Reanalysing a glacier mass balance measurement series - Ålfotbreen (1963-2010) and Hansebreen (1986-2010)

Bjarne Kjøllmoen

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Norges vassdrags- og energidirektorat
Middelthunsgate 29
Postboks 5091 Majorstua
0301 OSLO

Telefon: 22 95 95 95
Telefaks: 22 95 90 00
Internett: www.nve.no

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Reanalysing a glacier mass balance measurement series – Ålfotbreen (1963-2010) and Hansebreen (1986-2010)

Norwegian Water Resources and Energy Directorate
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Contents

Preface

This report documents the results from Ålfotbreen and Hansebreen and is part of a major reanalysis of mass balance measurements from ten glaciers in Norway with long time series. The time series are based on traditional glaciological observations using stakes and probings, as well as geodetic observations using aerial photogrammetry, laser scanning and maps.

This report is prepared and written by Bjarne Kjøllmoen. Contributions to the report were provided by Rune Engeset, Liss M. Andreassen and Hallgeir Elvehøy. The entire reanalysis is documented in a scientific paper in The Cryosphere: www.the-cryosphere.net/10/1/2016/.

We would like to thank Michael Zemp, Johannes Oerlemans, Andreas Kääb, Matthias Huss and Chris Nuth for valuable input during the progress of the analysis.

Oslo, March 2016

Morten Johnsrud
Director

Rune Engeset
Head of Section

Summary

The glaciological and geodetic methods provide independent observations of glacier balance. The glaciological method is based on annual surface mass balance measurements, whereas the geodetic method includes surface measurements, and internal and basal mass balance over a period of years. In this report, a re-analysing of Ålfotbreen and Hansebreen is performed. The analysed glaciological mass balance series cover the periods from 1963 to 2010 for Ålfotbreen and from 1986 to 2010 for Hansebreen. Within this period, usable Digital Terrain Models (DTMs) from 1968, 1988, 1997 and 2010 were available. Glaciological and geodetic mass balance were compared for the periods 1969-88, 1989-97 and 1998-2010 at Ålfotbreen and for the periods 1989-97 and 1998-2010 at Hansebreen.The re-analysis includes homogenization of both glaciological and geodetic observation series, uncertainty assessment, comparison of the glaciological and geodetic mass balance, and partly calibration of the mass balance series.

Three periods of data sets were compared and the results show significant discrepancies between the glaciological and geodetic methods for both Ålfotbreen and Hansebreen. Calibration was applied to Ålfotbreen 1998-2010 and Hansebreen 1989-1997 and 1998-2010, as the deviations were larger than the uncertainty.

The homogenized (1963-2010) and calibrated (1998-2010) glaciological cumulative mass balance for Ålfotbreen over 1963-2010 was −1.2 m w.e., while the original mass balance over the same period was +6.3 m w.e. The homogenized (1986-2010) and calibrated (1989-2010) glaciological cumulative mass balance for Hansebreen was −13.7 m w.e. The original mass balance over the same period was also −13.7 m w.e.

1 Introduction

1.1 Background

The Norwegian Water Resources and Energy Directorate (NVE) operate the Norwegian mass balance observation programme. The observations are both traditional field measurements, referred to as the “glaciological method” (also called direct, conventional or traditional method) and geodetic surveys, referred to as the “geodetic method” (Cogley et al., 2011. The programme includes 10 long time series: 6 glaciers with time series longer than 50 years and another 4 glaciers with time series with more than 23 years of data (Fleig et al., 2013) as per end of 2013.

In the joint paper from the workshop on “Measurement and Uncertainty Assessment of Glacier Mass Balance” at the Tarfala Research Station in northern Sweden (Nussbaumer et al., 2012) it is recommended that mass balance series longer than 20 year should be re-analysed (Zemp et al., 2013). This report describes the re-analysis of the Ålfotbreen and Hansebreen time series.

The glaciological mass balance method measures surface mass balance at point locations, and data are extrapolated over the entire glacier surface to obtain glacier-wide averages. The cumulative mass balance is the sum of the annual balances. In the geodetic method, cumulative balance is calculated from glacier surface elevations measured in different years by differencing digital elevation models (DEMs) and by converting the volume change to mass using a density conversion. The geodetic method is often used as check on the accuracy of annual measurements by the glaciological method (e.g. Andreassen, 1999 and Zemp, 2010). If a comparison between the direct and the geodetic method of a long-time series show great discrepancies, a calibration of the direct mass balance series is required.

1.2 Ålfotbreen

Ålfotbreen ice cap (61°45’N, 5°40’E) has an area of 10.6 km^2 (2010) and is, together with Blåbreen, the westernmost and the most maritime glacier in Norway (Fig. 1), except for some smaller ice patches to the west (Andreassen and Winsvold, 2012).

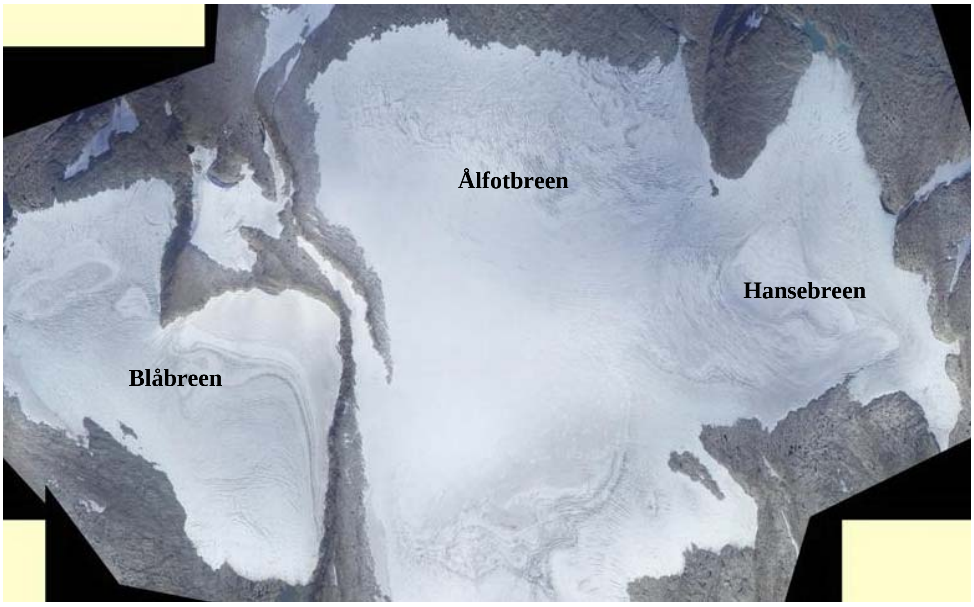


Figure 1
Orthophoto from 2010 of the Ålfotbreen ice cap, showing the two north-facing outlets Ålfotbreen and Hansebreen, and the nameless south-facing outlet. Blåbreen to the left is separated from Ålfotbreen.

It is a small north-facing ice cap resting on sandstones that have some characteristic staircase-like formations in the landscape (Fig. 2). The ice cap can be divided in three outlets, the two north-facing glaciers Ålfotbreen (4.0 km^2) and Hansebreen (2.8 km^2), and the nameless south-facing outlet (Fig. 1). Ålfotbreen extends from 1368 down to 890 m a.s.l., and Hansebreen from 1310 down to 927 m a.s.l. Both glaciers have a smooth and sloping surface with some crevasses.

Figure 2
The ice caps Blåbreen (in front) and Ålfotbreen. The characteristic staircase-like sandstone formations east of the ice caps can be seen in the background. Photo: Fjellanger Widerøe AS.

1.3 Previous results

The two adjacent glaciers Ålfotbreen and Hansebreen have been subject for annual direct glacier surface mass balance measurements since 1963 (Østrem and Liestøl, 1964) and

1986 (Laumann et al., 1988), respectively. The measurements at Ålfotbreen and Hansebreen are funded by Sogn og Fjordane Energi AS. The results for Ålfotbreen show a slight surplus from 1963 to 1988 and a heavy surplus from 1989 to 2000. Hansebreen was about in balance from 1986 to 2000. Over the years 2001-2013 however, the measurements reveal a distinct deficit.

Ålfotbreen/Hansebreen has been surveyed by aerial photography several years since 1945. Detailed glacier maps have been constructed from the photographs taken in 1968, 1988 and 1997. In 2010, the glaciers were surveyed with Light Detection And Ranging (LIDAR), also called laser scanning, combined with aerial photography.

Re-analysing glacier mass balance series is recommended as standard procedure for every mass balance monitoring programme with increasing importance for long time series (Zemp et al., 2013). Previous studies have shown that the direct/glaciological and geodetic method differs for Ålfotbreen (Østrem and Haakensen, 1999).

In this study the geodetic mass balance at Ålfotbreen and Hansebreen is determined from 1968 (photogrammetric), 1988 (photogrammetric), 1997 (photogrammetric) and 2010 LIDAR). The glaciological and geodetic mass balances were compared for the periods 1968-88, 1988-97, 1997-2010 and 1968-2010.

Following Zemp et al. (2013) the re-analyses include homogenization for identifying and removing artefacts and biases to achieve consistent observation series, uncertainty assessment of systematic and random errors for the glaciological and the geodetic balances, and, in cases of unexplained discrepancies, adjusting the (annual) glaciological to the (multi-annual) geodetic balances.

1.4 Outlook

In this report the mass balance measurements at Ålfotbreen and Hansebreen are reanalysed following the reanalyses scheme proposed by Zemp et al. (2013). The major steps are:

1. Analysis and scrutiny of glaciological and geodetic measurements (ch. 2)
2. Homogenization of glaciological and geodetic measurements (ch. 3)
3. Uncertainty assessment (ch. 4)
4. Validation of glaciological measurements against geodetic measurements (ch. 5)
5. Calibration of glaciological measurements (ch. 5)

The output of the re-analysis is a *homogenized glaciological mass balance time series* with an uncertainty assessment, and if calibration is required, a *calibrated glaciological mass balance time series*.

This report is part of an extensive work described in Andreassen et al. (2016).

2 Observations

2.1 Geodetic mass balance

Aerial photographs of Ålfotbreen were taken frequently between 1945 and 1997. The latest aerial photographs were taken in 2010. The quality of the photographs is most variable. Fresh snow on the glacier surface and lack of full coverage is typical for several of the photographing. Detailed topographic maps were constructed from photographs taken in July 1955, August 1968, September 1988 and August 1997. The latest map was constructed from laser scanning and aerial photographs taken in September 2010. The map from 1955 was constructed from photographs taken eight years before the mass balance measurements began and was, hence, not included in this study.

2.1.1 Mapping 1968

Vertical aerial photographs were taken from a flying height of 6200 m a.s.l. on 5th August 1968 by Widerøes Flyveselskap A/S (contract No. 3210). The photography was originally done for the Norwegian Geographical Survey in order to construct a topo-graphic map at scale 1:50.000. The photogrammetric plotting, however, was made solely for producing a glacier surface map. The original construction was made in 1969. Some years later however, it was proved that this construction was encumbered with some great errors related to the geodetic network. Therefore, a new digital map construction based on corrected control points was made by Fjellanger Widerøe AS (earlier Widerøes Flyve-selskap A/S) in 1992. At the time of photography, the entire glacier surface was still covered with snow from the last winter. Constructing contour lines over snow-covered surfaces are difficult, and can even be impossible if the surface is covered of fresh snow. Documentation describing the quality of the air photographs, taken with a mean scale of 1:33 000, is lacking, but since the glacier surface was covered with snow it is assumed that the quality is just passable for the purpose. However, for the original map plotting made in 1969, the height accuracy was reported to be better than ±2 m (relative) and ±5 m (absolute), respectively (brief comments on the printed map). Maximal error in horizontal direction was assumed as better than 5 metres.

Digital contour lines with 10 m contour intervals, and border outlines from the new constructed 1968-map were originally constructed in the Norwegian reference frames NGO 1948 (horizontally) and NN 1954 (vertically). Later the horizontal co-ordinate system was transformed to UTM, zone 32, Euref 89 datum. The data processing from digital contour lines to Digital Elevation Model (DEM) was performed using a Geographical Information System (GIS) with Surfer software version 8 (Golden Software, Inc. 1999). The gridding method applied to the 1968 data set was Kriging, which is a flexible method and useful for gridding almost any type of data set. The x and y co-ordinates in the regular grid end always in 0 (e.g. 324510, 6851560).

The glacier outlines plotted by the mapping company were some insufficient and was lately modified by NVE.

2.1.2 Mapping 1988

Vertical aerial photographs were taken from a flying height of 6200 m a.s.l. on 7th September 1988 by Fjellanger Widerøe AS (contract No. 9678). The photography was originally done for the Norwegian State Power Authority in connection with possible further power development in the area. The digital photogrammetric plotting, however, was made solely for glaciological purposes by Fjellanger Widerøe AS in 1992. The digital contour lines with 10 m contour intervals were constructed in the co-ordinate system UTM, zone 32, Euref 89 datum and height system NN 1954. The 1988 winter season was snow poor and the following summer season was warm. Accordingly, at the time of photography no snow from the last winter was remaining on the glacier surface. Hence, the quality of the air photographs, taken with a scale of 1:30.000, was considered to be excellent and appropriate for the purpose. The accuracy of the constructed contour lines was estimated to be better than 0.40 m in horizontal direction and better than 0.75 m in vertical direction (brief comments on the printed map). The data processing and method used from digital contour lines to DEM was similar to the 1968 mapping.

The glacier outlines were plotted by the mapping company.

2.1.3 Mapping 1997

Vertical aerial photographs were taken on 14th August 1997 (contract No. 12177). The photography and the digital photogrammetric plotting were made solely for glaciological purposes by Fjellanger Widerøe AS in 2001. While the 1968 and 1988 height data sets were digital contour lines, the 1997 data set contains interpolated data points regularly distributed in 10x10 m grid over the extent of the mapping area. The co-ordinate and height system is similar to the previous mappings. The quality of the air photographs, taken with a scale of 1:30.000, was considered to be fairly good. A snow-rich winter was compensated by a following warm summer. At the time of photography, the glacier surface was still snow covered over most of the area. In spite of that, the texture in the snow surface was distinct. The accuracy of the digital data set was not quantified, but it was considered to be reasonable. In the quotation, however, the mapping company estimates an expected accuracy of ±1.3 m dependent on the photograph quality and contrast. Whether this estimate is horizontal or vertical accuracy is not given. The data points (x and y co-ordinates) in the regularly grid supplied from the mapping company do not end in 0. Thus, the grid points were re-gridded ensuring co-ordinates ending in 0. Different gridding methods were tested with approximately similar result, and hence, the Kriging method was continued.

The glacier outlines were originally plotted by the mapping company. Due to several snow patches along the ice brim, some misinterpretations of the glacier outline were detected and modified.

2.1.4 Mapping 2010

Vertical aerial photographs and LIDAR data were recorded simultaneously on 2nd September 2010 by Terratec AS (Terratec AS, 2010).

The photographs were recorded by a Rollei metric ATC modular digital camera with a resolution of 60 megapixels. The mean flying height was 3000 m above ground level and the picture resolution is 25 cm GSD (Ground Sampling Distance). The resulting resolution of the orthophoto was given as 20 cm.

The LIDAR data was acquired using a Leica ALS50-II lidar instrument. The laser pulse rate was 81.100 Hz and the scan angle ±20 degrees, resulting in a mean point density of 0.5 points per m^2.

The digital data set supplied from the mapping company was point cloud, x-, y- and z-data in a regular grid (10x10 m), glacier outlines and orthophoto. The gridding method applied for the regular grid data was Triangulated Irregular Network (TIN). The expected accuracy of the LIDAR data was given as ±10-20 cm by the mapping company. The regular grid data set used in the following calculations, however, was based on the point cloud and gridded by Kriging.

The glacier outlines supplied from the mapping company were a little too much detailed, and hence, based on the orthophoto it was modified by NVE for the further calculations.

2.1.5 Density conversion factor

Determination of a density conversion factor is required in order to convert the volume change of snow, firn and ice to mass change. It is common to assume a constant density profile in the accumulation area, following Sorge's Law (Bader, 1954). Hence, density of glacier ice, 900- 917 kg m^{-3} (Cuffey and Paterson, 2010), is often used for the conversion (e.g. Haug et al., 2009 and Andreassen, 1999). This assumption however, is valid only under steady-state conditions and was considered to be a maximum estimate in this study. Assuming a value of 850 ±60 kg m^{-3} to convert volume change to mass change was found to be appropriate for a wide range of conditions (Huss, 2013). Hence, this value was used for the conversion of the volumetric changes into water equivalent.

2.1.6 Adjustment for different dates

Comparison of glaciological and geodetic mass balance required an adjustment because the field measurements and aerial surveys were acquired at different dates. The related error depends on the changes in surface elevation between the field and aerial surveys. Accordingly, increasing time span will result an increasing difference. The season (summer/autumn) and the general mass turn over will also influence the difference. Dates for field measurements and aerial surveys and corresponding adjustments are shown in table 1.

Table 1
Survey dates and adjustments for 1968, 1988, 1997 and 2010.

year	date		adjustment (m w.e.)		
	aerial survey	field survey	category	Ålfotbreen	Hansebreen
1968	5th Aug	30th Sep	melting	-1.46	
1988	7th Sep	22nd Oct	melting	-0.52	-0.57
1997	14th Aug	20th Nov	melting	-1.14	-1.09
2010	2nd Sep	28th Sep	melting	-0.45	-0.51

For all years, the aerial survey was performed some weeks before the ablation measurements in fall. The melting for the intermediate periods was estimated using a simple equation based on air temperature from a nearby climate station. Fresh snow at the time of ablation measurements was included in the winter balance for the subsequent year and was, hence not taken into account in this adjustment. According to the estimated melting from the aerial survey dates to the field survey dates, the geodetic mass balances were adjusted as:

$$– \text{date adj.}_{yearI} + \text{date adj.}_{yearII}$$

2.1.7 Glacier boundaries

As described in chapter 2.1.3 the hydrological and the glaciological basins were considered to be identical for both Ålfotbreen and Hansebreen and the ice divide from 2010 was used for all four DEMs. However, different interpretations and veritable changes of the ice margin reveal different drainage basins for the four years. The basin areas are 4.49 km^2 (1968), 4.17 km^2 (1988), 4.48 km^2 (1997) and 3.97 km^2 (2010) for Ålfotbreen and 3.42 km^2 (1968), 3.07 km^2 (1988), 3.18 km^2 (1997) and 2.75 km^2 (2010) for Hansebreen. For the geodetic volume change, calculations a combination of the glacier boundaries was used so that the analyses mask will surround both the glacier areas.

2.2 Glaciological mass balance

Glacier surface mass balance at Ålfotbreen has been monitored annually since 1963. The adjacent glacier in east, Hansebreen, has been measured since 1986. The measurements have been carried out by the Norwegian Water Resources and Energy Directorate (NVE). The extent of measurements has varied over time, and different methods of calculation have been used. The measurements and calculations are in principle based on methods from Østrem and Brugman (1991) and as described in Andreassen et al. (2005) and Kjøllmoen et al. (2011).

The measurements are reported in "Glaciological investigations in Norway", which are annual or multi-year reports published by NVE. The 2010 measurements were reported in Kjøllmoen et al. (2011). Measurements from 2011 and later are due for publication in a multi-year report when the re-analysis is finished.

