are coded in C and in the Livermore Solver for Ordinary Differential Equations (LSODE). ODE solver is written in Fortran. SoilFlow can be run on a daily or hourly time step. The model may be parametrized and validated with observed data (snow depth, frost depth, groundwater levels, soil temperature and soil water content). The main model outputs include soil water content, ground water level, soil temperature, snow temperature, snow depth, snow water equivalent, snowmelt, and soil frost depth.

SoilFlow is currently established at a number of NVEs ground water monitoring sites with ongoing measurement of soil water and/or ground water level (in total 42 sites). The simulations are useful as stand-alone simulations, but also useful as comparison and validation of the model simulations performed by the operational conceptual models currently available at NVE.

The user manual of the model is under preparation and will be available in the future.

1. The SoilFlow Model

1.1 Introduction

This document is the technical manual of the SoilFlow model (Soil Water Balance and Energy Model), The model is developed at the Hydrology department of the Norwegian Water Resources and Energy Directorate (NVE) to be used as a tool in landslide and flood forecasting, daily hazard evaluation and drought analysis. The development of the model was motivated by the need for a more realistic physical process understanding of soil water, soil heat transport and snow accumulation and melt in addition to the models currently being used.

The soil water and energy balance COUP model (Jansson and Karlberg 2011) has been used in NVE since 2003, among other things, for analyses of the effect of climate change and later as an analysis tool of the soil, ground water states and for extreme hydrological events. However, the COUP model requires many input parameters and is too complex to be used operationally. To be able to use a soil water and energy balance models as a forecasting tool, it was necessary to develop a simplified version of the model that can be run automatically. SoilFlow is the result of this work.

The SoilFlow model is currently used as a tool in landslide and flood forecasting, daily hazard evaluation and drought analysis at NVE.

1.2 Model description

The SoilFlow model is a one-dimensional physical soil water and energy balance model, that simulates water and heat transport simultaneously, in the snowpack and the soil, with both saturated and unsaturated flow, freezing and thawing, evapotranspiration, surface runoff and drainage. The model is a physical system that consists of a vertical, one-dimensional profile represented by integrating vegetation canopy, snow, residue, or soil surface to a specified depth within the soil (Fig. 1). The number of soil layers as well as snowpack thickness are defined based on the needs of the model.

In addition to weather data, plant growth and soil characteristics are also required inputs to the model. The model can be run on a daily or hourly time step. The model is parameterized and validated with a set of observed data as input (snow depth, frost depth, groundwater levels, soil temperature and water content). The main model outputs include soil water content, ground water level, soil temperature, snow depth, snow water equivalent, snowmelt, and soil frost depth. Derived outputs, such as water supply to the topsoil (snowmelt and rain), degree of soil saturation and frost depth, are intended to be tested as indicators of increased risk of debris flow and landslide hazard depending on time.

The equations used in SoilFlow are mostly adapted from the COUP model (Jansson and Karlberg 2011), the SHAW model (Flerchinger 2000). The equations used in the SoilFlow model includes three main modules (snow, waterflow in soil, and heat transport) and will be described further in this manual.

The SoilFlow model solves a coupled system of ordinary differential equations (ODEs) obtained by discretization of the partial differential equations describing heat and water transport in the snow and soil system. This coupled process is solved by using the Livermore Solver for Ordinary Differential Equations (LSODES, (Radhakrishnan and Hindmarsh 1993), and a package of FORTRAN subroutines designed for the numerical solution of the initial value problem for a system of ordinary differential equations. It is particularly well suited for stiff differential systems, for which the backward differentiation formula method of orders 1 to 5 is provided. This type of solving the ODEs, where all but one dimension is discretized, is called the "method of lines". The coupled occurrence of temperature and moisture gradients brings the combined transport of heat and moisture. Usually, this effect is ignored in nearly saturated soil and dry soils.

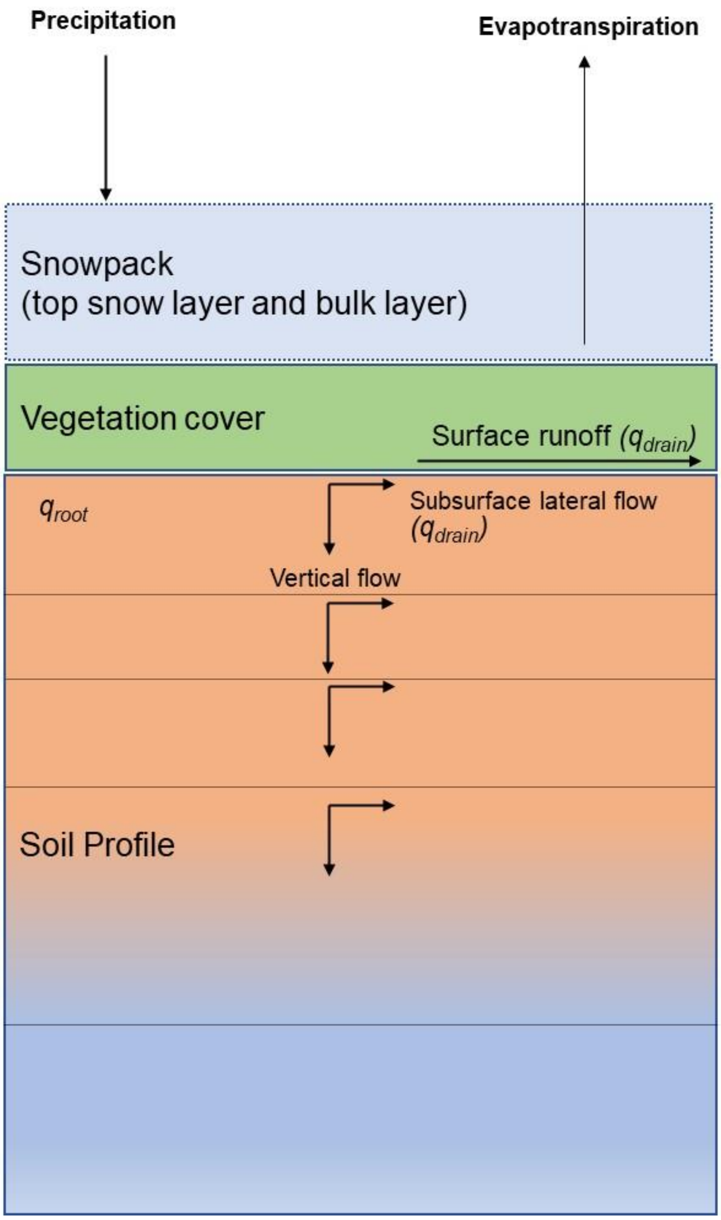

Figure 1. Flow chart of SoilFlow model process.