2.2.1 Monitoring program and field measurements

The mass balance measurements at Ålfotbreen started in May 1963 (Østrem and Liestøl, 1964) and annual measurements have been conducted every year since then. The direct mass balance records are based on annual measurements of winter balance and annual balance. The winter balance measurements were usually carried out from mid April to early May, while the annual balance was normally measured from late September to late October. Winter balance was measured using a number of stakes, as well as doing a number of snow depth soundings to the late-summer surface from previous year. In addition to snow depth, snow density was measured in one or more vertical profiles. Usually the snow density measurements were done at the same time as the snow depth measurements. In some years between 1966 and 1980 however, the snow density was measured two or three times during the winter season. As an example, for the winter season 1977-78, snow density was measured in December 1977 and January and April 1978. The density measurements in January 1978 included the snow accumulation from December 1977 and the density measurements in April 1978 included the accumulation from January. Annual balance was measured by stake readings.

The number of stakes and snow depth measurements has varied a considerably over the years from the beginning in 1963 to present.

In the 1960s and 1970s, a network of 13-19 stakes was maintained on Ålfotbreen (Fig. 3). The number of snow depth measurements varied from 20 in 1965 and 1967 to 266 in 1969 with an average of 124 measurements in the first two decades. The snow depths were sounded in more or less fixed profiles across the entire glacier surface. The snow density was measured in 1-3 locations over the first ten years, but solely in one position from 1973 to present, at about 1225 m a.s.l. (Fig. 3).

From 1982, the measurement programme at Ålfotbreen was simplified (Fig. 3). The number of stakes was then reduced to 4-8 and has been standardised to 4-6 stakes since 1989. The number of snow depth measurements was at a minimum of 32 in 1982, but increased gradually to 160 in 1988. At Hansebreen the stake network was rather sparse (1-3 stakes) already from the beginning in 1986 until 2000. Since then 4-5, stakes have been maintained. The number of snow depth measurements increased from 38 in 1986 to 112 in 1988.

Between 1989 and 1995, several winters were extremely snow-rich. Over these years maintaining the stake network was demanding and the snow depth measurements were difficult and time-consuming. Thus, the number of snow depth soundings was reduced to 21-73 on Ålfotbreen and 11-27 on Hansebreen (Fig. 3). The number of stakes, however, was maintained through these years as well.

From 1996 to 2008, the number of snow depth measurements varied between 47 and 138 on Ålfotbreen and between 42 and 80 on Hansebreen.

Up to 2008, the snow depths were measured in points along straight line profiles. From 2009, the profile system was replaced with a grid system of 250 x 250 metres amounting to ca. 75 gridpoints at Ålfotbreen and ca. 50 gridpoints at Hansebreen (Fig. 3).

As the snow density measured at Ålfotbreen was considered to be representative for both glacier basins, the density was not measured at Hansebreen.

Typical networks representing the periods 1963-81 (1972), 1982-88 (1987), 1989-95 (1993), 1996-2008 (2002) and 2009-13 (2012) are shown in figure 3.

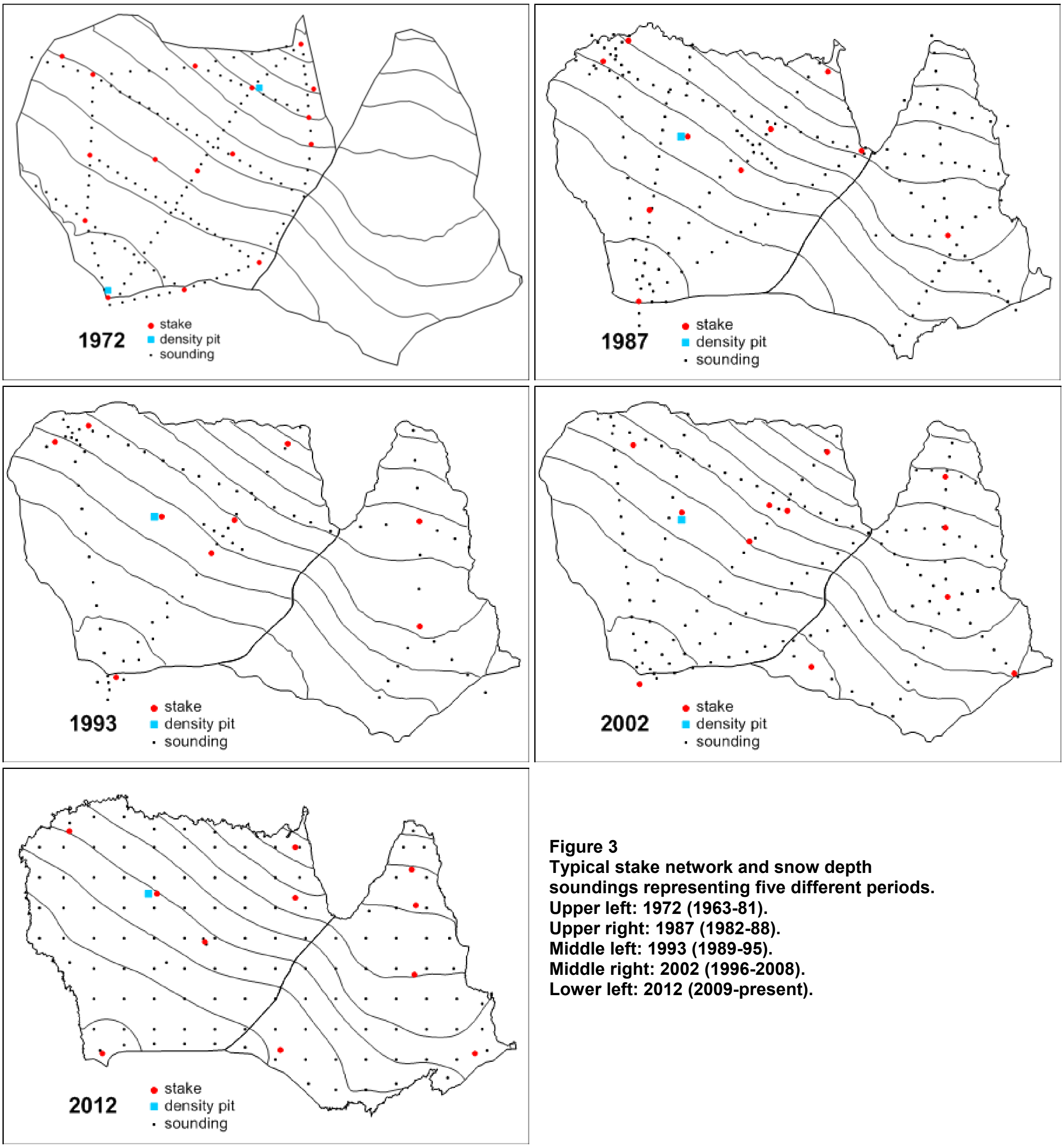


Figure 3
Typical stake network and snow depth soundings representing five different periods.
Upper left: 1972 (1963-81).
Upper right: 1987 (1982-88).
Middle left: 1993 (1989-95).
Middle right: 2002 (1996-2008).
Lower left: 2012 (2009-present).

2.2.2 Mass balance calculation

The mass balance was calculated using a stratigraphic method, i.e. between two successive "summer surfaces" (surface minima) as described in Østrem and Brugman (1991). The spatial interpolation of point measurements was done by making winter balance distribution map or estimating winter balance/annual balance in elevation intervals of 50 m vertical resolution.

From 1963 to the end of the 1980s, the spatial distribution of the mass balance was inter-/extrapolated manually from the point measurements. Isolines with a distance of 250, 500 or 1000 mm w.e. were drawn for both winter and summer balance (Fig. 4). The calculation of the total mass balance was based on the area between these lines. The areas between adjacent isolines within each height interval (50 m) were integrated using a planimeter and the total amount of accumulation and ablation was calculated for each height interval. Then altitudinal balance values $B_w(z)$, $B_s(z)$ and $B_a(z)$ were calculated.

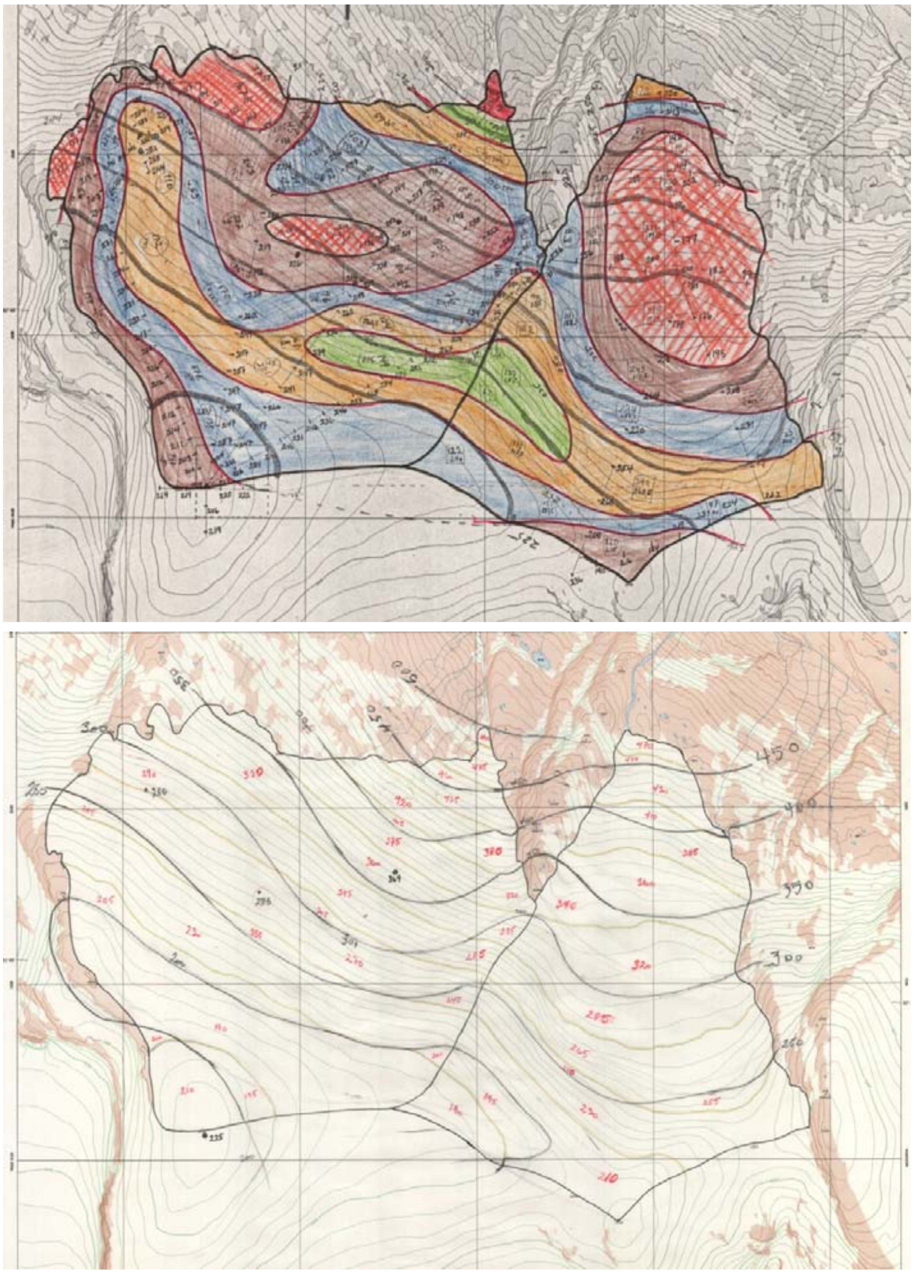


Figure 4
From 1963 to the end of the 1980s the spatial distribution of the mass balance was manually drawn for both winter and summer balance. The maps show the winter (upper) and summer balance (lower) from 1986.

Since 1989, the altitudinal mass balance curves have been made by plotting point measurements of winter, summer and annual balance versus altitude, and extracting representative values for each 50-m elevation interval from the scattered points (Fig. 5). The entire glacier surface was well represented of measurements most of the years.

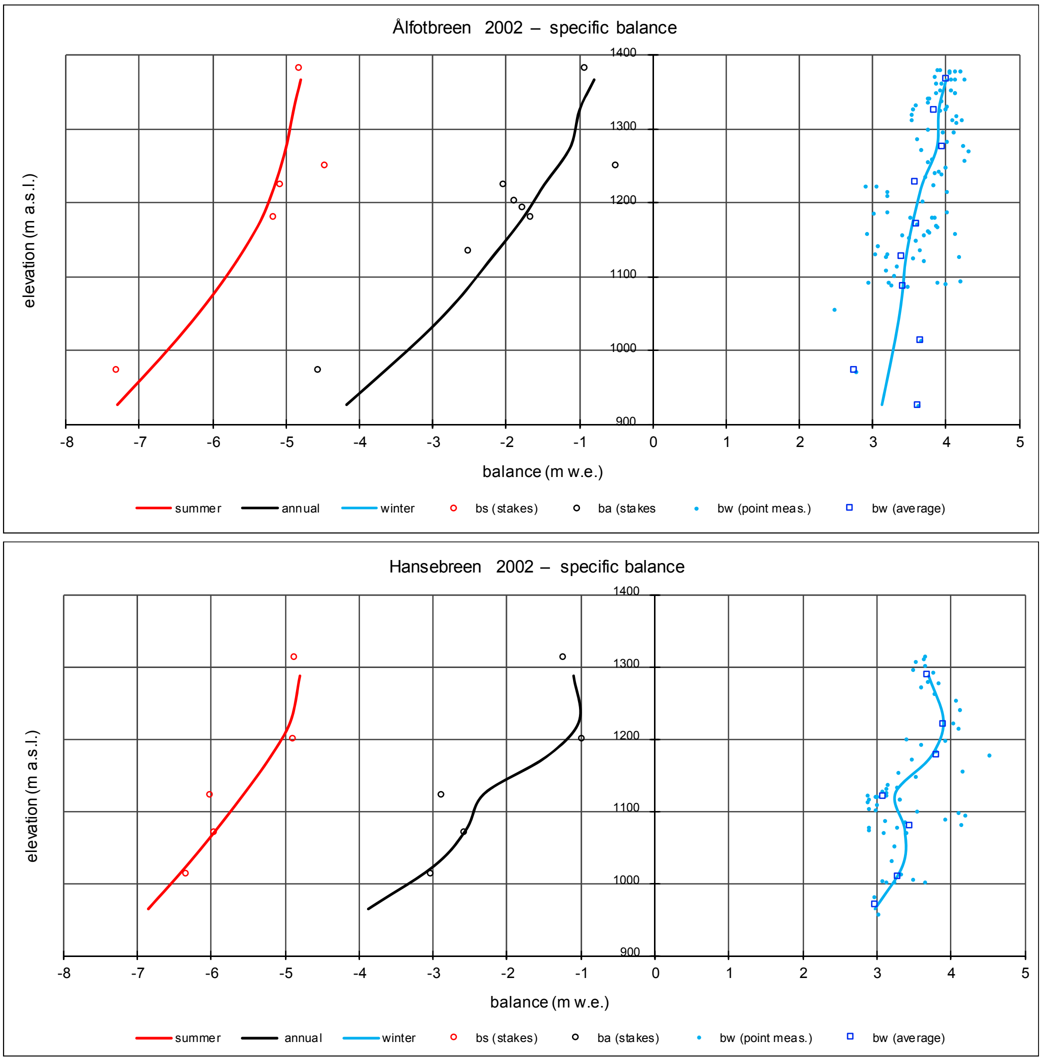


Figure 5
The altitudinal winter, summer and annual balance curves are plotted versus altitude. Point values for b_w (○), b_s (○) and b_a (•), together with average b_w (□) for each 50 m height interval are also plotted. This calculation method has been used since 1989. The diagrams above are from Ålfotbreen (upper) and Hansebreen (lower) in 2002.

2.2.3 Glacier boundaries

For both Ålfotbreen and Hansebreen the hydrological and glaciological basins were considered to be identical. Hence, the issue of which drainage basin used can be ignored. The drainage divides, or rather the ice divides, were solely calculated from the mapped glacier surface topography. The ice divide was calculated for the DEMs from 1955, 1968, 1988, 1997 and 2010. A comparison of these five ice divides showed only marginal differences. The ice divide from the DEM_{2010} was considered to be most accurate and was used in the further study. The glacier outlines and ice divides from the DEM_{2010} are shown in figure 6.

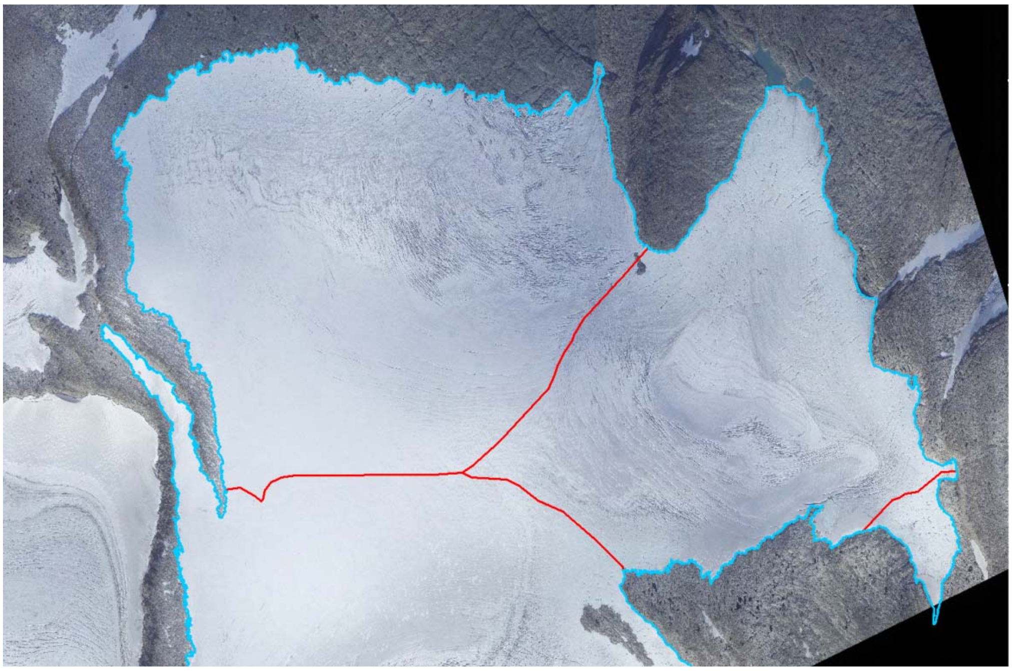


Figure 6
Glacier outlines (blue) and ice divides (red) for the northern part of the ice Ålfotbreen ice cap based on aerial photos and laser scanning (LIDAR) in 2010.

In the reported datasets as published up to and including 2010 (Kjøllmoen, 2011), the mass balance calculations are based on the height-area distribution from the 1955, 1968, 1988, 1997 and 2009 maps (Fig. 7). As shown in the figure, there are considerable time lags between the mass balance year and the reference area used for calculating mass balances. When a new map was available, it was used for the calculations from then and onwards.

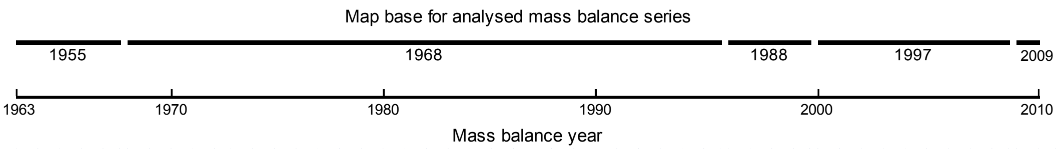


Figure 7
Upper line indicates map base used in the reported mass balance series. Years denote year of validity period for each map.

2.2.4 Glaciological mass balance series

Ålfotbreen

The original glaciological mass balance over the period 1963-2010 was calculated and reported to a surplus of 6.3 m w.e. The results show a distinct volume increase from 1971 to 1976 (+7.0 m w.e.) and a considerable increase from 1989 to 2000 (+11.6 m w.e.). Over the years 1963-70 and 1977-88, the measurements reveal only small changes amounted as −3.1 and −0.3 m w.e., respectively. From 2001 to 2010, the results show a distinct volume decrease of −8.8 m w.e. The mean winter, summer and annual balance for 1963-2010 is 3.70, −3.57 and +0.13 m w.e., respectively. The annual winter, summer and annual mass balance results from 1963 to 2010 are shown in figure 8.

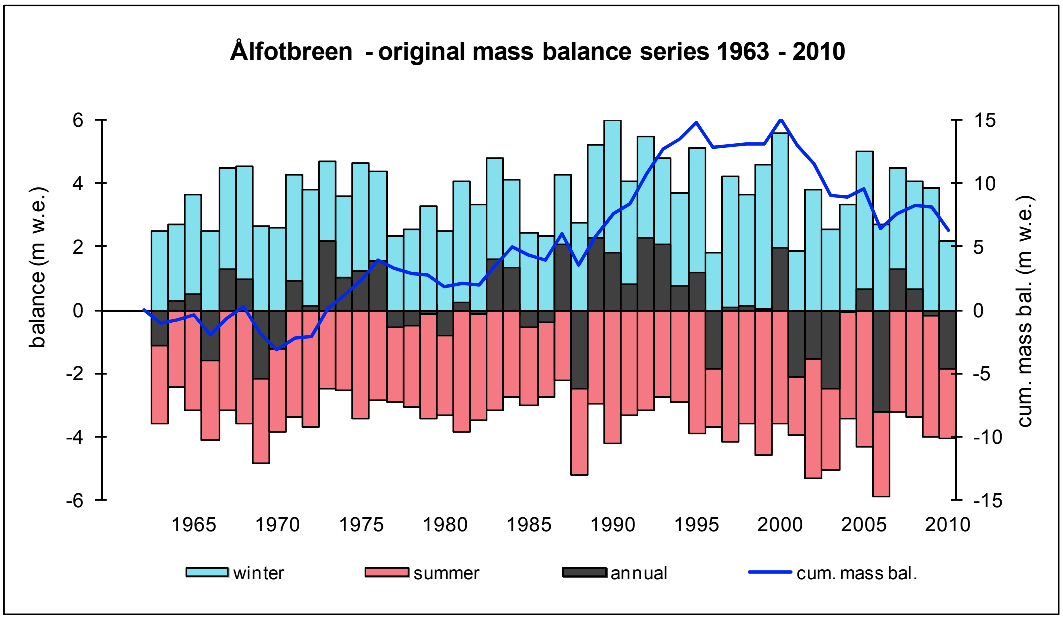


Figure 8
Original winter, summer and annual mass balance for Ålfotbreen over the period 1963-2010.

Hansebreen

The original glaciological mass balance over the period 1986-2010 was calculated and reported to a deficit of 13.7 m w.e. From 1986 to 2000, the cumulative mass balance was about 0. After 2000 however, the results show a distinct volume decrease up to 2010 with −14.0 m w.e. The mean winter, summer and annual balance for 1986-2010 is 3.42, −3.97 and −0.55 m w.e., respectively. The annual winter, summer and annual balance results from 1986 to 2010 are shown in figure 9.

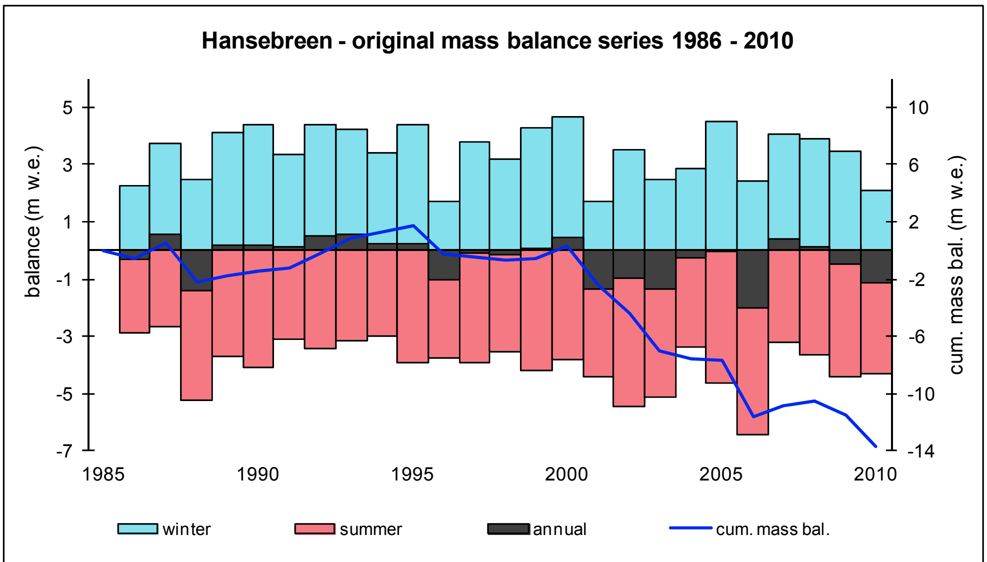


Figure 9
Original winter, summer and annual mass balance for Hansebreen over the period 1986-2010.

3 Homogenization

3.1 Geodetic mass balance

The accuracy of the final DEMs is principally influenced by the quality of the raw data and by the process from raw data to DEM. The raw data acquisition and the DEM processing were quite different for the years 1968, 1988, 1997 and 2010, respectively.

3.1.1 Mapping 2010

The 2010 DEM was based on raw data acquired by LIDAR (see chap. 2.1.4). Typical sources of errors are the accuracy of the ground control points, the glacier topography (steep slopes) and the meteorological/hydrological conditions (snow-covered surface). All these sources will influence the accuracy of the final DEM. The surface topography, which is general for all four years, is gently sloping over the entire area.

Generally, the accuracy of data sets acquired by LIDAR is of high quality and was estimated to be ±10-20 cm (Terratec, 2010). The accuracy of the 2010 LIDAR data was documented by comparing the original LIDAR data set with nine control points measured with GNSS on the glacier surface, and with three fixed points measured with GNSS in non-glacierized areas. As the x and y co-ordinates of the LIDAR data set versus GNSS points/fixed points were not exact equal, interpolated values from a 0.5x0.5 m grid were extracted from the LIDAR data set.

GNSS on the glacier surface

The control points on the glacier surface were measured on 12th August, while the LIDAR data was acquired on 2nd September. Based on stake readings on 12th August and 28th September, and air temperature from two climate stations, the elevation change for the control points were estimated for the period from 12th August to 2nd September. The impact of a potential vertical ice motion was not considered. The results from the comparison are shown in table 2 and figure 10. The accuracy of the measured control points was assumed as ±0.2 m. The differences (Diff. = $\text{Height}_{\text{adj.}} - \text{Height}_{\text{DEM}}$) were between +0.46 and +1.38 m with an average of +0.77 m.

Table 2
Comparison of the glacier surface elevation between control points measured with GNSS and interpolated values from a 0.5 x 0.5 m grid extracted from the original LIDAR data set. The surface elevations measured on 12th August ($\text{Height}_{\text{GNSS}}$) were adjusted to elevations related to 2nd September ($\text{Height}_{\text{adj.}}$).

Point No.	North	East	$\text{Height}_{\text{GNSS}}$	$\text{Height}_{\text{adj.}}$	$\text{Height}_{\text{DEM}}$	Diff. (m)
12-09	6 851 868.72	323 891.63	953.76	952.78	952.00	0.78
12-10	6 851 764.53	323 890.90	977.56	976.59	975.72	0.87
13-05	6 851 302.80	324 053.01	1 064.57	1 063.68	1 063.18	0.50
13-10	6 851 323.76	323 908.57	1 071.41	1 070.56	1 069.18	1.38
50-08	6 851 576.66	324 884.16	1 004.66	1 003.76	1 003.04	0.72
37-10	6 851 359.32	322 769.37	1 205.13	1 204.28	1 203.82	0.46
80-03	6 850 706.44	324 911.14	1 100.60	1 099.68	1 099.00	0.68
60-09	6 851 265.03	324 915.33	1 044.72	1 043.90	1 043.00	0.90
28-02	6 850 975.75	323 143.89	1 220.45	1 219.76	1 219.13	0.63

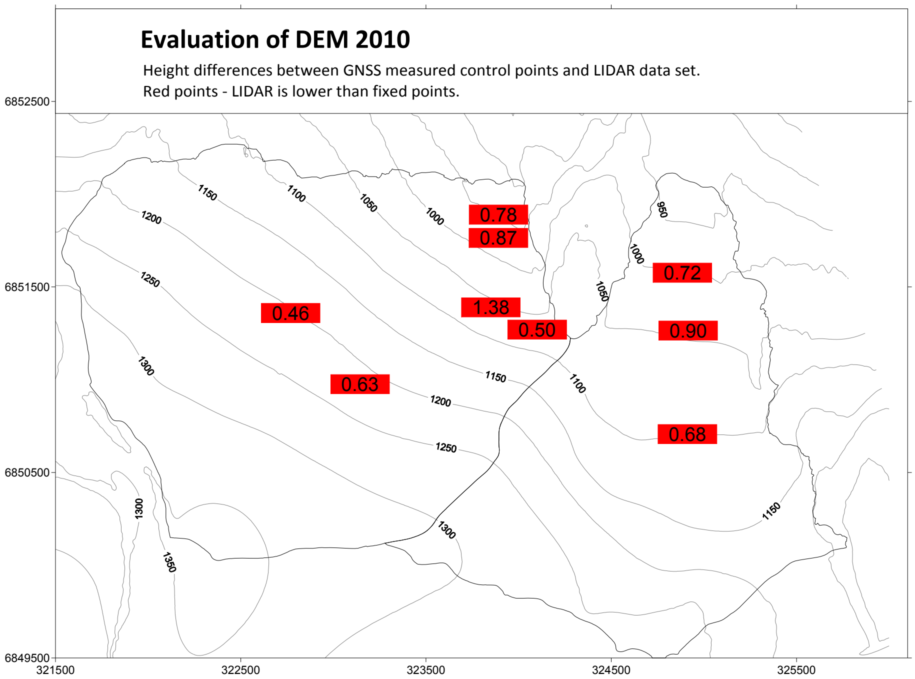


Figure 10
Spatial distribution of the nine control points measured on the glacier surface on 12th August.

Fixed points

Interpolated values from the 2010 LIDAR data were also compared with three fixed points in non-glacierized areas. The fixed points were measured with GNSS in August 2000 and the accuracy is estimated as ±0.03 m and ±0.05 m, respectively. The differences (Diff. = $\text{Height}_{\text{fix pt}} - \text{Height}_{\text{DEM}}$) were between +0.18 and +0.81 m. The accuracy of the control points was assumed to be better than ±0.15 m. The results are given in table 3 and figure 11.

Table 3
Comparison of three fix points with corresponding point positions extracted from the 2010 LIDAR data set in non-glacierized areas.

Point No.	North	East	$\text{Height}_{\text{fix pt}}$	$\text{Height}_{\text{DEM}}$	Diff. (m)
$\text{B29PNVE2}_{\text{signal}}$	6 852 179.2	324 537.3	989.14	988.96	0.18
B29T198	6 850 154.4	325 893.8	1 280.99	1 280.41	0.58
ÅLF100	6 852 194.2	322 065.0	1 166.21	1 165.40	0.81

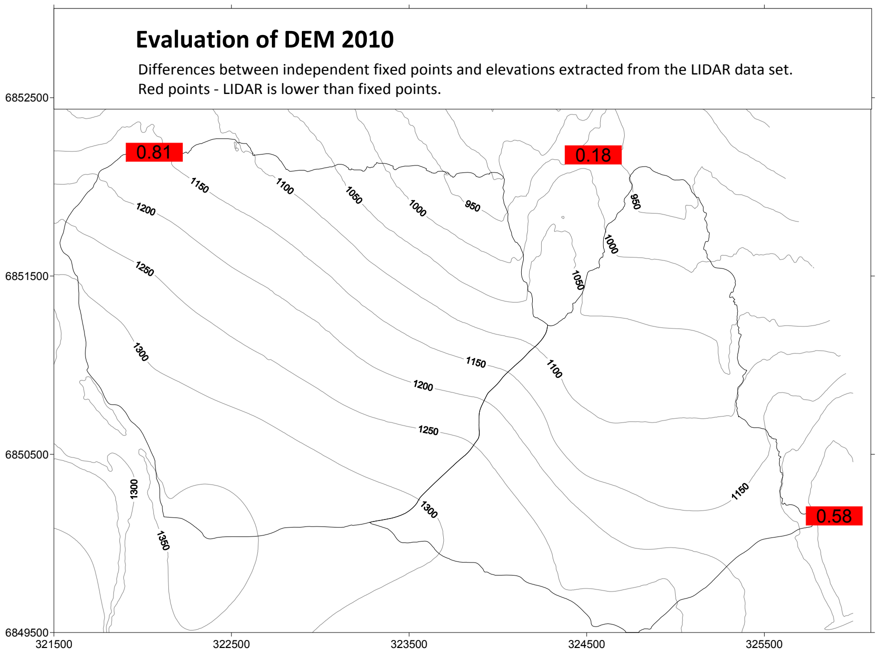


Figure 11
Spatial distribution of three fix points in non-glacierized areas compared with corresponding point positions extracted from the 2010 LIDAR data set.

Mapping 2010 – evaluation summary

The evaluation based on the GNSS measurements on the glacier surface revealed differences up to 1.38 m. Due to the time lag between the GNSS measurements and the LIDAR acquisition, the estimated surface elevation change from 12th August to 2nd September is an uncertain factor and can possibly explain some of the relative great differences. The comparison based on the fixed points gave differences up to 0.81 m.

Furthermore, the LIDAR points and the derived grid points will not match the ground control points. The control points in non-glacierized areas are typically located on the topmost point of a peak or a hill, usually represented by a bolt (3-5 cm high). The LIDAR points will only exceptionally hit the bolt. The impact of this mismatching will be lower elevation of the interpolated LIDAR points compared with the corresponding fixed points.

Generally the two independent evaluations showed usable accordance between the control points and the LIDAR data with mean differences less than 1 m. Accordingly, the quality of the LIDAR data was considered as good, and correction of the 2010 DEM was not necessary.

3.1.2 Mapping 1997

The 1997 DEM was based on raw data acquired by aerial photography. Typical sources of errors were the accuracy of the ground control points, the glacier topography (steep slopes) and the meteorological/hydrological conditions (snow-covered surface). All these sources will influence the accuracy of the final DEM.

The 1997 DEM was generated in 2001 and the generation was based on six ground control points with accuracy better than 0.05 m in both horizontal and vertical direction. At the time of photography, the glacier surface was still snow covered over most of the area. However, due to shadow effects, the snow surface was distinct and the photographs were considered to be of reasonable quality for the purpose.

The 1997 data set supplied from the mapping company was interpolated x, y and z data points in a regular grid of 10x10 m. As the veritable raw data set (irregularly distributed data points) was not available, a direct evaluation of the raw data was not possible without a new map construction, which was not reasonable in this study. The accuracy was not quantified properly, but only estimated to an expected accuracy of ±1.3 m.

At the time of photographing (14th August 1997) no independent surveying were done. However, the accuracy of the 1997 DEM was investigated by comparing the DEM with fixed points and with the 2010 DEM in non-glacierized areas.

Fixed points

The 10 x 10 m DEM was compared with three independent fixed points in non-glaciated areas. The fixed points and the method are similar to the evaluation of the 2010 DEM. The comparison showed differences (Diff. = $Height_{fix\ pt} - Height_{DEM}$) within ±1.9 m. Results from the comparison are shown in table 4 and figure 12.

Table 4
Comparison of three fixed points with corresponding co-ordinates extracted from the 1997 DEM in non-glaciated areas.

Point No.	North	East	$Height_{fix\ pt}$	$Height_{DEM}$	Diff. (m)
$B29PNVE2_{signal}$	6 852 179.2	324 537.3	989.14	989.56	-0.42
B29T198	6 850 154.4	325 893.8	1 280.99	1 279.94	1.05
ÅLF100	6 852 194.2	322 065.0	1 166.21	1 164.26	1.95

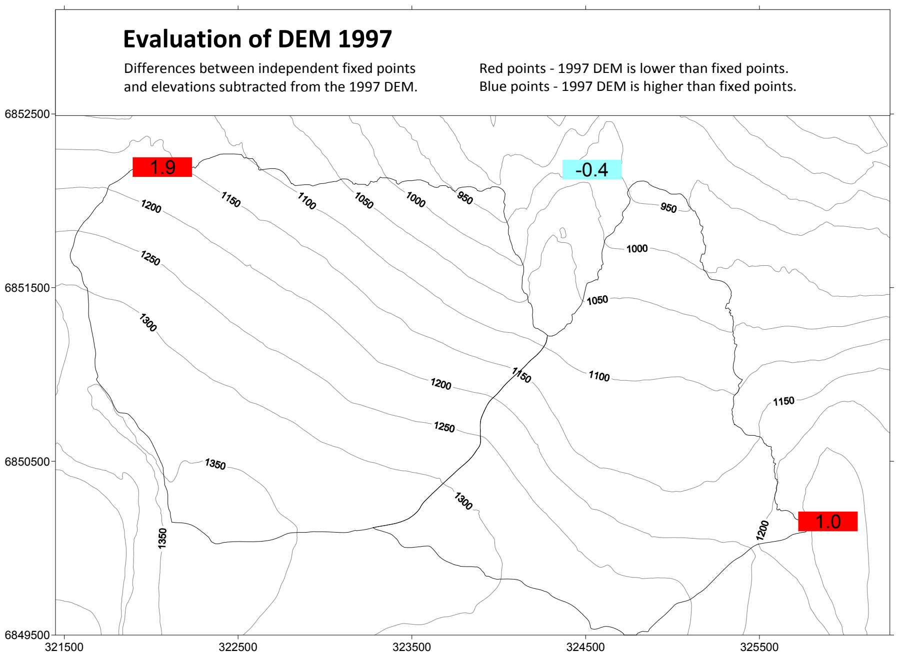


Figure 12
Spatial distribution of three fixed points in non-glaciated areas compared with corresponding co-ordinates extracted from the 1997 DEM.

DEM 2010

The quality of the 1997 DEM was also compared with the 2010 DEM in non-glaciated areas. For this evaluation, the 2010 DEM was considered as the reference DEM. Ideally the non-glaciated terrain from the 1997 DEM should correspond exactly with the 2010 DEM. Due to the inaccuracies, however, elevation differences will always occur when comparing two DEMs.

For the 1997 DEM, evaluation congruent non-glaciated terrain was located north and east of the glacier (Fig. 13). The grid elevation data from the 1997 DEM were subtracted from the corresponding grid elevation values extracted from the 2010 DEM. The results from 17405 points were first compared. The differences (Diff. = $DEM_{1997} - DEM_{2010}$) were between −12.9 and +8.6 m with an average of +0.8 metres.

Comparing elevation values in steep areas is considered to be very uncertain and should preferably be avoided. Thus, all difference values located in areas steeper than 30° were removed. Accordingly, the results from the 16378 remaining points showed differences from −12.2 to +9.3 m with an average of +0.7 m. Generally, the results indicated that the 1997 DEM is approximately ¾ m higher than the 2010 DEM (Fig. 13).