The software package MATLAB https://se.mathworks.com/help/matlab/ is used for pre- and post-processing of input and output data.

The following sections describe the three main modules in the SoilFlow model and notations and symbols.

2. The Snow Module

The snow functions are modified from the SHAW model, which in turn is based on (E. a. Anderson 1976)). In the SHAW model (Flerchinger, 2000) there is a changing number of numerical layers in the snowpack during a simulation, but in SoilFlow there are only two layers (Fig. 1): a bulk layer and a top snow layer that are dynamically adjusted. Separating out the top snow layer was necessary because of the calculation of the albedo, which will be highly influenced on newly fallen snow.

2.1 Heat Flux within the snowpack

The general energy balance within the snowpack is written as follows:

$$\underbrace{\rho_{sp} c_i \frac{\partial T}{\partial t}}_{(1)} + \underbrace{\rho_l L_f \frac{\partial w_{sp}}{\partial t}}_{(2)} = \underbrace{\frac{\partial}{\partial z}\left[k_{sp} \frac{\partial T}{\partial z}\right]}_{(3)} + \underbrace{\frac{\partial R_n}{\partial z}}_{(4)} - \underbrace{L_s \left(\frac{\partial q_v}{\partial z} + \frac{\partial \rho_v}{\partial t}\right)}_{(5)} \tag{2.1}$$

Where the terms in (W m^{-3}) represent, respectively:

(1) Specific heat term for change in energy stored due to a temperature increase.
(2) Latent heat required to melt snow.
(3) Net thermal conduction into a layer.
(4) Net radiation absorbed with a layer.
(5) Net latent heat of sublimation.

Symbols in the above equation are as follows:

ρ_{sp} snow density (g/cm^3)
C_i specific heat capacity of ice (J/g C)
T air temperature Celcius
t time
z depth in cm
ρ_l density of water (g/cm^3)
L_f latent heat of fusion and sublimation (J/g)
w_{sp} volumetric liquid water content (cm^3/cm^3)
k_{sp} thermal conductivity of the snow (J/cm^2)
Rn net downward radiation flux within the snow (W/m^2)
L_s laten heat of sublimation (J/g)
q_v vapor flux (g/s cm^2)
ρ_v vapor density within the snow (g/cm^3)

Heat transferred by liquid movement in the snowpack is not considered in the energy balance equation; at the end of each time step, a mass balance of the snowpack is computed to adjust the snowpack for melt, water percolation, and thermal advection. From equation 2.1, ignoring the contribution from net heat of sublimation (5) and rearranging the heat flux in and out of the snowpack the rate of change of thermal energy stored in snow can be expresed as follows, (using method of lines):

$$\frac{dH_{sp}}{dt} = Q_s \left(c_{pi} T_{air} - L_m\right) + F_{snow}\left(Q_r c_{pw} T_{air} + R_n + q_h[0]_{sp} + q_{hasp}\right) \tag{2.2}$$

Here, q_{hasp}, the heat conducted from air to snowpack, is given by [Wm-2]:

$$q_{hasp} = \begin{cases} \frac{T_{air}-T_{sp}}{0.5d_{sp}k_{sp}+r_{hsp}(\rho_a c_{pa})} & \text{if } T_{air} < 0 \\ - & \\ \frac{\rho_a c_{pa} T_{air}}{r_{hsp}} & \text{otherwise} \end{cases} \tag{2.3}$$

The heat flux [Wm-2] from the snowpack into the ground is:

$$q_h[0]_{sp} = \frac{T_{sp}-T[1]}{0.5d_{sp}/k_{snow}+0.5\Delta z[1]/k_h[1]}. \tag{2.4}$$

Where the the snow temperature of the snowpack T_{sp}, is given by [C]:

$$T_{sp} = \frac{H_{sp}+m_{spi}L_m}{c_{pi}\,\{_{0.1} m_{spi}\}} \tag{2.5}$$

Aerodynamic resistance of snow [s/m]:

$$r_{hsnow} = 0.89\frac{\left(\frac{max\left\{1,\ \ z_m-\frac{d_{sp}}{100}\right\}}{\text{snow\_roughness}}\right)^2}{86400\kappa^2 u_{wind}}$$
(2.6)

Snow thermal conductivity in J/cm.k.day

$$k_{sp} = 864\left(0.138 - 1.010\rho_{spi} + 3.233\rho_{spi}^2\right) \tag{2.7}$$

Snowpack depth [cm]:

$$d_{sp} = \frac{m_{spi}}{\rho_{spi}} \tag{2.8}$$

Snow grain size [mm] (only calculated for top snow layer):

$$d_{s(top)} = 0.16 + 0 \times \left(\frac{\rho_{spi(top)}}{\rho_l}\right)^2 + 110\left(\frac{\rho_{spi(top)}}{\rho_l}\right)^4 \tag{2.9}$$

The albedo of the snow (only top snow layer):

$$\alpha_{sp} = \left\{_{0.35}\, 1 - 0.206 \times 1.77\sqrt{d_{s(top)}}\right\} \tag{2.10}$$

Ice mass in snowpack per area [g/cm^2]:

$$m_{spi} = \left\{ {m_{sp} \atop 0} - \frac{H_{sp}}{L_m} \right\} \tag{2.11}$$

Rate of change of total snow mass (liquid + ice) [g/cm^2]:

$$\frac{dm_{sp}}{dt} = Q_s + F_{snow}\left(Q_r - q_{lsp}\right)$$

$$F_{snow} = \max(0,\min(1, m_{sp}/\mathrm{sl})) \qquad \text{where snow limit sl} = 0.1 \tag{2.12}$$

The snowpack mass can be written as a sum of frozen snow and water in the snowpack [g/cm^2]:

$$m_{sp} = m_{spi} + m_{spl}$$

Liquid flow from snowpack [g/cm^2/day] is calculated as:

$$q_{lsp} = 5\ \mathrm{day}^{-1}\left(m_{sp} - m_{spi} - m_{splh}\right) \times \left(1 + 10\frac{m_{spl}}{m_{sp}}\right) \tag{2.13}$$

The water holding capacity of the snowpack is ([g/cm^2], with default values from SHAW)

$$m_{splh} = \frac{m_{spi}}{\rho_{spi}}\ Max\left(0.03, 0.03 + (0.1 - 0.03) * \frac{0.2 - \rho_s}{0.2}\right) \tag{2.14}$$

The density of precipitating snow, ρ_s is in g/cm^3. The density of falling snow is related to the vertical temperature and moisture structure of the atmosphere and wind speed (at 2 m height) at the time snow is falling.

$$\rho_s = 50 + 1.7(T_{wb} + 15)^{1.5}\ /1000 \tag{2.15}$$

Where the wet-bulb temperature, T_{wb} is calculated as done in the SHAW model.