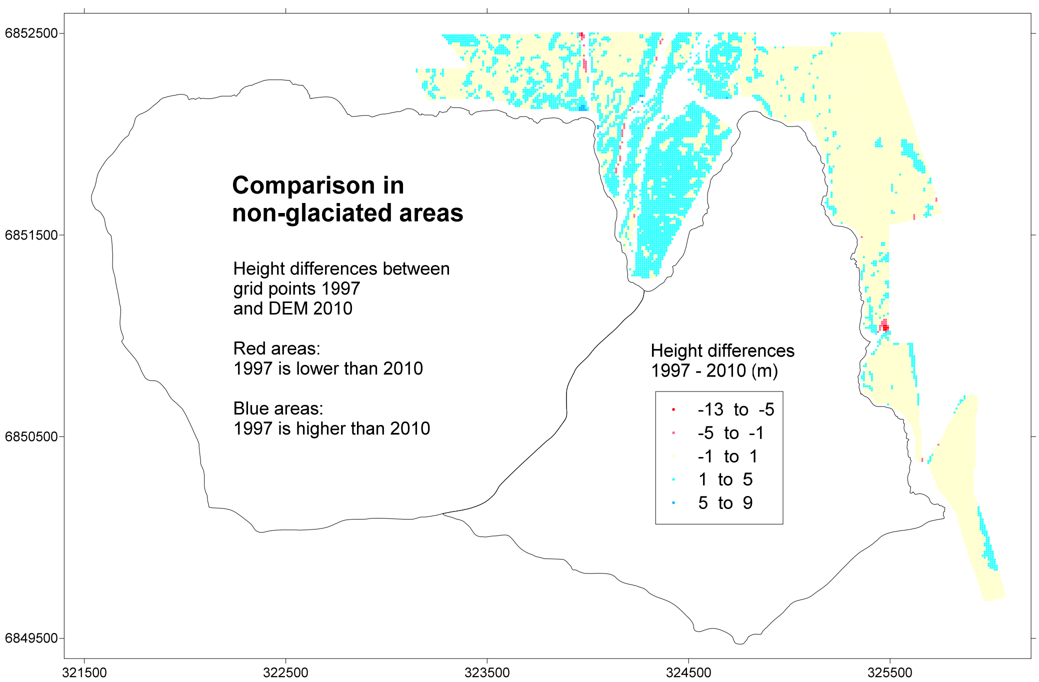


Figure 13
Spatial distribution of elevation differences in non-glaciated areas. The extracted grid points from the 1997 map were subtracted from the corresponding DEM values from 2010. Thus, blue areas indicated that the 1997 DEM was higher than the 2010 DEM and vice versa. Values in areas steeper than 30° were removed.

Terrain maps

The *x/y*-shift between two DEMs can be evaluated by checking the relationship between the elevation differences of the two DEMs and the direction of the terrain (aspect) (Nuth and Kääb, 2011). The 1997 DEM was compared with the 2010 DEM. The relationship can be described visually and schematic.

The elevation differences between 1997 and 2010 and the shaded relief of the terrain for 2010 are shown in figure 14, and the schematic drawing of the elevation differences in non-glacierized area is shown in figure 15.

The evaluation did not indicate any horisontal shift between the two DEMs.

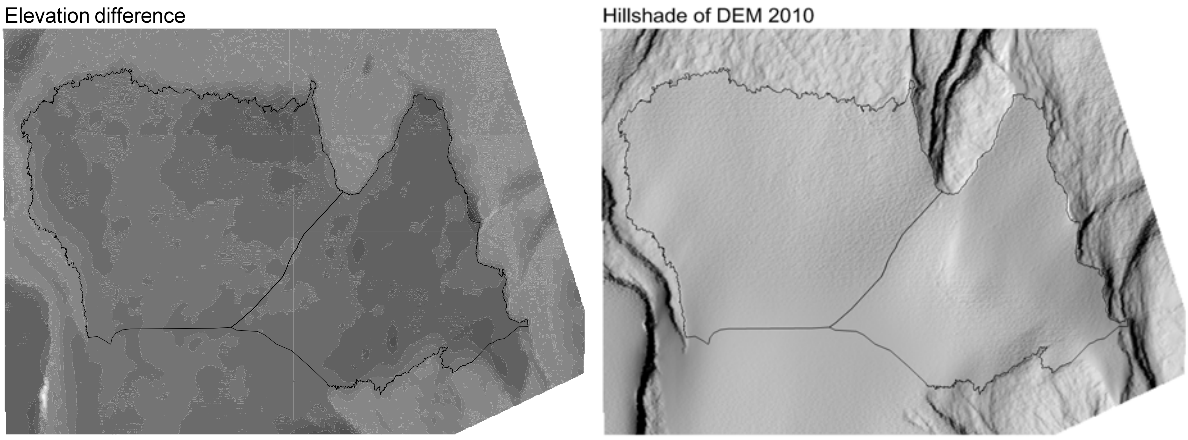


Figure 14
Elevation differences between 1997 and 2010 (left) and hillshade of the 2010 DEM (right).

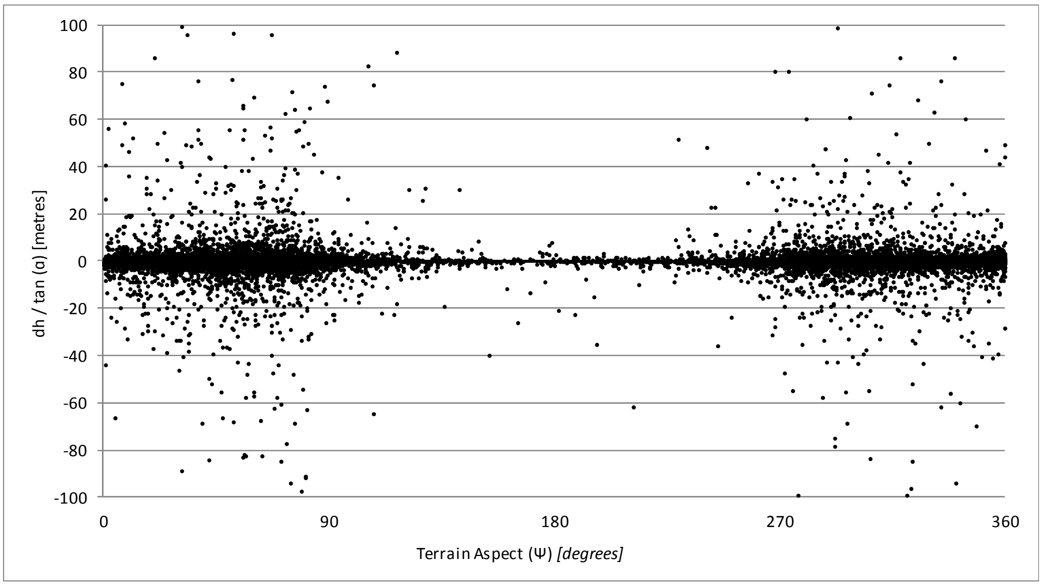


Figure 15
The scatter of elevation differences between the 1997 and 2010 DEMs showing the relationships between the vertical deviations normalized by the slope tangent (y-axis) and the terrain aspect (x-axis).

The uncertainty of the difference between the 1997 and the 2010 DEMs, where σ_{DEM} is the accuracy of a DEM, can be calculated as:

$$\sigma = (\sigma_{DEM2}{}^2 + \sigma_{DEM1}{}^2)^{½}$$

Using an elevation accuracy of the glacier surface of 0.30 m for the 2010 DEM and 0.7 m for the 1997 DEM gave an estimated uncertainty of the difference DEM of 0.8 m. The accuracy of the 1997 DEM may be better. If the accuracy was 0.5 m, the uncertainty of the difference DEM was reduced to 0.6 m.

Mapping 1997 – evaluation summary

The evaluation comparing fixed points in non-glacierized areas with the 10 x 10 m DEM was not considered to be a sound verification of the 1997 DEM quality. The evaluation was influenced by several elements of uncertainties. The accuracy of the control points was not known, only assumed (±0.15 m). The control points were located on topmost point of a peak, and the number of control points are only three. Due to all these uncertainties, an evaluation of the 1997 DEM based on the fixed points in non-glacierized areas was considered as unreliable and was not taken into account.

The comparison between the 1997 DEM and the 2010 DEM in non-glacierized areas indicated that the 1997 DEM was too elevated. The mean elevation difference was 0.7 m and the standard deviation of the elevation differences was calculated as 0.7 m. The number of independent samples was set to 100 (Koblet et al., 2010). The acceptable standard error at 95 % confidence level [1.96 x st.dev/sqr(n)] was calculated as 0.1 m. The mean of the elevation difference (0.7 m) was higher than the acceptable standard error (0.1 m), and hence, a requirement of a vertical correction of the 1997 DEM was indicated. As the mean elevation difference, however, was smaller than 1.0 m, a vertical correction was not necessary.

3.1.3 Mapping 1988

The 1988 DEM was based on raw data acquired by aerial photography.

The 1988 map was constructed in 1992. The construction was based on several control points and passpoints. Information about the quality of the points is not available. At the time of photography there was no snow remaining on the glacier surface from the last winter. Hence, the photographs were described as excellent for the purpose.

The 1988 data set supplied from the mapping company was digital contour line points. As described in chapter 2.1.2 the accuracy was estimated to be better than ±0.40 m in horizontal direction and better than ±0.75 m in vertical direction.

At the time of photographing (7th September 1988) no independent surveying were done. However, the accuracy of the 1988 DEM was investigated by comparing the 1988 contour lines with fixed points and with the 2010 DEM in non-glacierized areas.

Fixed points

A direct comparison of the contour line points with fixed points was not possible without a new map construction, which was not reasonable in this study. However, the 10x10 m DEM was compared with three independent fixed points in non-glaciated areas. The fixed points and the method were similar to the evaluation of the 1997 and 2010 DEMs with a minor correction. The marked fix point B29PNVE2 was some eccentric to the bolt. The comparison showed differences (Diff. = $Height_{fix\ pt} - Height_{DEM}$) within ±2.2 m. Results from the comparison are shown in table 5 and figure 16.

Table 5
Comparison of three fixed points with corresponding points extracted from the 1988 DEM in non-glaciated areas.

Point No.	North	East	$Height_{fix\ pt}$	$Height_{DEM}$	Diff. (m)
B29PNVE2	6 852 180.8	324 540.4	990.04	991.56	-1.52
B29T198	6 850 154.4	325 893.8	1 280.99	1 280.79	0.20
ÅLF100	6 852 194.2	322 065.0	1 166.21	1 168.39	-2.18

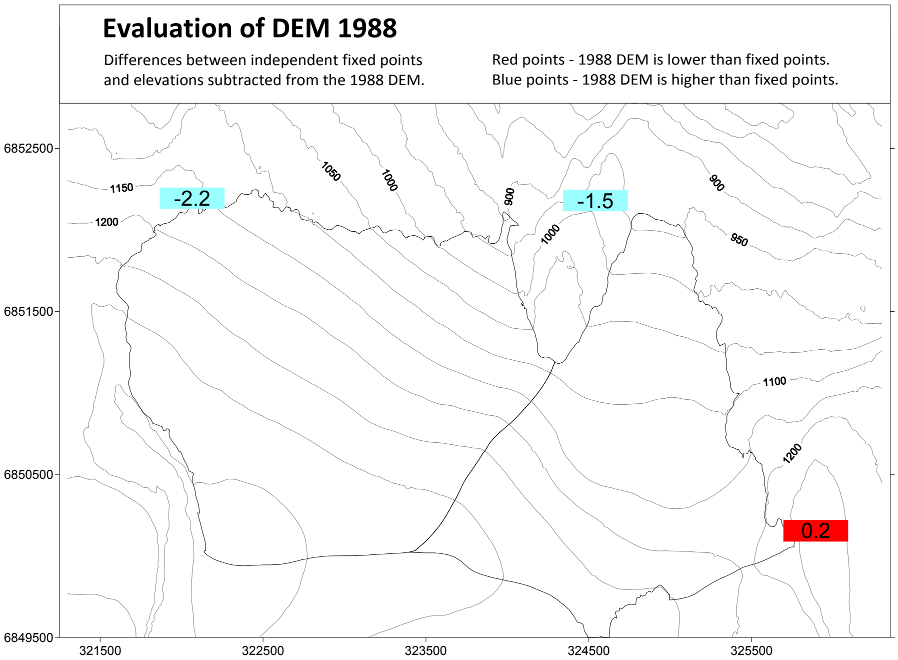


Figure 16
Spatial distribution of three fixed points in non-glaciated areas compared with corresponding points extracted from the 1988 DEM.

DEM 2010

The 1988 DEM was also evaluated by comparing with the 2010 DEM in non-glaciated areas. For the 1988, DEM evaluation congruent non-glaciated terrain was located north and east of the glacier. The contour line points from the 1988 map were subtracted from the corresponding values extracted from the 2010 DEM. The results from 6551 points were first compared. The differences (Diff. = $DEM_{1988} - DEM_{2010}$) were between −2.8 and +16.0 m with an average of +3.4 m. When removing all difference values located in areas steeper than 30°, the 6440 remaining points showed differences between −2.8 and +11.0 m with an average of +3.3 m. Generally, this evaluation indicated that the 1988 DEM was approximately 3 m higher than the 2010 DEM (Fig. 17).

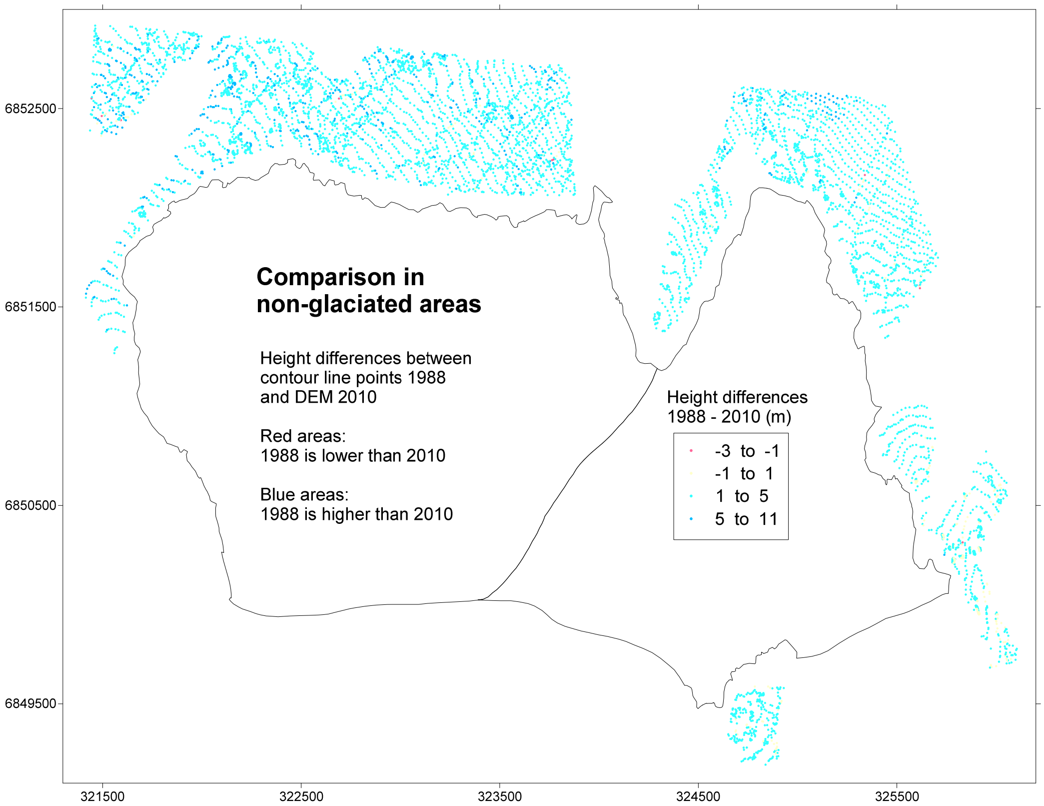


Figure 17
Spatial distribution of elevation differences in non-glaciated areas. The contour line points from the 1988 map were subtracted from the corresponding DEM values from 2010. Thus, blue areas indicated that the 1988 DEM was higher than the 2010 DEM and vice versa. Values in areas steeper than 30° were removed.

Terrain maps

As for the 1997 DEM, the 1988 DEM was also checked for horizontal shift. The relationship and hence, the similarity of elevation differences between 1988 and 2010 with the shaded relief of the terrain for 2010 are shown in figure 18. Figure 19 shows the schematic drawing of the elevation differences with a function that was based upon the terrain slope and aspect.

The evaluation did not indicate any shift between the two DEMs.

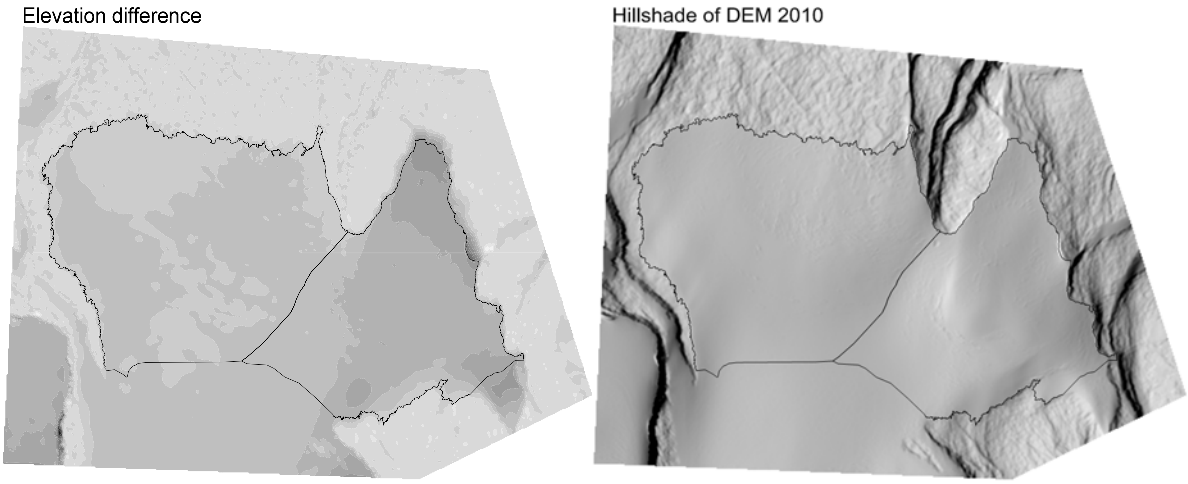


Figure 18
Elevation differences between 1988 and 2010 (left) and hillshade of the 2010 DEM (right).

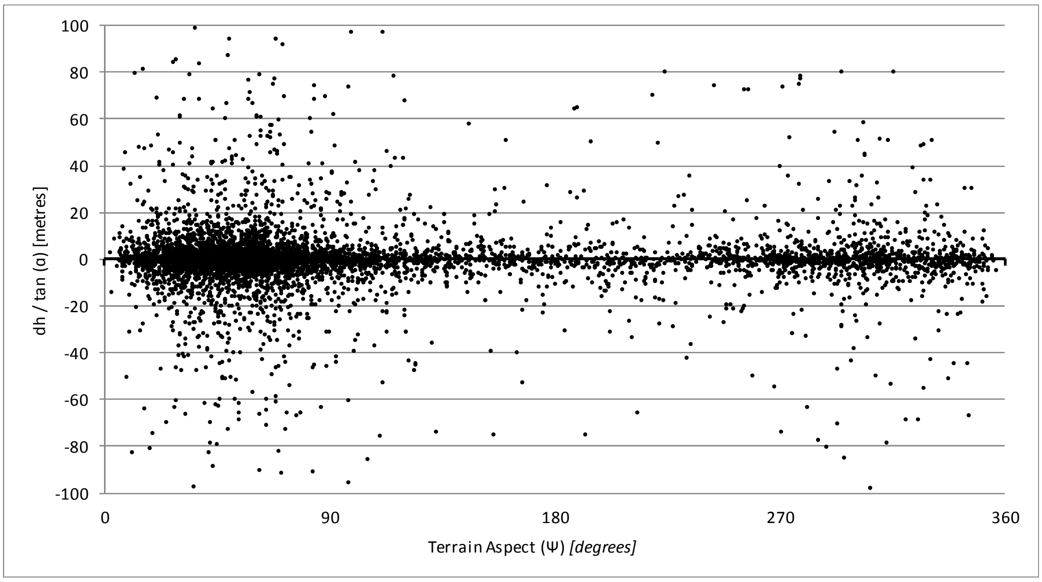


Figure 19
The scatter of elevation differences between the 1988 and 2010 DEMs showing the relationships between the vertical deviations normalized by the slope tangent (y-axis) and the terrain aspect (x-axis).