2.2 Settling and compaction of the snowpack

After snow falls, metamorphosis of the ice crystals in the snowpack as they change shape, causes the snowpack to settle. This process is relatively independent of snow density up to a value, ρ_d, of about 150 kg/m^3. Anderson (1976) suggested the following relation for fractional increase in density due to settling, ρ_{sp}, snowpack density [kg/m^3]:

$$\frac{1}{\rho_{sp}}\frac{\partial \rho_{sp}}{\partial t} = C3\ \exp(C4\ T)\ \exp\left[-46(\rho_{sp} - \rho_d)\right] \qquad \text{for } \rho_{sp} > \rho_d \tag{2.16}$$

$$\frac{1}{\rho_{sp}}\frac{\partial \rho_{sp}}{\partial t} = C3\ \exp(C4\ T) \qquad \text{for } \rho_{sp} < \rho_d \tag{2.17}$$

Where *C3* is the fraction rate of settling at 0 °C for densities, less than ρ_d, and *C4* is an empirical coefficient (taken as 0.04 °C). The presence of liquid water will increase the rate of settling. When liquid water is present in the snow, the fractional rate of settling computed from this equation is multiplied by a factor, *C5* assumed equal to 2.0 (E. a. Anderson 1976).

The total rate of change of snow density in the main snowpack is calculated as follows:

$$\frac{d\rho_{spi}}{dt} = \left\{\begin{matrix} 10 \\ -10 \end{matrix}\ \frac{(\rho_{Qs} - \rho_{spi})Q_s\{_{0.001}\rho_{spi}\}}{\{_{0.001}m_{spi}\}\rho_s} + sp_{settling} + sp_{compaction}\right\} \tag{2.18}$$

Similarly for topsnow:

$$\frac{d\rho_{spi_{(top)}}}{dt} = \begin{Bmatrix} 10 & \frac{(\rho_s - \rho_{spi(top)})Q_s\{0.001\rho_{spi(top)}\}}{\{{}^{1.0}_{0.001} m_{spi}\}\rho_s} + sp_{settling} \\ -10 & \end{Bmatrix} \tag{2.19}$$

Rate of change of snow density due to snow settling $sp_{settling}$:
sp_{water} = *sp - spi*

sp_f = min(1.0 + 10*sp_{water}/*sp*, 2.0)

$sp_{settling}$= 24*ρ_s**0.01*exp(0.04*T_s)

if (ρ_s > ρ_d)
$sp_{settling}$ = $sp_{settling}$* exp(-46*(ρ_s - ρ_d))

$$sp_{settling} = 0.24\ \mathrm{day}^{-1}\rho_{spi} * \min\left(1.0 + 10\frac{m_{spl}}{m_{sp}}, 2.0\right) * \exp(0.04\ T_s) \tag{2.20}$$

or

if ρ_{psi} > ρ_d

$sp_{settling}$ = $sp_{settling}$ * exp(-46*($\rho_{psi} - \rho_d$))

Rate of change of snow density due to snow compaction: $sp_{compaction}$

$sp_{compaction}$ = ρ_s*0.01*24*sp/2*exp(0.08*T_s - 21.0*ρ_s);

$$sp_{compaction} = 0.24\rho_{spi}\frac{m_{sp}}{2\mathrm{g/cm^2}}\exp\left(0.08\ T_s - 21.0\ \rho_{spi}\right) \tag{2.21}$$

3. Soil water flow

The soil water equation used in SoilFlow is based on Richard equation. The equation for water in the soil assumes that the flow is laminar and thus follows Darcy's law.

$$q_w = -k(\psi)\frac{\partial\psi}{\partial z} + k(\psi)$$

or

$$q_w = D(\theta)\frac{\partial\theta}{\partial z} + k(\theta) \tag{3.1}$$

Where :

$$D(\theta) = -\frac{k(\theta)\,\partial\psi}{\partial\theta}$$

k is the unsaturated hydraulic conductivity [cm/s], Z is depth of the soil layer [cm], ψ is the water tension [cm], D is the diffusion coefficient $[cm^2/s]$ and θ is the water content $[cm^3/cm^3]$. The hydraulic conductivity values fall in a wide range; generally, K can range from 10-9 cm/s for clays, to 15 cm/s for sandy soils.

Mass conservation gives the generalized equation.

$$\frac{\partial\theta}{\partial t} = -\frac{\partial q_w}{\partial z} + s_w$$

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial\theta}{\partial z} + K(\theta)\right] \tag{3.2}$$

θ, the water content and S_w an external water source term for water exchange purposes [cm/s]. This equation cannot be solved without additional information, because we have two unknown variables θ and ψ. In addition, the parameter k_w (cm/s) is dependent on θ. In other words, we need to define two more equations between θ, ψ and k_w to solve the differential equation. We usually call these equations the retention curve $\psi = f(\theta)$, and the unsaturated conductivity function $k_w = f(\theta)$ or alternatively $k_w = f(\psi)$.

3.1 Water retention curve

There are several water retention curves available in the literature. In the SoilFlow model this is basically done by implementing the water retention function described by van Genuchten (Genuchten 1980).

The matric potential is modelled for unsaturated soil with the van Genuchten equation, with some changes. The modification is with regard to the the ODE solver, which performes poorly for the regular Van Genuchten close to saturation and close to the residual water content. At these water contents the graph of the potential approaches vertical lines towards zero or minus infinity, respectively. With the modifications, below a certain water content, the relationship between water content and potential is linear, and close to saturation there is (over a very narrow interval) a hyperbolic relation which joins the van Genuchten equation in a smooth transition to the positive potential above saturation. Above saturation, the potential increases very steeply and linearly, due to compressibility of water and residual air.