The uncertainty of the difference between the 1988 and the 2010 DEMs, where σ_{DEM} is the accuracy of a DEM, can be calculated as:

$$\sigma = (\sigma_{DEM2}^2 + \sigma_{DEM1}^2)^{½}$$

Using an elevation accuracy of the glacier surface of 0.30 m for the 2010 DEM and 3.5 m for the 1988 DEM gives an estimated uncertainty of the difference DEM of 3.5 m. The accuracy of the 1988 DEM may be better. If the accuracy is 2.0 m, the uncertainty of the difference DEM is reduced to 2.0 m.

Mapping 1988 – evaluation summary

As for the 1997 DEM evaluation, the simple evaluation comparing fixed points in non-glacierized areas with extracted values from the 1988 DEM was not considered to be a sound verification of the 1988 DEM quality. Hence, this evaluation was considered as unreliable and was not taken into account.

The comparison between the 1988 DEM and the 2010 DEM in non-glacierized areas indicated that the 1988 DEM was too elevated. The mean elevation difference was 3.3 m and the standard deviation of the elevation differences was calculated as 1.4 m. The number of independent samples was set to 52, which was the number of independent contour lines. The acceptable standard error at 95 % confidence level [1.96 x st.dev/sqr(n)] was calculated as 0.4 m. As the mean of the elevation difference (3.3 m) was greater than 1.0 m and higher than the acceptable standard error (0.4 m), a vertical correction was necessary. For the following calculations, the 1988 DEM was lowered 3.3 m.

3.1.4 Mapping 1968

The 1968 DEM was based on raw data acquired by aerial photography.

Originally, the 1968 map was constructed in 1969. Due to great errors in the geodetic network, the map was re-constructed in 1995 based on four or five corrected control points. Information about the quality of the control points was not available. At the time of photography, the entire glacier surface was still covered with snow from the last winter.

The 1968 data set supplied from the mapping company was digital contour line points. As described in chapter 2.1.1 the height accuracy for the original plotting was reported to be better than 2 m (relative) and 5 m (absolute), and the horizontal accuracy to be better than 5 m.

At the time of photographing (5th August 1968) no independent surveying was done. However, the accuracy of the 1968 DEM was investigated by comparing the 1968 contour lines with fixed points and with the 2010 DEM in non-glacierized areas.

Fixed points

A direct comparison of the contour line points with fixed points was not possible without a new map construction, which was not reasonable in this study. However, the 10x10 m DEM was compared with three independent fixed points in non-glaciated areas. The fixed points and the method were similar to the evaluation of the 1988 DEM. The comparison showed height differences (Diff. = $Height_{fix\ pt}$ – $Height_{DEM}$) within ±2.3 m. Results from the comparison are shown in table 6 and figure 20.

Table 6
Comparison of three independent fixed points with corresponding points extracted from the 1968 DEM in non-glaciated areas.

Point No.	North	East	$Height_{fix\ pt}$	$Height_{DEM}$	Diff. (m)
B29PNVE2	6 852 180.8	324 540.4	990.04	991.21	-1.17
B29T198	6 850 154.4	325 893.8	1 280.99	1 279.37	1.62
ÅLF100	6 852 194.2	322 065.0	1 166.21	1 168.47	-2.26

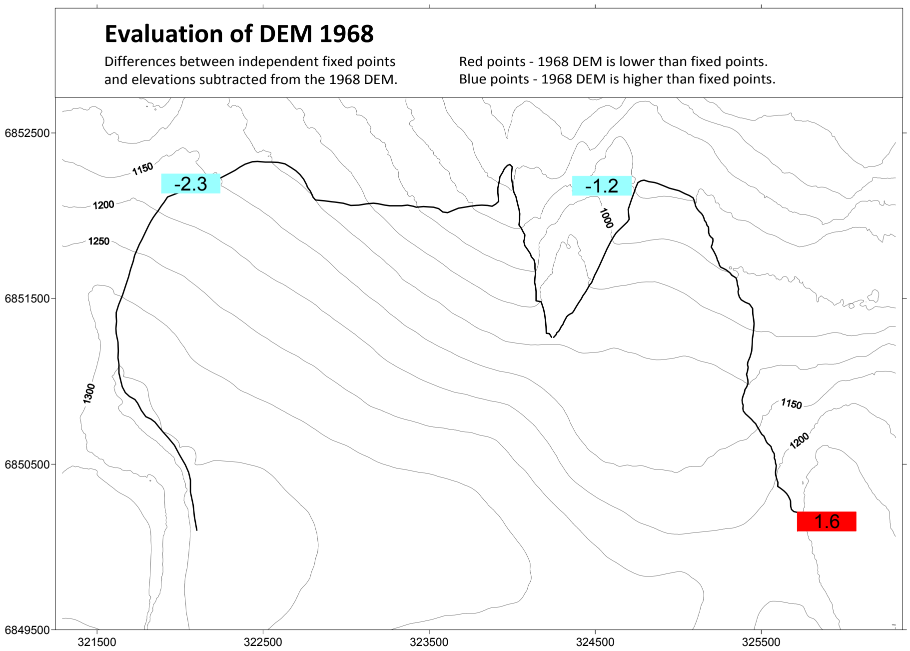


Figure 20
Spatial distribution of three independent fixed points in non-glaciated areas compared with corresponding points extracted from the 1968 DEM.

DEM 2010

The 1968 DEM was also evaluated by comparing with the 2010 DEM in non-glaciated areas. For the 1968 DEM evaluation, congruent non-glaciated terrain was mainly located north of the glacier. The contour line points from the 1968 map were subtracted from the corresponding values extracted from the 2010 DEM. The results from 1362 contour line points were first compared. The differences (Diff. = DEM_{1968} –DEM_{2010}) were between −3.7 and +17.3 m with an average of +5.3 m. When removing all difference values located in areas steeper than 30°, the remaining 1208 points showed differences from −2.1 to +11.3 m with an average of +5.1 m. Generally, this evaluation indicated that the 1968 DEM was approximately 5 m higher than the 2010 DEM (Fig. 21).

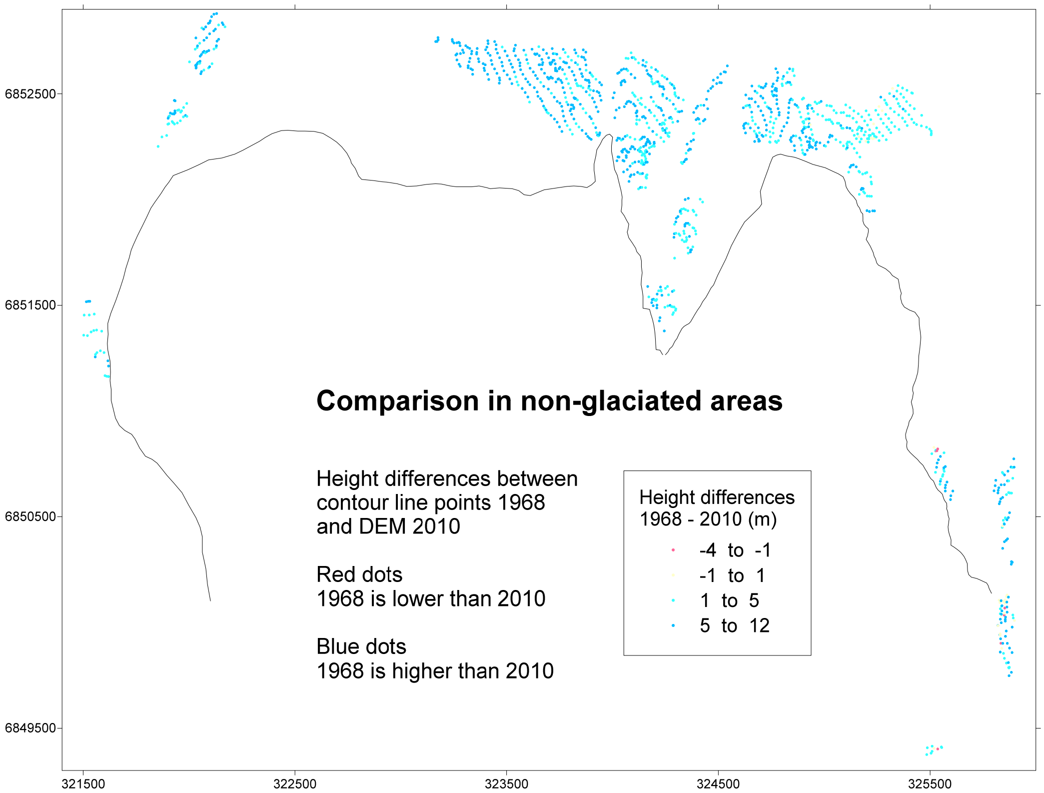


Figure 21
Spatial distribution of elevation differences in non-glaciated areas. The contour line points from the 1968 map were subtracted from the corresponding DEM values from 2010. Thus, blue areas indicated that the 1968 DEM was higher than the 2010 DEM and vice versa. Values in areas steeper than 30° were removed.

Terrain maps

As for the 1997 and 1988 DEMs, the 1968 DEM was also checked for horizontal shift. The relationship and hence, the similarity of elevation differences between 1968 and 2010 with the shaded relief of the terrain for 2010 are shown in figure 22. Figure 23 shows the schematic drawing of the elevation differences with a function that was based upon the terrain slope and aspect.

The evaluation did not indicate any shift between the two DEMs.

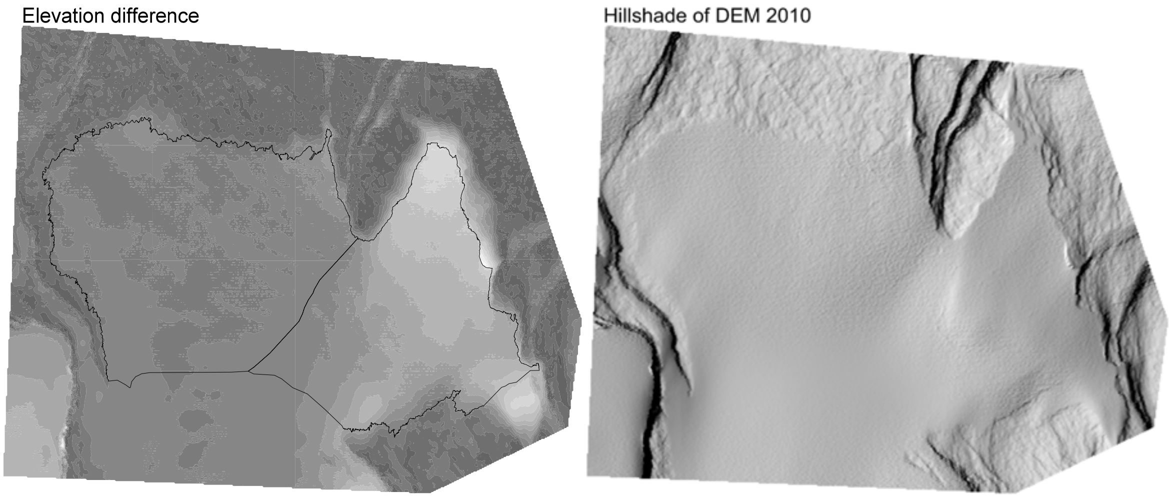


Figure 22
Elevation differences between 1968 and 2010 (left) and hillshade of the 2010 DEM (right).

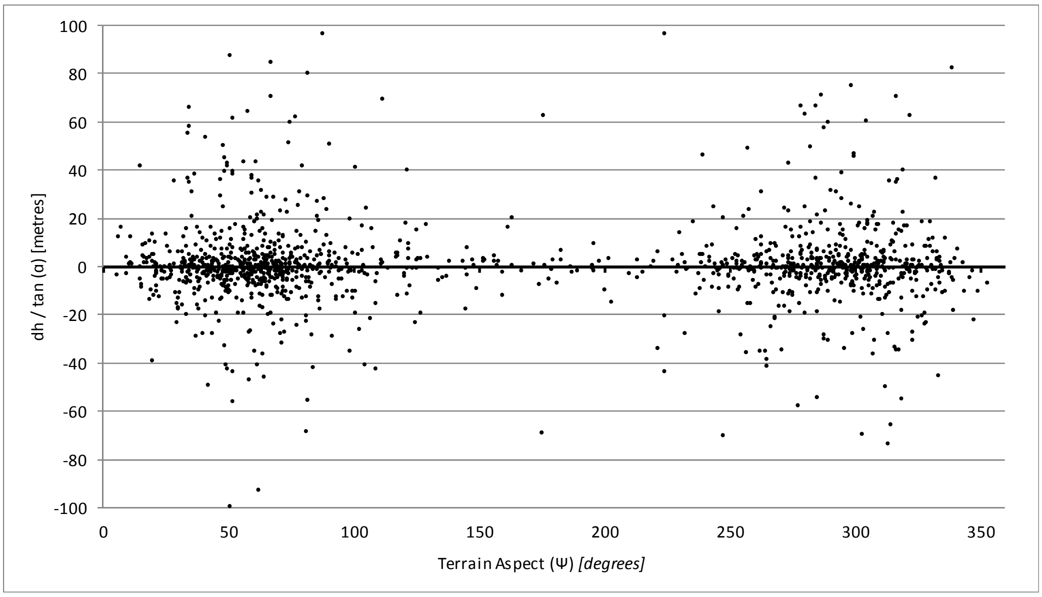


Figure 23
The scatter of elevation differences between the 1968 and 2010 DEMs showing the relationships between the vertical deviations normalized by the slope tangent (y-axis) and the terrain aspect (x-axis).

The uncertainty of the difference between the 1968 and the 2010 DEMs, where σ_{DEM} is the accuracy of a DEM, can be calculated as:

$$\sigma = (\sigma_{DEM2}{}^2 + \sigma_{DEM1}{}^2)^{½}$$

Using an elevation accuracy of the glacier surface of 0.30 m for the 2010 DEM and 5.0 m for the 1968 DEM gives an estimated uncertainty of the difference DEM of 5.0 m. The accuracy of the 1968 DEM may be better. If the accuracy is 2.0 m, the uncertainty of the difference DEM is reduced to 2.0 m.

Mapping 1968 – evaluation summary

As for the 1997 and 1988 DEM evaluations, the simple evaluation comparing fixed points in non-glacierized areas with extracted values from the 1968 DEM was not considered to be a sound verification of the 1968 DEM quality. Hence, this evaluation was considered as unreliable and was not taken into account.

The comparison between the 1968 DEM and the 2010 DEM in non-glacierized areas indicated that the 1968 DEM was too elevated. The mean elevation difference was 5.1 m and the standard deviation of the elevation differences was calculated as 2.0 m. The number of independent samples was set to 44, which was the number of independent contour lines. The acceptable standard error at 95 % confidence level [1.96 x st.dev/sqr(n)] was calculated as 0.6 m. As the mean of the elevation difference (5.1 m) was greater than 1.0 m and higher than the acceptable standard error (0.6 m), a vertical correction was necessary. For the following calculations the 1968 DEM was lowered 5.1 m.

3.1.5 Mass change 1968-1988

The spatial distribution of thickness changes at Ålfotbreen and Hansebreen from 5th August 1968 to 7th September 1988 is shown in figure 24. The geodetic mass balance over the period 1968(69)-1988 was calculated using grid size 10 x 10 m. Average volume change was multiplied with the density conversion factor (850 kg m^{-3}), divided with the mean area for 1968 and 1988 and adjusted for additional melting both years. The results are given in table 7.

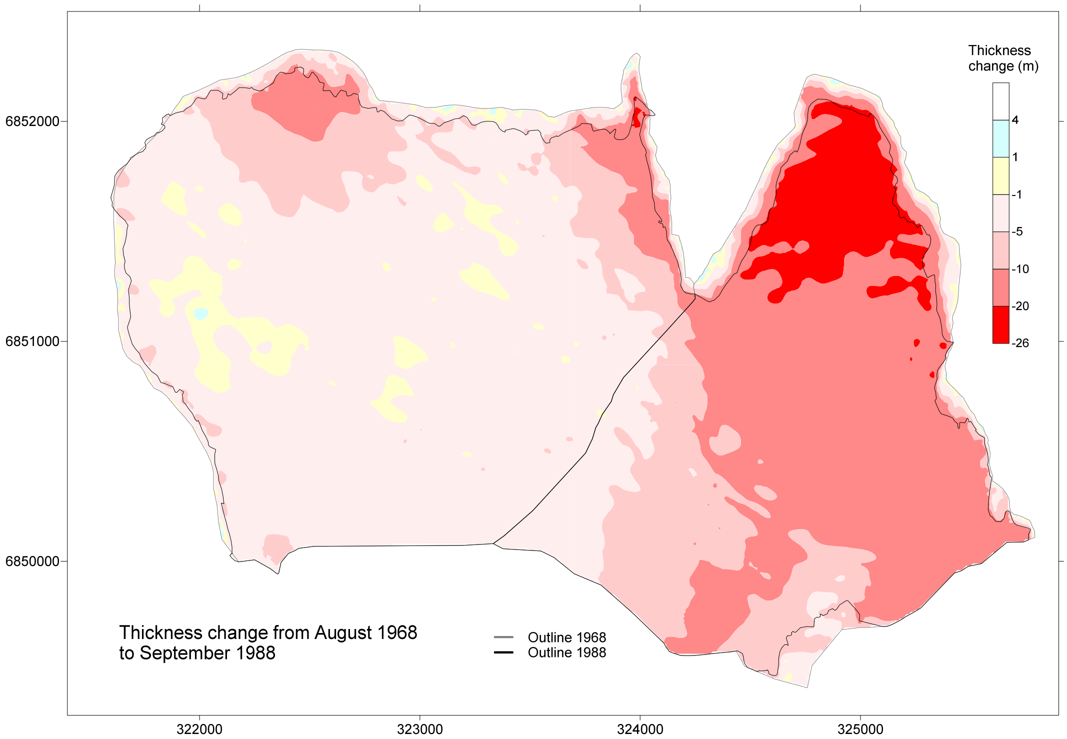


Figure 24
DEM differences of Ålfotbreen and Hansebreen from 5th August 1968 to 7th September 1988.

Geodetic mass balance over 1968-1988 was −2.43 m w.e. for Ålfotbreen and −10.47 m w.e. for Hansebreen. In accordance with the geodetic mass balance deficit, the ice thickness change was significant negative (<−1 m) for 99 % of the surveyed glacier area at Hansebreen and for 93 % at Ålfotbreen (Fig. 24). The ice thickness change was within ±1 m for 1 % at Hansebreen and 7 % at Ålfotbreen. Accordingly, the thickness change was significant positive (>1 m) for less than 1 % of the surveyed glacier area at both glaciers. Mean thickness change was −3.8 m for Ålfotbreen and −12.5 m for Hansebreen.

3.1.6 Mass change 1988-1997

The spatial distribution of thickness changes at Ålfotbreen and Hansebreen from 7th September 1988 to 14th August 1997 is shown in figure 25. The geodetic mass balance over the period 1988(89)-1997 was calculated using grid size 10 x 10 m. Average volume change was multiplied with the density conversion factor (850 kg m^{-3}), divided with the mean area for 1988 and 1997 and adjusted for additional melting both years. The results are given in table 7.

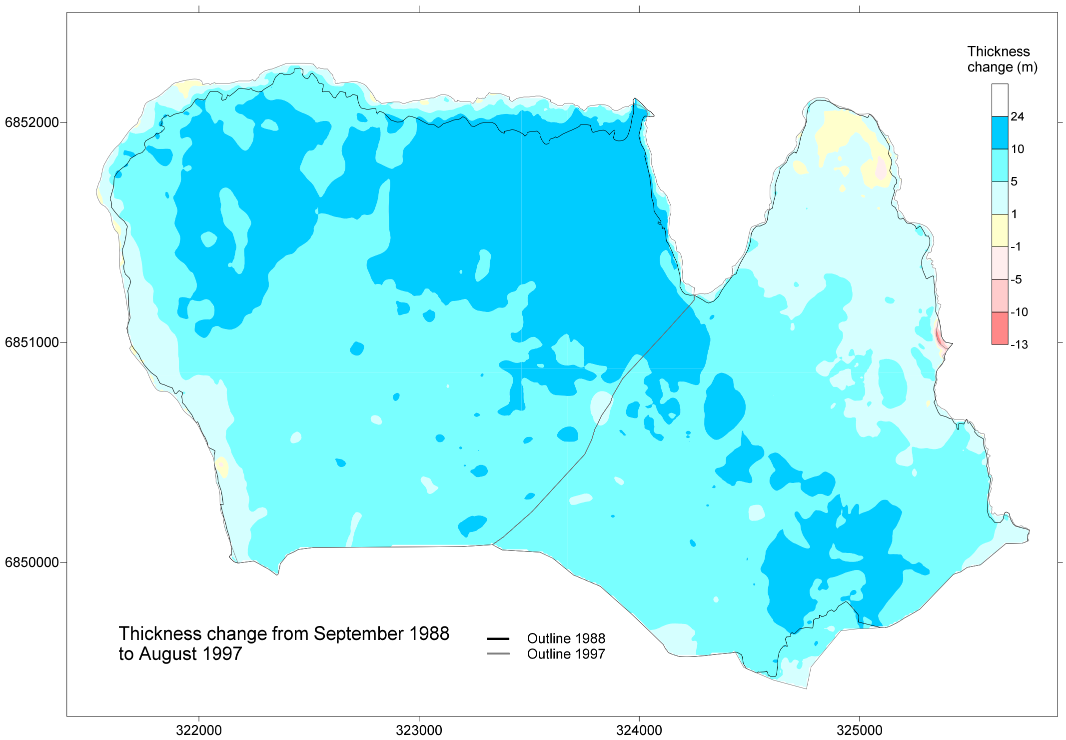


Figure 25
DEM differences of Ålfotbreen and Hansebreen from 7th September 1988 to 14th August 1997.

Geodetic mass balance over 1988-1997 was +7.81 m w.e. for Ålfotbreen and +5.45 m w.e. for Hansebreen. In accordance with the geodetic mass balance surplus, the ice thickness change was significant positive (>1 m) for 98 % of the surveyed glacier area at Hansebreen and for 99.5 % at Ålfotbreen (Fig. 25). The ice thickness change was within ±1 m for 2 % at Hansebreen and 0.5 % at Ålfotbreen. Accordingly, the thickness change was significant negative (<−1 m) for less than 1 % of the surveyed glacier area at both glaciers. Mean thickness change was +9.6 m for Ålfotbreen and +6.9 m for Hansebreen.

3.1.7 Mass change 1997-2010

The spatial distribution of thickness changes at Ålfotbreen and Hansebreen from 14th August 1997 to 2nd September 2010 is shown in figure 26. The geodetic mass balance over the period 1997(98)-2010 was calculated using grid size 10 x 10 m. Average volume change was multiplied with the density conversion factor (850 kg m^{-3}), divided with the mean area for 1997 and 2010 and adjusted for additional melting both years. The results are given in table 7.

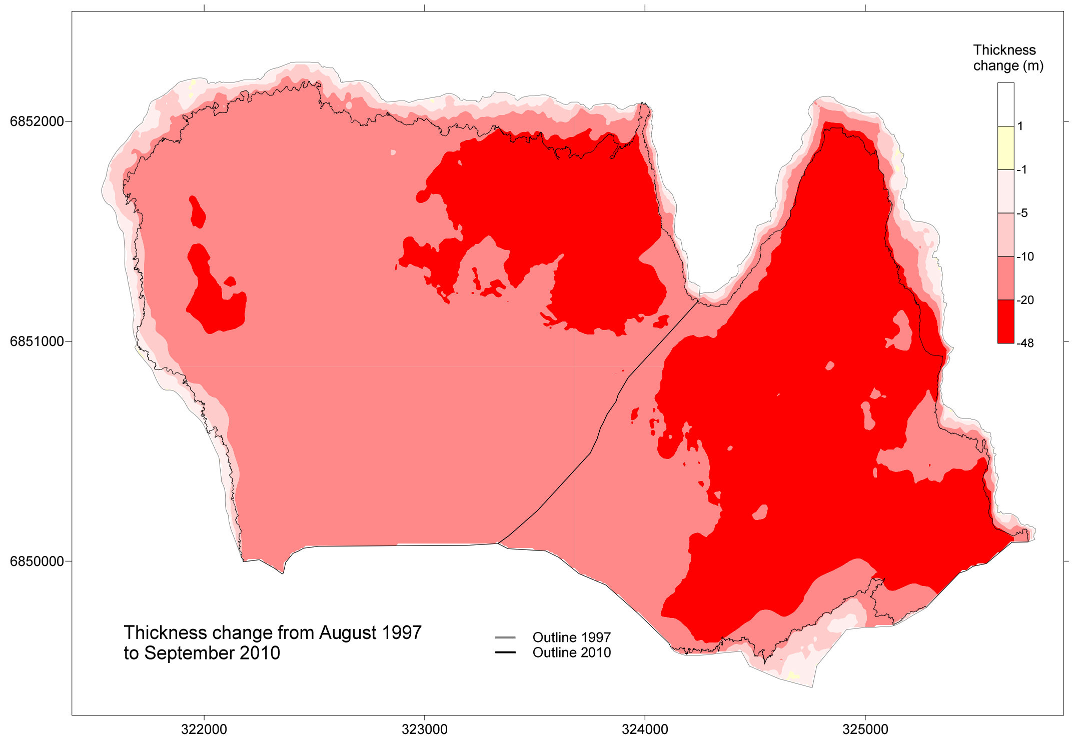


Figure 26
DEM differences of Ålfotbreen and Hansebreen from 14th August 1997 to 2nd September 2010.

Geodetic mass balance over 1997-2010 was −13.59 m w.e. for Ålfotbreen and −17.42 m w.e. for Hansebreen. In accordance with the geodetic mass balance deficit, the ice thickness change was significant negative (<−1 m) for 99.8 % of the surveyed glacier area at Hansebreen and for 99.9 % at Ålfotbreen (Fig. 26). No areas had positive thickness change. Mean thickness change was −15.8 m for Ålfotbreen and −19.7 m for Hansebreen.

3.1.8 Mass change Ålfotbreen 1968-2010 and Hansebreen 1988-2010

In order to include as much as possible of the glaciological mass balance time series the mass change was also calculated for the periods 1968-2010 for Ålfotbreen and 1988-2010 for Hansebreen.

Mass change Ålfotbreen 1968-2010

The spatial distribution of thickness changes at Ålfotbreen and Hansebreen from 5th August 1968 to 2nd September 2010 is shown in figure 27. The geodetic mass balance for Ålfotbreen over the period 1968(69)-2010 was calculated using grid size 10 x 10 m. Average volume change was multiplied with the density conversion factor (850 kg m^{-3}), divided with the mean area for 1968 and 2010 and adjusted for additional melting both years. The results are given in table 7.

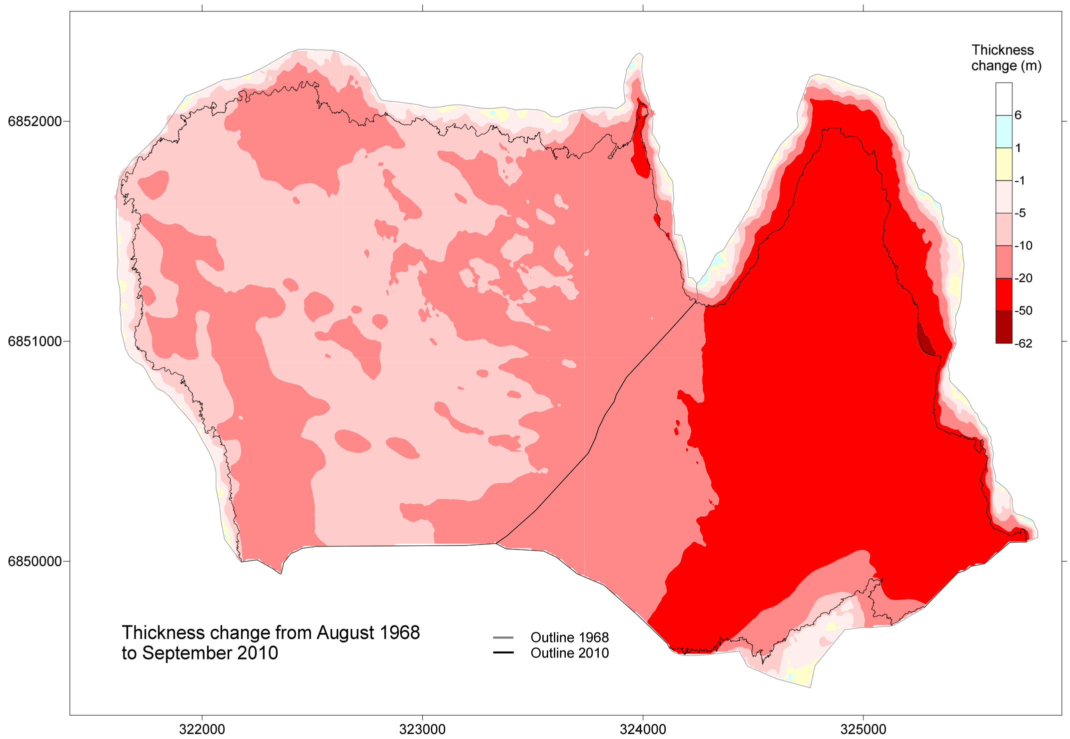


Figure 27
DEM differences of Ålfotbreen and Hansebreen from 5th August 1968 to 2nd September 2010.

Geodetic mass balance for Ålfotbreen over 1968-2010 was −8.08 m w.e. In accordance with the geodetic mass balance deficit, the ice thickness change was significant negative (<−1 m) for 99 % of the surveyed glacier area at Ålfotbreen (Fig. 27). Less than 0.1 % of the area had significant (>1 m) positive thickness change. Mean thickness change for Ålfotbreen was −10.1 m.

Mass change Hansebreen 1988-2010

The spatial distribution of thickness changes at Ålfotbreen and Hansebreen from 7th September 1988 to 2nd September 2010 is shown in figure 28. The geodetic mass balance for Hansebreen over the period 1988(89)-2010 was calculated using grid size 10 x 10 m. Average volume change was multiplied with the density conversion factor (850 kg m^{-3}), divided with the mean area for 1988 and 2010 and adjusted for additional melting both years. The results are given in table 7.

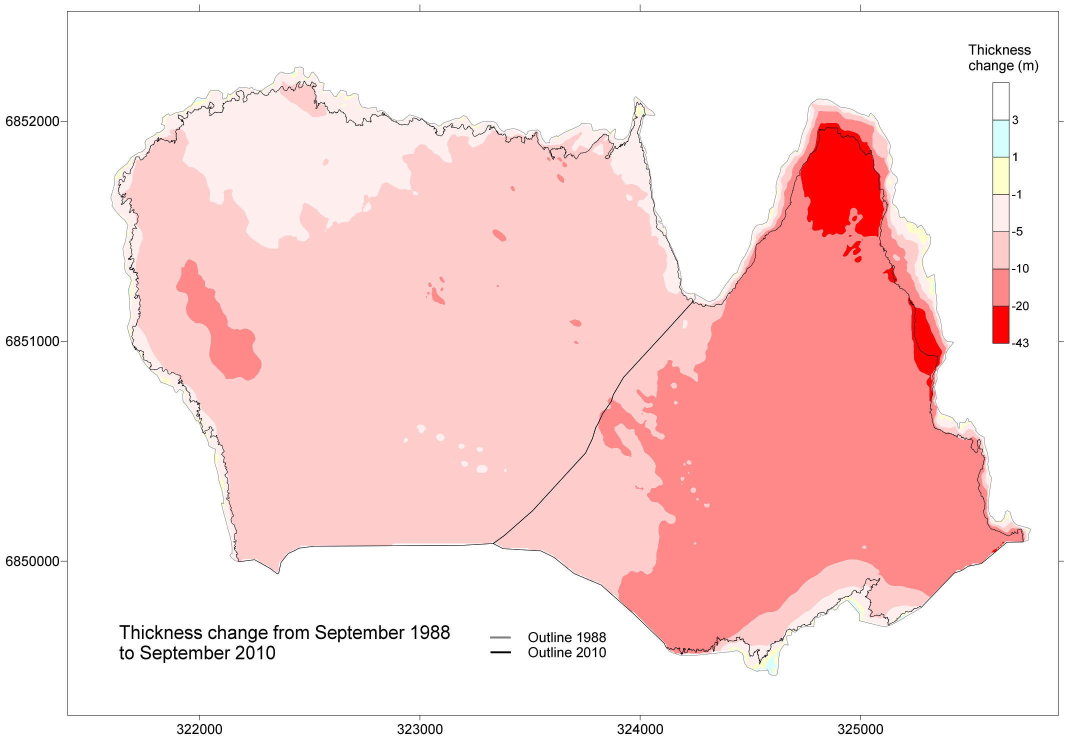


Figure 28
DEM differences of Ålfotbreen and Hansebreen from 5th August 1968 to 2nd September 2010.

Geodetic mass balance for Hansebreen over 1988-2010 was −11.83 m w.e. In accordance with the geodetic mass balance deficit, the ice thickness change was significant negative (<−1 m) for 99 % of the surveyed glacier area at Hansebreen (Fig. 28). Less than 0.3 % of the area had significant (>1 m) positive thickness change. Mean thickness change for Hansebreen was −13.3 m.

Table 7
Volume change and geodetic mass balance for Ålfotbreen (1968-88, 1988-97, 1997-2010 and 1968-2010) and Hansebreen (1968-88, 1988-97, 1997-2010 and 1988-2010).

glacier	$area_{year I}$	$area_{year II}$	vol.ch.	dens. fac.	date adj. (m w.e.)		geod. mb. (m w.e.)	
period	(km^2)	(km^2)	(mill. m^3)	(kg m^{-3})	yearI	yearII	acc.	ann.
Ålfotbreen								
1968-1988	4.49	4.17	-17	850	-1.46	-0.52	-2.43	-0.12
1988-1997	4.17	4.48	43	850	-0.52	-1.14	7.81	0.87
1997-2010	4.48	3.97	-71	850	-1.14	-0.45	-13.59	-1.05
1968-2010	4.49	3.97	-45	850	-1.46	-0.45	-8.08	-0.19
Hansebreen								
1968-1988	3.42	3.07	-43	850	-1.29	-0.57	-10.47	-0.52
1988-1997	3.07	3.18	22	850	-0.57	-1.09	5.45	0.61
1997-2010	3.18	2.75	-63	850	-1.09	-0.51	-17.42	-1.34
1988-2010	3.07	2.75	-41	850	-0.57	-0.51	-11.83	-0.54

3.2 Glaciological mass balance

As the methodology of the mass balance calculations has changed through the years since the beginning in 1963 a homogenization of the series of Ålfotbreen and Hansebreen were necessary. Four major factors considered and are described in the following.

3.2.1 Contour line method (1963-1988)

From 1963 to 1988 the winter and summer balances were calculated using the contour line method (Fig. 4). From 1989 the altitudinal mass balance curves have been made by plotting point measurements versus altitude, the profile method (Fig. 5). Accordingly, the homogenization involved re-calculation of the years 1963-1988 using the profile method.

3.2.2 Height-area distribution (1963-1968, 1978-2000, 2004-2009)

The mass balance calculations were based on height-area distribution from five DEMs (1955, 1968, 1988, 1997 and 2010). There were considerable time lags between the mass balance data and the map used for the calculations (Fig. 7). Over the years from 1963 to 2010, Ålfotbreen had periods of both shrinking and growing. Hence, the period between two mappings are divided in two, and each DEM was applied to half of the period before the mapping year and half of the period after the mapping year (Fig. 29). Another method for assessing the height-area distribution is an annual linear change-over from DEM_I to DEM_{II}. This method will be most appropriate if the changes over a period are solely increasing, or decreasing. The lowering of the 1968 and 1988 DEMs from chapter 3.1 will also influence the height-area distribution and was, hence, included in the homogenization that involved re-calculation of the period 1963-2010 using the proper DEM.

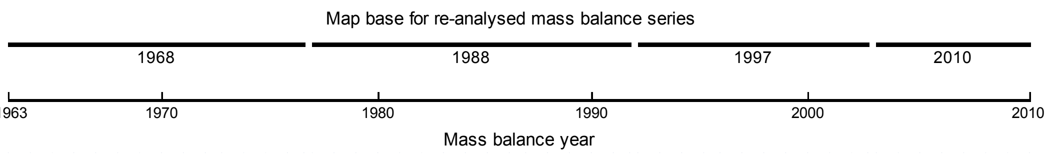


Figure 29
Upper line indicates map base for the re-analysed mass balance series. Years denote year of validity period for each map.

3.2.3 Converting from snow depth to water equivalent

Winter balance calculations are based on measurements of snow depths and snow density (described in chapter 2.2). The converting procedure from snow depth to water equivalent has varied through the ages. For the first four decades (1963-1999) a precise documentation of the converting procedure is lacking. However, for some of the years, it seems as if an average density (ρ_{av}) of the snow pack was used for each point measurement (c_a) expressed as: $b_w = c_a$ (m)*ρ_{av} (kg m^{-3})/1000. For some other years, it seems as if an unique snow density for each snow depth was estimated based on the measured average density. From 2006 the current converting method was adopted. The snow density and, hence, the water equivalent, was calculated for each snow sample part. The accumulated water equivalent was plotted with increasing snow depth and a mathematical trend line and function was formatted. Usually a polynomial of degree three or two was used expressed as: $b_w = a*c_a^3 + b*c_a^2 + c*c_a + d$ (a, b, c and d are coefficients). An example from 2007 is given in figure 30. Exceptionally, when a polynomial function was illogical, a power function (1963) and a linear function (2008) was used. In the homogenization this converting method was implemented for 38 of the 48 year period. Of different reasons the original water equivalent values (b_w) was continued for ten of the years. For the years 1965, 89 and 90, snow depth data was not available. For the years 1978-80 snow density was partially measured two or three times over the winter season, and for the years 1966-68 and 1976 the density measurements and the snow depth measurements were performed at different dates.