S_e is the effective saturation and is given by :

$$S_e = \frac{1}{(1+(\alpha\psi)^n)^m} \tag{3.1.1}$$

$$S_e = \frac{\theta-\theta_r}{\theta_s-\theta_r} \tag{3.1.2}$$

Where *α, n* and *m* are empirical parameters and *θ* is the water content $[cm^3/cm^3]$, θ_r is the residual water content (minimum), and θ_s is the saturated water content (maximum) and equal to the porosity. However, in many cases the effective saturation is avoided by setting the two equations together and solving it for the water content:

$$\theta = \theta_r + \frac{\theta_s-\theta_r}{(1+(\alpha\psi)^n)^m} \tag{3.1.3}$$

3.2 Unsaturated conductivity function

Van Genuchten gives the unsaturated conductivity equation as:

$$k_w^* = k\,\frac{\left[1-(\alpha\psi)^{n-1}\,[1+(\alpha\psi)^n]^{-m}\right]^2}{\left[1-(\alpha\psi)^n\right]^{\frac{m}{2}}} \tag{3.2.1}$$

With the same parameters as in the retention equation 3.1.1. This equation is in most cases written in a simpler form by using the definition of effective saturation S_e and by reducing the number of parameters involved by setting:

$$m = 1 - {}^{1}\!/_{n} \tag{3.2.2}$$

$$k_r = \frac{k_w^*}{k} = S_e^{\frac{1}{2}}\left[1-\left(1-S_e^{\frac{1}{m}}\right)^m\right]^2 \tag{3.2.3}$$

The "*" in k_w^* denotes that this approximates k_w. The relative conductivity k_r is mostly encountered in hydrogeological literature where *k* is simply the hydraulic conductivity. Soil scientists prefer to use k_{mat} (instead of *k*) and call it the saturated matrix conductivity. Theoretically *k* and k_{mat} are the same parameter but one should be aware of that soil scientists and hydrogeologists will not necessarily estimate it in the same way.

The rate of change of water content is approximated by:

$$\frac{d\theta[i]}{dt} = \frac{q_w[i+1]-q_w[i]}{\Delta z} - q_{root}[i] - q_{drain}[i] \tag{3.2.4}$$

The matric potential (h_m) is given by:

$$h_m = \begin{cases} \text{linear function of } \theta & if\ \theta < \theta_{low} \\ -\frac{1}{\alpha_{vg}}\left(\left(\frac{\theta-\theta_r}{\theta_s-\theta_r}\right)^{-\frac{1}{m}} - 1\right)^{\frac{1}{n}} & if\ \theta_{low} \le \theta \le \theta_s - \theta_{hypstart} \\ \text{hyperbolic and linear function of } \theta & \text{otherwise} \end{cases} \tag{3.2.5}$$

The total potential is them given by:

$$h = h_m + z$$

Relative Hydraulic conductivity (k_{rel}) ϵ [0,1]:

For $\theta \leq \theta_s - \theta_{hypstart}$

$$k_{rel} = \frac{k_w}{k_{sat}} = 1 - \left(1 - \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{\frac{1}{m}}\right)^m \tag{3.2.6}$$

At $\theta = \theta_s$, $k_{rel} = 1$.

3.3 The Evapotranspiration module

The potential evapotranspiration, E_0 is calculated with the Penman equation(R. G. Anderson and French 2019)

$$E_0 = 100 \frac{\frac{\frac{de_s}{dT}(R_n - G)}{100} + \frac{86400\rho_{air}c_p(e_s - e_a)}{r_a}}{\left(\frac{de_s}{dT} + 0.665 \times 10^{-3}\left(1 + \frac{r_s}{r_a}\right)\right)L_v} \tag{3.3.1}$$

Where Rn is the net radiation [W/m^2], G is the soil heat flux [W/m^2], $(e_s - e_a)$ represents the vapor pressure deficit of the air, ρ_a is the mean air density at constant pressure, c_p is the specific heat of the air, $\frac{de_s}{dT}$, represents the slope of the saturation vapor pressure temperature relationship, and r_s and r_a are the bulk surface and aerodynamic resistances[s/m]

The surface resistance r_s is given by [s/m]:

$$\mathrm{r_s} = \frac{\mathrm{r_1}}{\mathrm{LAI_{active}}}$$

Where r_1 [s/m] is the daily bulk stomatal resistance of the well-illuminated leaf, LAI_{active} [m^2/m^2] is the active (sunlit) leaf area index. The active LAI is the index of the leaf area that actively contributes to the surface heat and vapor transfer. A general equation for LAI_a [m^2/m^2]is given by:

$LAI_{a = 0.5 * LAI}$

Which takes into consideration the fact that generally only the upper half of dense clipped grass is actively contributing to the surface heat and vapor transfer(R. G. Anderson and French 2019) proposed equations to estimate bulk surface resistance to water flux based on the crop height of grass in terms of the estimate crop LAI. For clipped grass (<0.15 m) a general equation for LAI is:

LAI = 24*h_c

Where h_c=0.12 m is the reference crop height. For unclipped grass:

LAI = 5.5 + 1.5 * ln(h_c)

The bulk stomatal resistance, r_1, is the average resistance of an individual leaf, and has a value of about 100 s/m under well-watered conditions.

By assuming a crop height of 0.12 m, the surface resistance for a reference crop, approximates to 70 s/m and 45 s/m for a 0.50 m crop height.

$$r_s = \frac{100}{0.5 \times 24 \times h_{plants}} \quad (3.3.2)$$

The aerodynamic resistance:

$$r_a = \frac{\log\frac{z_m - \frac{2h_{plants}}{3}}{.123h_{plants}} \times \log\frac{z_h - \frac{2h_{plants}}{3}}{.0123h_{plants}}}{0.41^2 u} \quad (3.3.3)$$

Where r_a aerodynamic resistance, z_m height of wind measurement, h_{plants} plant height, z_h height of humidity measurement, u wind speed at height z, and r_s surface resistance.
Air water pressure is estimated from relative humidity data, and saturated vapor pressure calculated from air temperature.

The equation used for this purpose is:

$$S = 0.6108 \cdot e^{\left(\frac{17.27*T}{T+237.3}\right)}$$

$$EA = 0.01 \cdot RH \cdot S$$

Which gives the Air water pressure 'EA' in kPa. 'S' is the saturated vapor pressure in kPa while the input temperature *T* is in °C. RH is the relative humidity in percent.

3.4 Root distribution and water uptake by roots

The root distribution can be exponential, linear or constant and that can be choosen in the model setup. A depth can be set below which there are no active roots, i.e. maximum root depth (Skaggs et al. 2006). The root uptake is given as:

$$q_{root} = D_{roots} f(h_m) \quad (3.4.1)$$

Where the $f(h_m)$ is a function that restricts the root water uptake rate at very high and low potentials (Fig. 2). $f(h_m)$ is piecewise linear reduction function parameterized by four critical values of the water pressure head, h4 < h3 < h2 < h1 as shown below.

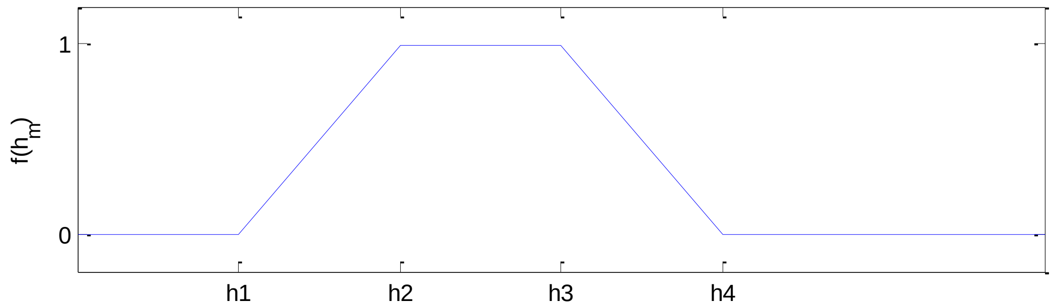

Figure 2. The root uptake function *f (h_m)*.