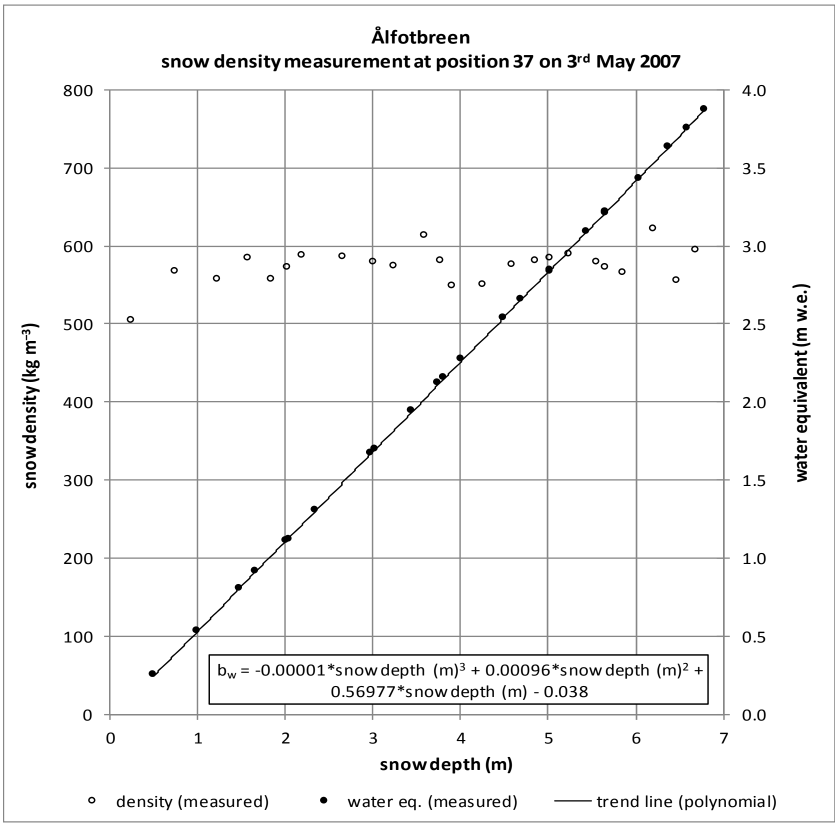


Figure 30
Snow density measurements from 2007. The snow density for each sample part is shown on the left axis and the accumulated water equivalent is shown on the right axis. The converting function is shown in the frame.

3.2.4 Ice divide

The ice divide was originally constructed for each map. As described in chapter 2.1.3 the different ice divides showed only marginal differences. The ice divide from the 2010 map was considered to be most accurate and approximately unchanged over the entire period. Accordingly, the homogenization involved re-calculation of the period 1963-2009 using the ice divide from 2010.

3.2.5 Results

Homogenizing by re-calculation of the mass balance series for Ålfotbreen (1963-2010) and Hansebreen (1986-2010) ensure a uniform methodology, data processing and interpretation of the calculation process from field data to the final balance values. All point measurements of snow depths and stakes were identified and given current positions and heights. The re-calculation was based on the profile method applying the current DEM and the ice divide from 2010. The review of the historic data sets and the re-calculation process also revealed some errors in the original mass balance calculations.

These errors were corrected in the re-calculations. Some of the years had great corrections. A qualitative assessment of six problematic years, where winter or summer balance adjustments were particularly great, follows:

1963: The original summer balance for Ålfotbreen was −3.59 m w.e., while the homogenized summer balance was −3.21 m w.e. The ablation measurements are well documented and summer balance data for twelve stakes are available. However, the re-calculation revealed a disparity in the data set. The summer balance values for each stake exist both in a stake form and in a diagram and these two data sets do not correspond exactly. Generally, the values from the form are lower than the diagram shows. The reason for this disparity was not found. The plotted summer balance diagram however, is usually a product of stake values. As the stake values were the basic data set and there was no indication of any error, the summer balance values from the stake form were considered more reliable. Accordingly, the homogenized summer mass balance was based on the values from the stake form.

1975: The original winter balance for Ålfotbreen was 4.64 m w.e., while the homogenized winter balance was 4.32 m w.e. In the upper areas, the snow depth measurements were rather sparse. The height intervals above 1250 m a.s.l. were represented with only four point measurements. The original winter balance curve was drawn irrespective of these four point measurements in the upper areas. The curve in the upper height intervals was rather controlled by the curve pattern below 1250 m elevation. There was, however, no indication in the data set or in the general documentation that any point measurements should be omitted from the calculations. Accordingly, the homogenized winter mass balance included all data and the winter balance curve was modified in the upper areas.

1990: The original winter balance for Hansebreen was 4.42 m w.e., while the homogenized winter balance was 4.08 m w.e. Great snow depths (up to 10 m) and few stakes made snow surveying by probing, core sampling and stake readings difficult, and hence, snow depth measurements were rather sparse. The winter balance curve was based on only thirteen snow depth measurements and was certainly subject of some interpretations. However, as the curve was drawn biased compared with the point measurements, the original winter balance was obviously overestimated. Accordingly, the homogenized winter balance gave lower values in all height intervals.

1991: The original winter and summer balances for Ålfotbreen were 4.09 and −3.30 m w.e., while the homogenized balance values were 3.44 and −2.87 m w.e. The original winter balance for Hansebreen was 3.37 m w.e., while the homogenized winter balance was 2.93 m w.e. After the snow accumulation measurements in April, there was a heavy snowfall in late April/early May. The additional snow accumulation was estimated based on precipitation and temperature data from nearby climate stations and earlier experiences. In the original mass balance calculations, this additional snow accumulation, estimated as 0.3-0.5 m w.e., was included in both winter and summer mass balance. According to the current practice, this ungauged additional snow accumulation was not included in the homogenized mass balance.

2000: The original winter balance for Ålfotbreen was 5.57 m w.e., while the homogenized winter balance was 5.17 m w.e. The homogenization revealed an error in the converting procedure from snow depths to water equivalents. The converting error resulted in overestimated water equivalents for snow depths greater than 7 m. The overestimation was amounted to 0.3 m w.e. for snow depth 10 m increasing to 0.7 m w.e. for snow depth 12 m. Snow depths between 10 and 12 m included 45 % of all snow depth measurements. Accordingly, the homogenization gave lower winter balance.

2008: The original winter balance for Ålfotbreen was 4.04 m w.e., while the homogenized winter balance was 4.57 m w.e. The original winter balance for Hansebreen was 3.90 m w.e., while the homogenized winter balance was 4.25 m w.e. The homogenization revealed that the original converting function (from snow depths to water equivalents) was some incorrect. Generally, the snow depth varied between 7 and 10 m at Ålfotbreen and Hansebreen this year. The snow density was measured down to 6.22 m depth (Fig. 31). The measurements showed a gradual increasing density from 285 kg m^{-3} in the topmost half-meter to 579 kg m^{-3} at 4.66 m depth. The measurements below 4.66 m depth, however, showed a distinct shift (Fig. 31). The density of the eight lowest samples was between 523 and 544 kg m^{-3}. The original trend line and function, was a polynomial of degree three. After a thorough evaluation a linear trend line was considered to be more appropriate for the extrapolated snow depths (below 7 m depth). Accordingly, the homogenization gave higher winter balance, and, hence higher summer balance.

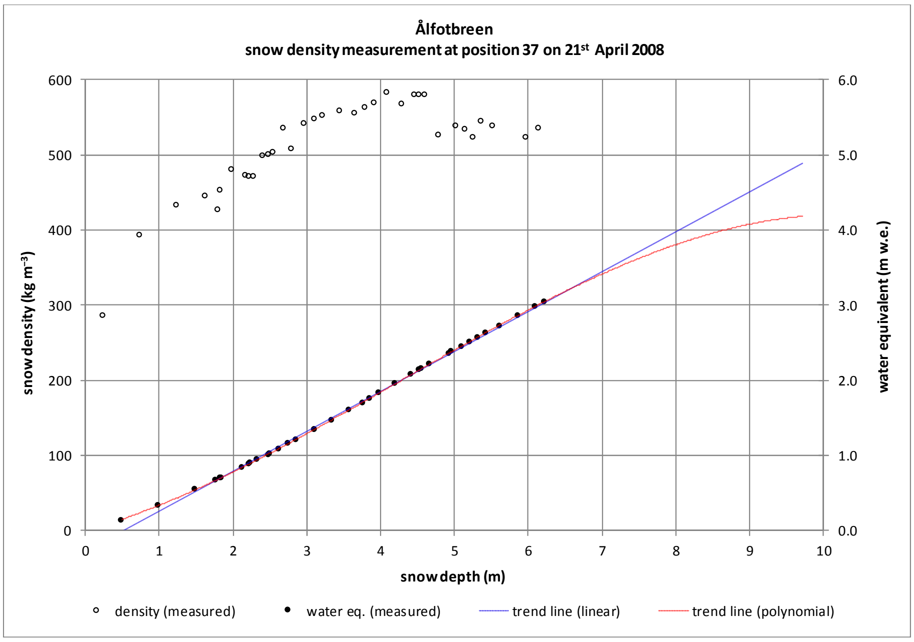


Figure 31
The snow density measurements (○) and the derived water equivalent values (●) at Ålfotbreen in 2008. When homogenizing the mass balance series the original trend line, a polynomial of degree three (red line) was replaced with a linear trend line (blue line).

The original and homogenized mass balance series for Ålfotbreen and Hansebreen are shown in tables 8 and 9 and figure 32.

The homogenized mass balance series for Ålfotbreen over the period 1963-2010 shows a positive cumulative mass balance of 4.7 m w.e., which is 1.6 m w.e. less surplus than the original series for the same period. The cumulative winter balance was reduced with 1.8 m w.e. and the cumulative summer balance was reduced with 0.2 m w.e.

The homogenized mass balance series for Hansebreen over the period 1986-2010 shows a negative cumulative mass balance of 15.1 m w.e., which is 1.4 m w.e. greater deficit than the original series for the same period. The cumulative winter balance was reduced with 1.6 m w.e. and the cumulative summer balance was reduced with 0.2 m w.e.

Generally the homogenized mass balance series over 1963-2010 (Ålfotbreen) and 1986-2010 (Hansebreen) gave a lower annual winter balance than the original series, while the annual summer balances were both lower and greater than the original values. The annual mean winter balance decrease was 0.036 m w.e. for Ålfotbreen and 0.065 m w.e. for Hansebreen. The corresponding decrease of annual summer balance was 0.003 m w.e. and 0.010 m w.e. The individual impact of the four major changes in methodology was not tested thoroughly, but some few spot checks indicated rather small changes of the annual balances, typical within ±0.1 m w.e. The greatest contributions to the cumulative mass balance changes were ascribed to the individual errors described and to different approaches and interpretations of the field data and the mass balance calculations.

Table 8
Original and homogenized mass balance series for Ålfotbreen over the period 1963-2010. Homogenization of the year 2010 was not required.

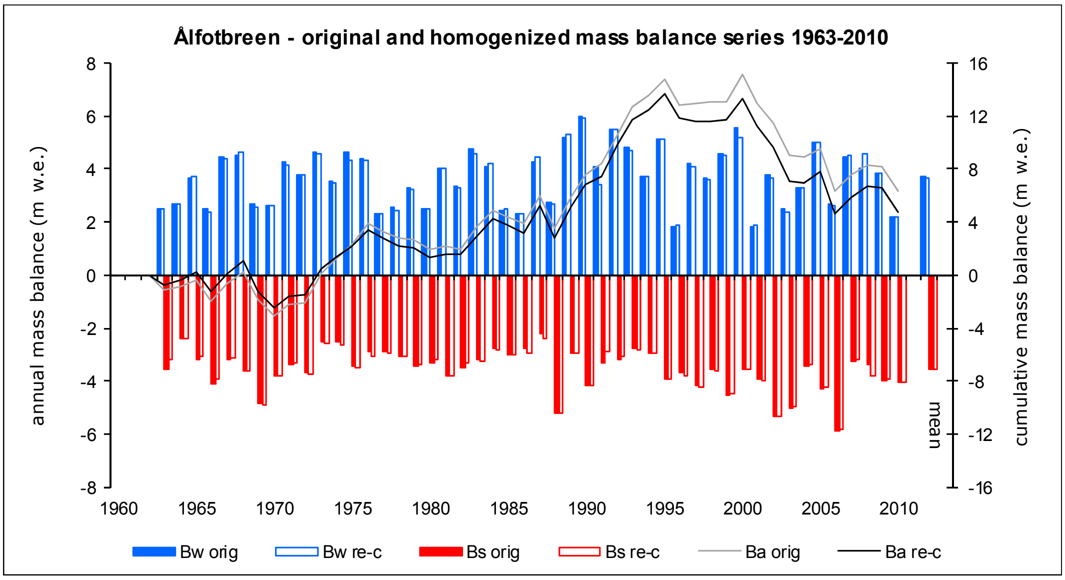
	Original mass balance series								Homogenized mass balance series								Hom.gen. with regard to			
Year	B_w	B_s	B_a	ΣB_a	ELA	AAR	DEM	Area	B_w	B_s	B_a	ΣB_a	ELA	AAR	*DEM	Area	A-dist	DEM	Dvd	Dens
1963	2.49	-3.59	-1.09	-1.09	1300	27	1955	4.75	2.52	-3.21	-0.70	-0.70	1270	37	1968	4.49	X	X	X	X
1964	2.69	-2.41	0.28	-0.82	1140	71	1955	4.75	2.66	-2.38	0.29	-0.41	1165	69	1968	4.49	X	X	X	X
1965	3.64	-3.16	0.48	-0.34	1150	69	1955	4.75	3.75	-3.07	0.68	0.27	1105	83	1968	4.49	X	X	X	
1966	2.47	-4.08	-1.61	-1.95	>1380	0	1955	4.75	2.40	-3.93	-1.53	-1.26	>1380	0	1968	4.49	X	X	X	
1967	4.46	-3.18	1.28	-0.66	950	98	1955	4.75	4.43	-3.12	1.30	0.04	1020	94	1968	4.49	X	X	X	
1968	4.55	-3.60	0.95	0.29	1075	84	1955	4.75	4.65	-3.64	1.01	1.06	1090	86	1968	4.49	X	X	X	
1969	2.66	-4.83	-2.17	-1.89	>1380	0	1968	4.82	2.58	-4.92	-2.34	-1.28	>1380	0	1968	4.49	X	X	X	X
1970	2.60	-3.83	-1.23	-3.12	>1380	0	1968	4.82	2.65	-3.78	-1.13	-2.41	>1380	0	1968	4.49	X	X	X	X
1971	4.29	-3.35	0.94	-2.18	1140	75	1968	4.79	4.14	-3.31	0.83	-1.58	1130	77	1968	4.49	X	X	X	X
1972	3.82	-3.70	0.12	-2.06	1195	61	1968	4.79	3.78	-3.74	0.04	-1.54	1205	58	1968	4.49	X	X	X	X
1973	4.67	-2.49	2.18	0.13	<870	100	1968	4.79	4.57	-2.56	2.01	0.47	<869	100	1968	4.49	X	X	X	X
1974	3.57	-2.54	1.03	1.15	1065	90	1968	4.79	3.49	-2.63	0.85	1.32	1080	87	1968	4.49	X	X	X	X
1975	4.64	-3.43	1.21	2.37	1050	92	1968	4.79	4.32	-3.50	0.82	2.14	undef.		1968	4.49	X	X	X	X
1976	4.40	-2.87	1.53	3.89	<870	100	1968	4.79	4.37	-3.06	1.31	3.45	<869	100	1968	4.49	X	X	X	
1977	2.33	-2.89	-0.56	3.33	1280	34	1968	4.79	2.31	-2.94	-0.63	2.82	undef.		1968	4.49	X	X	X	X
1978	2.56	-3.07	-0.51	2.82	1290	30	1968	4.82	2.46	-3.08	-0.62	2.20	1340	8	1988	4.17	X	X	X	
1979	3.28	-3.41	-0.13	2.70	1240	47	1968	4.82	3.20	-3.36	-0.15	2.05	1250	44	1988	4.17	X	X	X	
1980	2.51	-3.30	-0.79	1.90	1275	35	1968	4.82	2.48	-3.21	-0.73	1.32	1295	28	1988	4.17	X	X	X	
1981	4.04	-3.82	0.22	2.12	1210	56	1968	4.82	4.04	-3.81	0.24	1.56	1195	62	1988	4.17	X	X	X	X
1982	3.35	-3.48	-0.13	1.99	1240	46	1968	4.81	3.30	-3.31	-0.01	1.54	undef.		1988	4.17	X	X	X	X
1983	4.79	-3.19	1.60	3.60	1010	96	1968	4.81	4.61	-3.23	1.38	2.93	1005	97	1988	4.17	X	X	X	X
1984	4.09	-2.77	1.32	4.92	1050	92	1968	4.81	4.22	-2.85	1.38	4.30	undef.		1988	4.17	X	X	X	X
1985	2.44	-3.00	-0.56	4.36	1290	30	1968	4.82	2.48	-2.98	-0.50	3.81	1295	28	1988	4.17	X	X	X	X
1986	2.35	-2.76	-0.41	3.95	1255	42	1968	4.82	2.34	-2.95	-0.61	3.19	undef.		1988	4.17	X	X	X	X
1987	4.29	-2.22	2.07	6.02	<870	100	1968	4.82	4.45	-2.38	2.07	5.27	995	98	1988	4.17	X	X	X	X
1988	2.73	-5.21	-2.48	3.54	>1380	0	1968	4.82	2.69	-5.18	-2.50	2.77	>1376	0	1988	4.17	X	X	X	X
1989	5.20	-2.93	2.27	5.81	1030	94	1968	4.82	5.29	-2.95	2.34	5.11	1035	95	1988	4.17		X	X	
1990	5.98	-4.19	1.79	7.61	995	97	1968	4.82	5.96	-4.19	1.76	6.88	995	98	1988	4.17		X	X	
1991	4.09	-3.30	0.79	8.40	1035	93	1968	4.82	3.44	-2.87	0.57	7.44	1085	89	1988	4.17		X	X	X
1992	5.48	-3.19	2.29	10.69	1050	92	1968	4.82	5.48	-3.08	2.39	9.84	1020	96	1988	4.17		X	X	X
1993	4.81	-2.74	2.07	12.76	<870	100	1968	4.82	4.69	-2.82	1.87	11.71	<903	100	1997	4.48		X	X	X
1994	3.71	-2.92	0.79	13.54	925	99	1968	4.82	3.72	-2.92	0.80	12.51	975	98	1997	4.48		X	X	X
1995	5.10	-3.90	1.20	14.75	1120	80	1968	4.82	5.14	-3.91	1.23	13.74	1105	84	1997	4.48		X	X	X
1996	1.83	-3.71	-1.88	12.87	>1380	0	1968	4.82	1.87	-3.82	-1.94	11.80	>1383	0	1997	4.48		X	X	X
1997	4.22	-4.14	0.08	12.95	1200	63	1988	4.36	4.09	-4.25	-0.16	11.64	1220	55	1997	4.48		X	X	X
1998	3.66	-3.55	0.11	13.06	1240	50	1988	4.36	3.58	-3.60	-0.03	11.61	1225	53	1997	4.48		X	X	X
1999	4.61	-4.55	0.06	13.11	1245	48	1988	4.36	4.55	-4.47	0.08	11.69	1220	55	1997	4.48		X	X	X
2000	5.57	-3.58	1.99	15.10	1025	96	1988	4.36	5.17	-3.58	1.59	13.28	1055	91	1997	4.48		X	X	X
2001	1.86	-3.95	-2.09	13.01	>1382	0	1997	4.50	1.90	-3.97	-2.07	11.21	>1383	0	1997	4.48		X	X	X
2002	3.78	-5.31	-1.53	11.48	>1382	0	1997	4.50	3.69	-5.30	-1.62	9.59	>1383	0	1997	4.48		X	X	X
2003	2.52	-5.03	-2.50	8.98	>1382	0	1997	4.50	2.41	-4.98	-2.57	7.02	>1383	0	1997	4.48		X	X	X
2004	3.32	-3.42	-0.10	8.88	1225	53	1997	4.50	3.32	-3.35	-0.03	6.99	1225	51	2010	3.97		X	X	X
2005	4.99	-4.32	0.67	9.55	1135	78	1997	4.50	4.99	-4.21	0.77	7.77	1050	93	2010	3.97		X	X	X
2006	2.69	-5.88	-3.19	6.36	>1382	0	1997	4.50	2.65	-5.85	-3.19	4.57	>1368	0	2010	3.97		X	X	
2007	4.49	-3.22	1.27	7.63	1000	97	1997	4.50	4.49	-3.17	1.32	5.89	990	98	2010	3.97		X	X	
2008	4.04	-3.35	0.68	8.31	1130	79	1997	4.50	4.57	-3.78	0.79	6.68	1120	82	2010	3.97		X	X	X
2009	3.84	-4.00	-0.17	8.14	1240	48	1997	4.50	3.83	-3.95	-0.13	6.55	1235	48	2010	3.97		X	X	
2010	2.19	-4.03	-1.84	6.30	>1368	0	2010	3.97	2.19	-4.03	-1.84	4.71	>1368	0	2010	3.97				

*The ice divide from 2010 was used for all DEMs.
A-dist means whether the re-calculation of winter and summer mass balance was changed from area distributed (planimeter) to height distributed values.
DEM means whether the map base of the re-calculated mass balance series was changed.
Dvd means whether the ice divide from 2010 was used for the re-calculation.
Dens means whether the converting from snow depth to water equivalent is based on a mathematical trend line/function.