3.5 Soil surface ponding (surfaces water storage)

A factor F_{pond} is used to make a continuous transition between conditions with and without ponding:

F_{pond} = max(0.0, min(10.0*H_{pond}, 1.0)) (3.5.1)

The flow of water to the pond or soil surface is (cm):

$$q_{wtop} = (1 - F_{snow})Q_r + F_{snow}q_{lsp} \quad (3.5.2)$$

The maximum water flow the soil surface can absorb:

$$q_w[0]_{max} = k_{sat}\frac{h_{pond} - h_0}{z[0]} \quad (3.5.3)$$

Water absorbed by soil surface:

$$-q_w[0] = F_{pond}q_w[0]_{max} + (1 - F_{pond})min\{flux\ to\ soil, q_w[0]_{max}\} \quad (3.5.4)$$

A parameter "pondshortcut" [1/cm] can be used to make it possible for water to flow directly from the pond to the lowest layer of the profile. (Without this parameter, frost can totally block infiltration.)

$$q_{shortcut} = F_{pond}(h_{pond} - h[\mathrm{nlay}])k_w[\mathrm{nlay}] * pondshortcut \quad (3.5.5)$$

Rate of change of pond depth:

$$\frac{\mathrm{dh_{pond}}}{\mathrm{dt}} = \mathrm{F_{pond}}(\mathrm{q_{wtop}} - \mathrm{q_w}[0] - \mathrm{k_{pond}}\mathrm{h_{pond}}) - \mathrm{q_{shortcut}} + (1 - \mathrm{F_{pond}})\{flux\ to\ soil, -q_w[0]_{\mathrm{max}}\} \quad (3.5.6)$$

Vertical water flow, $-k_w \nabla h$ is approximated by:

$$q_w[i] = -k_{mid}\frac{h[i] - h[i-1]}{z[i] - z[i-1]} \quad (3.5.7)$$

where k_{mid} is weighted average of $k_w[i]$ and $k_w[i-1]$.

4. Thermal energy and heat of soil

4.1 The soil heat process

Heat flow in the soil is the sum of conduction, the first term, and convection, the last two terms:

$$q_h = -k_h \frac{\partial T}{\partial z} + C_w T q_w + L_v q_v \qquad (4.1.1)$$

Where the indices *h*, *v* and *w* mean heat, vapor and liquid water, *q* is flux, *k* is conductivity, *T* is soil temperature, *C* is heat capacity, *L* is latent heat and *z* is depth. The first convective term, *CwTqw*, may or may not be included in the solution. Normally this convective term is important at high flow rates e.g., during heavy snow melt infiltration.

The law of energy conservation is given by:

$$\frac{\partial(CT)}{\partial t} - L_f \rho \frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial z}(-q_h) - s_h \qquad (4.1.2)$$

Where the indices *f* and *i* mean freezing and ice. C, T, θ and L are heat capacity, temperature, volumetric water content and latent heat.

The heat equation solved by SoilFlow is based on a similar formulation as used in the SOIL model(Jansson 1998). The equation has the usual components of a thermal distribution equation but in addition, it incorporates effects from ice, water, and vapor (subscripts *i*, *w* and *v*) in the soil pores.

$$\frac{\partial(CT)}{\partial t} - L_f \rho \frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - C_w T \frac{\partial q_w}{\partial z} - L_v \frac{\partial q_v}{\partial z} - s_h \qquad (4.1.3)$$

In this equation *t* and *z* are time and depth, respectively. The equation describes the temperature distribution *T*(z,t) which depends on the thermal conductivity *k* and the heat capacity *C*. Additional convection terms have been added for the transport of temperature through water movement through the soil q_w, while latent heat effects (L_f and L_v) from vapor movements q_v and ice formation θ_i have been added. The last term s_h is an external heat source term added for heat exchange purposes.

The rate of change of thermal energy storage, using the continuity equation for the conservation of energy, can be approximated by:

$$\frac{dH[i]}{dt} = \frac{q_h[i+1] - q_h[i]}{\Delta z} - T[i] c_{pw}(q_{root}[i] + q_{drain}[i]) \qquad (4.1.4)$$

Where *H* is the storage of heat [W/m^2], left hand side of the heat equation.

4.2 Thermal properties of soil

Volumetric heat capacity C_v [MJ/ m^3] can be calculated using empirical formula used in (Boguslaw and Lukasz 2004)

$C_v = (2.0x_s + 2.51x_o + 4.19x_w) \cdot 10^6$

or by Sikora and Kossowski (1993):

$Cv = (cs + 4190\ \theta w)\ \rho$

where: x_s , x_o , x_w [m^3/m^3] are volumetric contributions of mineral and organic components and water, respectively, c_s is specific heat of soil solid (713 J/kg K), θw is soil water content [g/g], ρ is soil bulk density [Mg/cm^3].

$$\theta_{ice} = \max\left(0, \min\left(-\frac{H}{Lm}\right)\right) \quad (4.2.1)$$

The soil (C) temperature is:

$$T = \frac{H + \theta_{ice} L_m}{c_{pi}\theta_{ice} + c_{pw}(\theta - \theta_{ice}) + C_b} \quad (4.2.2)$$

Thermal conductivity of soil empirical function is adapted from (Jansson and Karlberg 2011)

For unfrozen soil conditions:

$$\lambda = (a_1 \log \frac{\theta}{\rho_b} + a_2) \times 10^{b_4 \rho_b} \quad (4.2.3)$$

For frozen soil:

$$\lambda = b_1 \times 10^{b_2 \rho_b} + b_3 \frac{\theta}{\rho_b} \times 10^{b_4 \rho_b} \quad (4.2.4)$$

Typical values used are a1= (0.1-0.9), a2=0.06 – (-0.2) , b1=0.002-.01 , b2=1.3-1.4 , b3 =0.004-0.004 b4 =0.9-.92.

Flux of heat into uppermost layer:

$$q_h[0] = F_{snow} q_h[0]_{sp} + (1 - F_{snow})\left(k_h[0]\left(\frac{T_{air} - T_0}{\Delta z[0]/2}\right) + Q_r T_{air} c_{pw}\right) \quad (4.2.5)$$

Heat conductivity between layer i and $i + 1$:

$$k_{hmid}[i] = \sqrt{k_h[i] k_h[i+1]} \quad (4.2.6)$$

Heat flux between layers (positive upwards); $-k_h \nabla T + c_{pw} T q$ is approximated by:

$$q_h[i] = k_{hmid}[i] \frac{T[i+1] - T[i]}{z[i] - z[i+1]} + q[i] c_{pw} \frac{T[i] + T[i+1]}{2} \quad (4.2.7)$$

4.3 Radiation

Extraterrestrial radiation R_a [MJ/m^2.day]is calculated based on day of the year and latitude of the weather station with the following expression (Allen, R.G., *et al* (1998))

$$R_a = \frac{1366.7}{\pi} \cdot \delta_r \cdot \left[\omega \cdot \sin\left(\frac{lat \cdot \pi}{180}\right) \cdot \sin(\delta) + cos\left(\frac{lat \cdot \pi}{180}\right) \cdot \cos(\delta) \cdot \sin(\omega)\right] \quad (4.3.1)$$

where ,

$$\delta = 0.409 \cdot \sin\left(\frac{2\pi\tau}{365.2425} - 1.39\right) \quad (4.3.2)$$

$$\delta_r = 1 + 0.033 \cdot \cos\left(\frac{2\pi\tau}{365.2425}\right) \quad (4.3.3)$$

$$\omega = arccos\left[-\tan\left(\frac{lat \cdot \pi}{180}\right) \cdot \tan(\delta)\right] \qquad (4.3.4)$$

lat is the latitude in degrees, and τ is the Julian day number.

The emission factor (F) is calculated from the air water pressure and cloudiness by [kPa]:

$$F = \sigma\left(0.34 - 0.14\sqrt{EA}\right)\left(1.335\frac{R_s}{R_{so}} - 0.35\right) \qquad (4.3.5)$$

Where the ratio of R_s shortwave radiation and R_{so} the clear-sky shortwave radiation, is estimated from:

$$\frac{R_s}{R_{so}} = \left[\frac{0.25 + 0.5(1 - CC)}{0.75}\right]$$

Where *CC* is the cloud cover (from 0 to 1) and *EA* the air water pressure [kPa]. If *CC* measurements are not available, one can use radiation measurements instead, or maximum and minimum temperature (daily or hourly). Extraterrestrial radiation R_a, *CC*, R_s and R_{so} are related to each other through the following equations:

$$\frac{R_s}{R_a} = 0.25 + 0.5(1 - CC) \qquad (4.3.6)$$

$$R_{so} = 0.75R_a \qquad (4.3.7)$$

5. Model implementation

The main part of the SoilFlow model consists of seven coupled differential equations for soil water, surface water, snow and heat transport which are solved in 1D for a given soil profile as illustrated in Figure 1. Any soil type can be simulated, with diverse vegetation covers or bare soil. The model consists of modules, that allow the snowpack and soil profile to interact with the atmosphere, permitting the processes such as snowmelt, precipitation, and evapotranspiration to be dynamically simulated. MATLAB scripts are used for pre-processing and post-processing of input and output data whereas the evaluating functions, described below, are written in C, and the ODE solver (Livermore Solver for Ordinary Differential Equations (LSODE)) is written in Fortran (Radhakrishnan and Hindmarsh 1993).

The System of differential equations that describes the model are summarised as follows:

The rate of change of water content is approximated by:

$$\frac{d\theta[i]}{dt} = \frac{q_w[i+1]-q_w[i]}{\Delta z} - q_{root}[i] - q_{drain}[i] \quad (5.1)$$

The rate of change of thermal energy is approximated by

$$\frac{dH[i]}{dt} = \frac{q_h[i+1]-q_h[i]}{\Delta z} - T[i]c_{pw}(q_{root}[i] + q_{drain}[i]) \quad (5.2)$$

Rate of change of thermal energy stored in snow

$$\frac{dH_{sp}}{dt} = Q_s\left(c_{pi}T_{air} - L_m\right) + F_{snow}\left(Q_r c_{pw} T_{air} + R_n + q_h[0]_{sp} + q_{hasp}\right) \quad (5.3)$$

Rate of change of total snow mass (liquid + ice):

$$\frac{dm_{sp}}{dt} = Q_s + F_{snow}\left(Q_r - q_{lsp}\right) \quad (5.4)$$

The total rate of change of snow density is then:

$$\frac{d\rho_{spi}}{dt} = \left\{ \begin{matrix} 10 \\ \\ -10 \end{matrix} \frac{(\rho_s - \rho_{spi})Q_s\{_{0.001}\rho_{spi}\}}{\{_{0.001}m_{spi}\}\rho_s} + \text{settling} + \text{compaction} \right\} \quad (5.5)$$

Similarly for topsnow:

$$\frac{d\rho_{spi_{(top)}}}{dt} = \left\{ \begin{matrix} 10 \\ \\ -10 \end{matrix} \frac{(\rho_s - \rho_{spi(top)})Q_s\{_{0.001}\rho_{spi(top)}\}}{\{^{1.0}_{0.001}m_{spi}\}\rho_s} + \text{settling} \right\} \quad (5.6)$$

Rate of change of surface soil pond depth:

$$\frac{\mathrm{dh_{pond}}}{\mathrm{dt}} = \mathrm{F_{pond}}\left(\mathrm{q_{wtop}} - \mathrm{q_w}[0] - \mathrm{k_{pond}h_{pond}}\right) - \mathrm{q_{shortcut}} + \left(1 - \mathrm{F_{pond}}\right)\{\text{flux to soil}, -\mathrm{q_w}[0]_{\max}\} \quad (5.7)$$

5.1 Input data

The following input data are required to run the SoilFlow model (daily or hourly timesteps)

Weather data:

- Precipitation
- Temperature
- Global radiation
- Wind speed
- Relative humidity
- Cloud cover (optional)
- Ground water table (optional)

Plant data:

- Root depth
- Leaf Area Index
- Growth factor
- Height and albedo

Soil data:

- Soil density
- Soil porosity
- Heat capacity of soil
- Hydraulic conductivity
- Soil thermal conductivity
- Soil water characteristic curve

Boundary and Initial Condition

The following state variables need to be initialized from observed values:

- Soil temperature
- Soil water content
- Snow mass, density and temperature

6. Example of model simulation

The SoilFlow model is tested and applied at a number of places in Norway where there are ground water measurements, and at some soil moisture observation stations (Fig. 3). Green circles in Figure 3a show the location of ground water stations, which are part of the NVE's network while the blue circles show the stations where SoilFlow model is applied. The model was tested at Groset, which boundaries are shown in Figure 3b. The list of soil properties used for the Groset area are given in Table 1, while the input used for the model are listed in Table 2. The results from model simulation for Groset for the period 1.06.2019 – 15.12.2020 are shown in Table 3 and Figure 4.

An exampel of how the model is parametrised and validated for Groset station is presented in Colleuille et al. (2007). The report presents the results done with the COUP-model, but the principle is almost the same with SoilFlow.

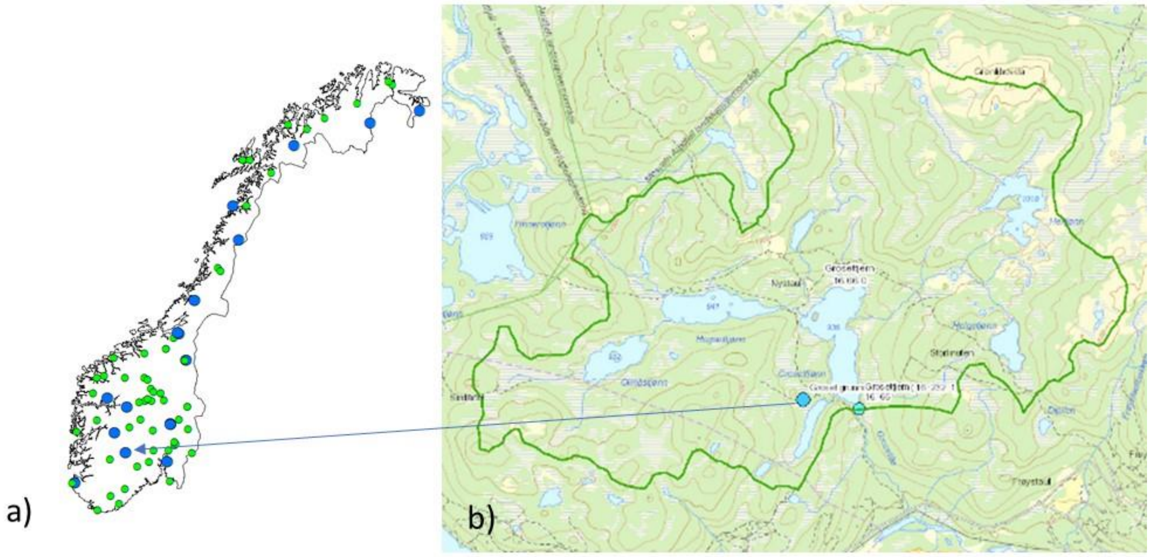

Figure 3. Locations where the SoilFlow model has been tested and applied. a) NVEs ground water network: green circles, all stations; blue circles, stations where SoilFlow is applied. b) Boundaries of Groset catchment (green line). The blue circle indicates the location of the ground water station, while the light green circle indicates the location of the water discharge station (Lat. 59.84°, Lon. 8.31°, MSL 970m).

Table 1. Soil profile properties used at Groset station.

Parameter	Depth 1	Depth 2	Depth 3	Depth 4
θ_r	0.08	0.08	0.08	0.08
θ_s	0.45	0.3	0.25	0.25
α	1	0.9	0.2	0.2
n	1.2	1.2	1.6	1.6
m	1-1/n	1-1/n	1-1/n	1-1/n
hs	0	0	0	0
k_{sat}	300	300	300	300
k_{sat_h}	300	3	1	1
Cs	12	12	2	0.8
ρ_s	2.7	2.7	2.7	2.7
porosity	0.4	0.4	0.3	0.3
heat_k_a1	0.1	0.1	0.01	0.1
heat_k_a2	0.058	0.058	0.0058	0.058
heat_k_a3	0.6245	0.62	0.062	0.62
heat_k_b1	0.00158	0.00158	0.00158	0.00158
heat_k_b2	1.336	1.336	1.336	1.336
heat_k_b3	0.00375	0.00375	0.00375	0.00375
heat_k_b4	0.9118	0.9118	0.9118	0.9118

Parameter	Value
all_snow_temperature	2
no_snow_temperature	3
soiltype	5
soildepth1	30
soildepth2	40
soildepth3	80
soildepth3	150
surfacewater_distance	11
surfacewater_height	-3.8
soil_albedo	0.25
ponddk	1
pondshortcut	1
rootfactor	1
ks_factor	1
qfakt	30
tcorr	1
pcorr	1.2

Table 2. Model input sample.
ppt=precipitation, temp=temperature, gwt=ground water level, rad=global radiation, wnd=wind, cld= cloud cover, snd=snow depth swe=snow water equivalent, tmax maximum temperatures, tmin= minimum temperatures.

Date	ppt	temp	gwt	rad	rhd	wnd	cld	pres	snd	swe	tmax	tmin
01.06.2019	3.7	5.2	-0.418	1950	621	427	50	0	0	0	8.5	2.2
02.06.2019	0	5.9	-0.462	2110	84	5	91	0	0	0	8.1	4.1
03.06.2019	9.6	7.9	-0.302	610	81	3	87	0	0	0	10.7	5.8
04.06.2019	1.3	8.4	-0.373	1330	81	4	86	0	0	0	12.6	5
05.06.2019	0.1	7.9	-0.376	800	68	4	74	0	0	0	10.9	5.2
06.06.2019	3.9	7.4	-0.404	650	93	2	94	0	0	0	8.7	5.7
07.06.2019	5.2	12.9	-0.405	700	88	3	75	0	0	0	16.5	9.5
08.06.2019	0	11.5	-0.426	1250	77	2	75	0	0	0	15.5	6.3
09.06.2019	13.4	7.1	-0.356	3000	84	3	89	0	0	0	10.5	6.4

Table 3. Model output sample.
ppt=precipitation, temp=temperature, gwt=ground water level, gwtobs= ground water level observed, soildf=soil water deficit, snd=snow depth, swe=snow water equivalent, lwc = liquid water content in snowpack, frost= frost depth (0 indicates no frost).

Date	ppt	temp	gwt	gwtobs	soildf	snd	swe	lwc	frost
01.06.2019	0.44	5.2	-57.56	-41.8	75.49	0	0.01	0.01	0
02.06.2019	0	5.9	-57.08	-46.2	75.44	0	0	0	0
03.06.2019	1.15	7.9	-57.38	-30.2	75.67	0	0	0	0
04.06.2019	0.16	8.4	-55.4	-37.3	75.97	0	0	0	0
05.06.2019	0.01	7.9	-55.43	-37.6	75.66	0	0	0	0
06.06.2019	0.47	7.4	-56.38	-40.4	75.54	0	0	0	0
07.06.2019	0.62	12.9	-56.45	-40.5	75.73	0	0	0	0
08.06.2019	0	11.5	-55.98	-42.6	75.63	0	0	0	0
09.06.2019	1.61	7.1	-56.13	-35.6	76.05	0	0	0	0

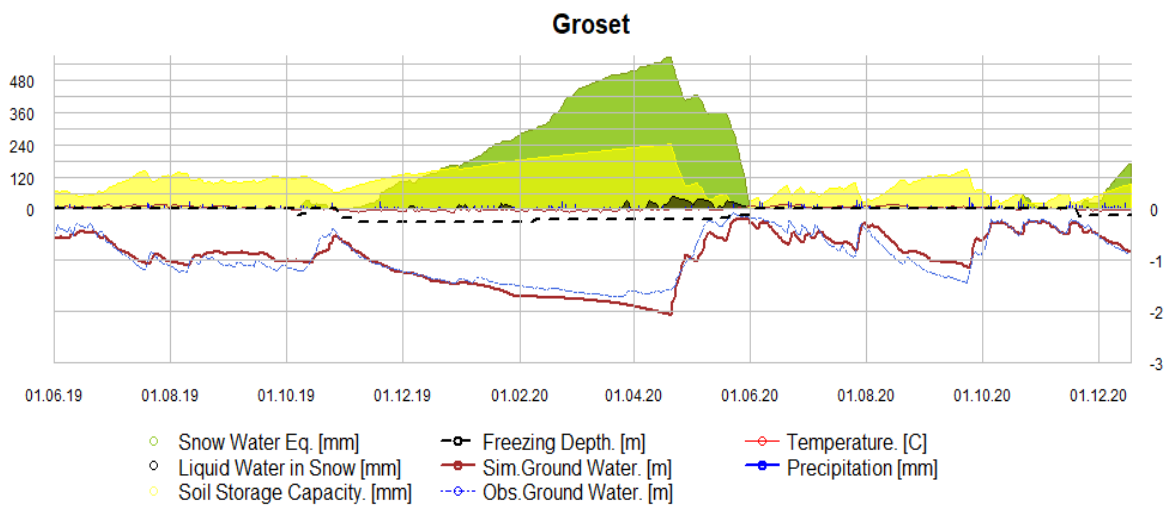

Figure 4. Simulation result plot at Groset ground water station.

7. Model Overview

MATLAB can call C/C++ or Fortran code directly by using its MEX (MATLAB executable) compiler. MEX files are dynamically loaded and allow external functions to be invoked from MATLAB, that means MEX files serve as wrapper for C or Fortran code to be used as MATLAB routines. The SoilFlow model runs in a MATLAB environment, where its main evaluating functions are coded in C and in the Livermore Solver for Ordinary Differential Equations (LSODE), ODE solver written in Fortran. These external functions are coupled with MATLAB by the MEX compiler (see the model overview, Fig. 5). The following steps shall be followed to install and run SoilFlow (assuming MATLAB is installed on your windows PC):

1. Install MinGW with the gcc and gfortran compiler and add the MINGW\bin directory to the windows path. Copy the SoilFlow files into a directory (for example C:\soilflow)
2. >> cd c:\soilflow
3. >> start

This should compile the C and Fortran files.

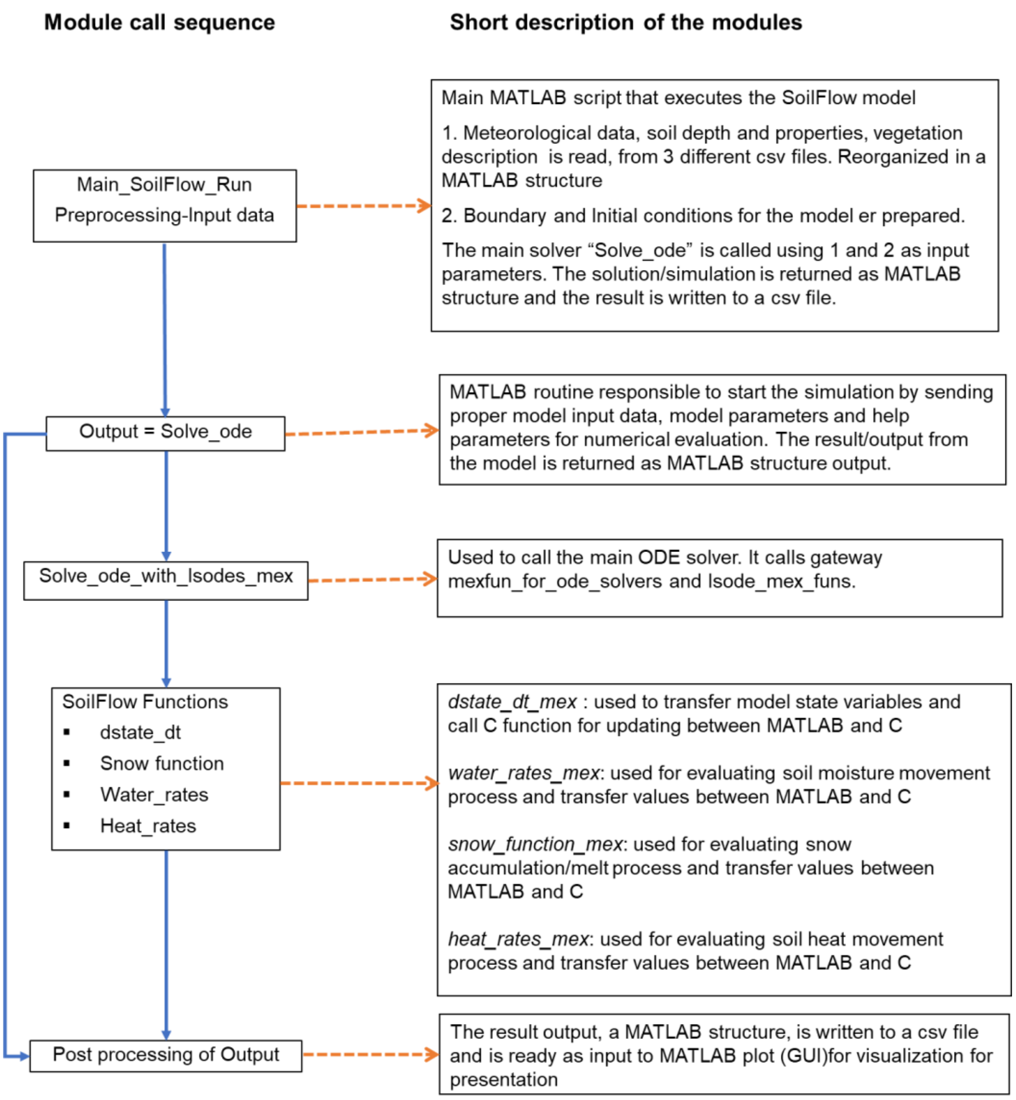

Figure 5. SoilFlow module overview.

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