Table 9
Original and homogenized mass balance series for Hansebreen over the period 1986-2010. Homogenization of the year 2010 was not required.

	Original mass balance series								Homogenized mass balance series								Hom.gen. with regard to			
Year	Bw	Bs	Ba	ΣBa	ELA	AAR	DEM	Area	Bw	Bs	Ba	ΣBa	ELA	AAR	*DEM	Area	A-dist	DEM	Dvd	Dens
1986	2.28	-2.87	-0.58	-0.58	1200	35	1968	3.32	2.16	-2.86	-0.70	-0.70	undef.		1988	3.07	X	X	X	X
1987	3.76	-2.63	1.13	0.54	1100	73	1968	3.32	3.50	-2.62	0.88	0.18	1110	69	1988	3.07	X	X	X	X
1988	2.50	-5.25	-2.75	-2.21	>1320	0	1968	3.32	2.48	-5.23	-2.75	-2.57	>1318	0	1988	3.07	X	X	X	X
1989	4.13	-3.71	0.42	-1.79	1140	57	1968	3.32	4.06	-3.68	0.39	-2.19	1130	60	1988	3.07		X	X	
1990	4.42	-4.10	0.32	-1.47	1140	57	1968	3.32	4.08	-4.16	-0.09	-2.27	1155	50	1988	3.07		X	X	
1991	3.37	-3.11	0.26	-1.21	1125	63	1968	3.32	2.93	-3.02	-0.10	-2.37	1145	54	1988	3.07		X	X	X
1992	4.41	-3.43	0.97	-0.23	1125	63	1968	3.32	4.24	-3.57	0.67	-1.69	1130	60	1988	3.07		X	X	X
1993	4.23	-3.15	1.08	0.85	<925	100	1968	3.32	4.18	-3.27	0.91	-0.78	1075	82	1997	3.18		X	X	X
1994	3.39	-2.97	0.43	1.28	1120	65	1968	3.32	3.23	-2.86	0.36	-0.42	1115	69	1997	3.18		X	X	X
1995	4.38	-3.90	0.48	1.76	1140	57	1968	3.32	4.32	-3.80	0.52	0.09	1150	55	1997	3.18		X	X	X
1996	1.74	-3.76	-2.02	-0.26	>1320	0	1968	3.32	1.73	-3.54	-1.81	-1.72	>1325	0	1997	3.18		X	X	X
1997	3.77	-3.92	-0.15	-0.41	1160	47	1988	2.91	3.58	-3.80	-0.22	-1.94	1175	47	1997	3.18		X	X	X
1998	3.21	-3.51	-0.30	-0.71	1170	44	1988	2.91	3.17	-3.48	-0.32	-2.25	1185	44	1997	3.18		X	X	X
1999	4.30	-4.19	0.11	-0.60	1155	49	1988	2.91	4.31	-4.17	0.14	-2.11	1150	55	1997	3.18		X	X	X
2000	4.69	-3.82	0.86	0.26	1075	81	1988	2.91	4.63	-3.79	0.84	-1.27	1065	84	1997	3.18		X	X	X
2001	1.71	-4.43	-2.72	-2.46	>1327	0	1997	3.06	1.71	-4.46	-2.75	-4.02	>1325	0	1997	3.18		X	X	X
2002	3.51	-5.44	-1.93	-4.39	>1327	0	1997	3.06	3.54	-5.39	-1.85	-5.87	>1325	0	1997	3.18		X	X	X
2003	2.45	-5.12	-2.67	-7.06	>1327	0	1997	3.06	2.38	-4.99	-2.61	-8.48	>1325	0	1997	3.18		X	X	X
2004	2.87	-3.38	-0.50	-7.56	1220	31	1997	3.06	2.96	-3.60	-0.64	-9.12	1230	24	2010	2.75		X	X	X
2005	4.52	-4.61	-0.09	-7.65	1150	53	1997	3.06	4.53	-4.62	-0.09	-9.22	1145	52	2010	2.75		X	X	X
2006	2.45	-6.43	-3.98	-11.63	>1327	0	1997	3.06	2.41	-6.50	-4.08	-13.30	>1310	0	2010	2.75		X	X	
2007	4.07	-3.23	0.85	-10.79	1042	89	1997	3.06	4.07	-3.17	0.90	-12.40	1025	93	2010	2.75		X	X	
2008	3.90	-3.65	0.26	-10.53	1125	64	1997	3.06	4.25	-3.80	0.46	-11.95	1105	68	2010	2.75		X	X	X
2009	3.45	-4.42	-0.97	-11.50	>1327	0	1997	3.06	3.48	-4.39	-0.91	-12.86	>1310	0	2010	2.75		X	X	
2010	2.10	-4.31	-2.22	-13.71	>1310	0	2010	2.75	2.10	-4.31	-2.22	-15.08	>1310	0	2010	2.75				

*The ice divide from 2010 was used for all DEMs.
A-dist means whether the re-calculation of winter and summer mass balance was changed from area distributed (planimeter) to height distributed values.
DEM means whether the map base of the re-calculated mass balance series was changed.
Dvd means whether the ice divide from 2010 was used for the re-calculation.
Dens means whether the converting from snow depth to water equivalent is based on a mathematical trend line/function.



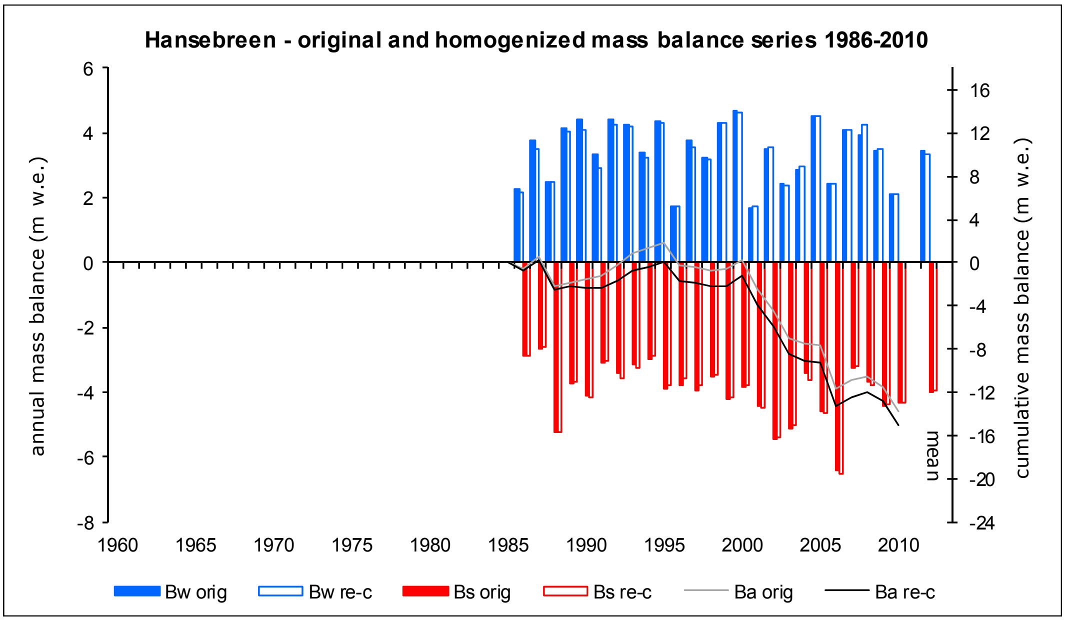


Figure 32
Original and homogenized mass balance series for Ålfotbreen (above) over 1963-2010 and Hansebreen (below) over 1986-2010.

4 Uncertainty

4.1 Glaciological mass balance

The accuracy of the glaciological mass balance measurements depends on several factors. There are three main sources of random and systematic errors in the glaciological method; the field measurements at point locations, the spatial averaging of these results over the entire glacier, and the changes of glacier in area and elevation (Zemp et al., 2013). In the following, some elements which can influence the measuring result are described.

4.1.1 Factors influencing uncertainty

The winter balance measurements are principally composed of assessing snow depth and measuring snow density.

Snow depth are mainly surveyed by soundings and often verified by stake readings and core drillings. The certainty of identifying the summer surface in each point by soundings is influenced of the snow depth, the snow density and the consistence of the summer surface. Great snow depths, high snow density and a soft summer surface will complicate the snow depth measurements. Sensing the summer surface by probing can then be very difficult, often resulting in overestimated snow depths. Stake readings are usually reliable, but errors can occur. Core drillings can be a proper method for measuring snow depth if probing is difficult or impossible. Generally, stake readings and core drillings are time-consuming, and hence these two methods are considered to be only a supplement to probing.

The uncertainty of snow density measurements can generally be influenced by several factors. When using a coring auger the core diameter can vary and is therefore often difficult to determine. Weighing the snow will also represent an uncertainty. Using either an analogue spring scale or a digital scale, it is important that the scale is calibrated, unless a biased error will occur. Measuring only the upper part of the snow pack (often in winters with heavy snowfalls) and how representative the location is will always affect the snow density measurements.

The quality of the winter balance measurements is also influenced of the density ratio of point measurements, where they are located, whether they are covering the entire glacier area and height intervals or not, and how representative they are. The smoothness of the snow layer will vary from glacier to glacier and from year to year. Hence, it is of great importance to define a proper measurement design for each glacier.

The summer and annual mass balance measurements consist mainly of stake readings. The density of remaining snow in the autumn has exceptionally been measured. Although the implementation of stake readings is quite simple, errors can occur. Erroneous reading of the folding rule, mistakes of stake IDs, stakes sinking, tilting to one side or melting out, and highly extended towers are sources of errors. The density of remaining snow in the autumn was normally assumed as 600 kg m^{-3}. The density of melted firn was, depending on the age, usually assumed to be between 650 and 800 kg m^{-3}, while the density of melted ice was always taken as 900 kg m^{-3}. These assumptions can of course be some divergent from the truth. The accuracy of the summer balance is also affected of the number of ablation stakes, the height distribution and how representative the stake locations are.

4.1.2 Field point measurements

Average snow depth on the glacier plateau is approximately 7 meters. In winters with heavy snowfalls 8-10 m can be measured, even up to 13 m was measured in some few extreme winters. Generally, the snow pack will be more solid with increasing snow depth. Probing through 10 m of snow can be difficult if the snow pack is solid. Verifying the summer surface, hence, is even more difficult. The summer surface will usually appear as a solid layer. After a cool summer, often combined with a great amount of remaining snow, this layer can hardly be perceptible with the probe. Generally, with suchlike conditions the probe will often be penetrated through the summer surface layer. Consequently, when the snow depth is great, the snow pack is solid or the summer surface is indistinct, measured snow depth will typically be higher than the true value. These situations are not unusual at Ålfotbreen.

4.1.3 Spatial interpolation

Accumulation

The spatial distribution and the number of point measurements will be also a source for uncertainty. The glacier surface at Ålfotbreen is quite even without any icefalls or heavy crevassed areas. Hence, the annual measurements were generally well distributed through the entire period of investigations. The number of snow depth measurements, however, has varied from year to year.

The glaciological mass balance was measured and calculated following the direct glaciological method as described in Østrem and Brugman (1991). However, systematic errors in the field data surveying can at least be an important contribution to the distinct discrepancy. The in situ measurements were basically comprised of data acquisition of winter accumulation and summer ablation. Assessing the winter *accumulation*, or rather the winter *balance*, requires measurements of snow depth and snow density. The summer balance was generally based on stake readings.

Another factor is the risk of overestimating snow depth during the implementation of probing. The summer surface can often be difficult to define, particularly after a cool summer in the previous year, or after a snow rich winter season. Thus, determination of summer surface will be a matter of experience and knowledge. All these indications will result in overestimated winter balance and underestimated summer balance and can, hence, be a significant contribution to the great discrepancy.

Ablation

Ablation has been measured on varying number of stakes. Over the first two decades, 12 to 19 stakes were usually maintained. From 1983 the number of stakes has generally varied between four and eight at Ålfotbreen and between two and five at Hansebreen. Independent of the number of stakes, a proper distribution of the stakes has always been ensured. As the Ålfotbreen ice cap has a large mass turnover with huge snow depths and great melting, maintaining the stakes can often be a challenge. It is not unusual that stakes were covered with snow in winter season and melted out in summer season.

The density of remaining snow in the autumn was measured for nine years (1964, 65, 68, 74, 75, 76, 83, 87 and 1993) on Ålfotbreen over the period 1963-2013. The mean density for the remaining snow pack over these years was 618 kg m^{-3}. Density of remaining snow was generally estimated as 600 kg m^{-3}. This estimate was considered to be sufficient for this purpose.

4.1.4 Glacier reference area changing over time

The glacier surface is continuously changing, thus affecting the elevation of all measurements and the relationship between area and elevation used for spatial interpolation. The fact that DEMs are available at certain points in time only, introduce an uncertainty factor in the estimates of glaciological mass balance from reference glacier area-elevation functions. Large changes and few DEMs give the high uncertainties, while frequent sampling or constant change give smaller uncertainties.

4.1.5 Quantification of uncertainty

Limited data exist to quantify the uncertainty in glaciological mass balance measurements. An error budget was estimated based on expert opinion from two glaciologist, one responsible for the observations and one fairly independent of the measurements, considering the main factors assumed to influence the error budget.

The uncertainty in the glaciological mass balance was estimated to be 0.32 m w.e. a^{-1}. These were the contributing factors, all in in m w.e. a^{-1}:

ε.glac.point.a	**0.26**
probing to summer surface	0.15
stakes and towers	0.20
density snow	0.05
density firn	0.02

ε.glac.spatial.a	**0.19**
interpolation stakes (no. of stakes per 100 m in elevation)	0.15
interpolation probings (no. of probings per 100 m in elevation)	0.10
ice fall and crevassed area not measured	0.05

ε.glac.ref.a	**0.05**
ice divide	0.01
DEM	0.05

4.2 Geodetic mass balance

The accuracy of the geodetic mass balance is influenced by the accuracy of the DEM's, the density conversion factor and the correction required because the field and aerial surveys were not carried out on the same date. In the following, these three elements are described.

4.2.1 Accuracy of the DEM's

The accuracy of the final DEM's depends on the mapping method (aerial photographs or LIDAR), the DEM processing, the topography (steep or flat slopes) and the meteorological conditions (snow cover and clouds). All these elements are described in chapter 2.1. and a qualitative evaluation of the DEM's is described in chapter 3.1.

4.2.2 Density conversion factor

The chosen density conversion factor (described in chapter 2.1.5.) was 850 kg m^{-3} with an uncertainty measure as ±60 kg m^{-3}. Hence, the maximum/minimum estimates were 790 and 910 kg m^{-3}, respectively.

4.2.3 Survey dates

The adjustments for different survey dates are described in chapter 2.1.6.

4.2.4 Quantification of uncertainty

The uncertainties for the geodetic mass balances were estimated to be

- 0.09 m w.e. a^{-1} for 1968-1988
- 0.08 m w.e. a^{-1} for 1988-1997 and
- 0.04 m w.e. a^{-1} for 1997-2010.

These estimates were derived using the formula for error propagation, considering these uncertainties and the number of years between the mapping:

- 1968-DEM:
  - Acceptable standard error at 95% confidence level: 0.60 m.
  - Sensor uncertainty: 2.00 m (large glacier with low contrast photos, snow)
- 1988-DEM:
  - Acceptable standard error at 95% confidence level: 0.37 m.
  - Sensor uncertainty: 0.50 m (large glacier with high contrast photos, little snow)
- 1997-DEM:
  - Acceptable standard error at 95% confidence level: 0.14 m.
  - Sensor uncertainty: 0.50 m (large glacier with high contrast photos, little snow)
- 2010-DEM:
  - Acceptable standard error at 95% confidence level: 0.10 m
  - Sensor-related uncertainty: 0.1 m (laser)

Acceptable standard error at 95% confidence level was calculated from the standard deviation of elevation difference divided by the number of independent samples. The elevation differences were between the DEM and the reference DEM (which was 2010 laser DEM) outside the glacier.

The senor-related uncertainty was determined, by categorizing the sources of the DEMs into three categories with associated uncertainty levels:

- Laser scanning: 0.1 m
- Good aerial photos (small glacier or large with high contrast, little snow): 0.5 m
- Poor aerial photos (large glacier with low contrast, snow): 2.0 m.

4.3 Internal mass balance

Internal and basal balances are not observed. Thus, these terms were calculated for this study using the methods described in Oerlemans (2013) and Alexander et al. (2011). Basal mass balance was included in the calculation of internal balance. Ablation inside and underneath the glacier due to heat of dissipation was calculated. Ablation due to rain was considered negligible, as most of this melting affects snow, firn and ice at the surface, rather than the subglacial and basal system. Other terms such as geothermal heat and refreezing of melt water below the previous summer' surface were considered negligible. These factors are believed to be less influential in this climate.

Ablation due to heat of dissipation was calculated for each elevation interval used in surface mass balance for the given glacier. Melt by dissipation of energy, M, was calculated by the formula

$$M = \frac{\sum_h g * ph * ah * (h - bL)}{A * Lm}$$

where g is the acceleration of gravity, h is mean elevation of elevation interval used in surface mass balance calculations, ph is precipitation at h, ah is glacier area of elevation interval h, bL is bed elevation at glacier snout, A is total glacier area and Lm is latent heat of fusion. Precipitation was defined as a linear function of elevation. Daily precipitation was extracted from data version 1.1.1 at www.senorge.no (Saloranta, 2014a and 2014b) at the highest elevation of the glacier, and the gradient was selected to give an annual precipitation 1.5 times the measured winter balance.

The internal mass balance melt by dissipation was quantified as −0.06 m w.e. a^{-1} for Ålfotbreen and −0.04 m w.e. a^{-1} for Hansebreen.

The uncertainty was assumed to be one third of the calculated internal mass balance, which amounts to 0.02 m w.e. a^{-1} for Ålfotbreen and 0.01 m w.e. a^{-1} for Hansebreen.

5 Comparison and calibration

5.1 Comparison of glaciological and geodetic mass balances

Results from the glaciological, geodetic and internal mass balance calculations as well as the uncertainties are shown in table 10.

Table 10
Results of the uncertainty analysis. B is (glaciological (B glac), geodetic (B geod) and internal (B int)) mass balance and σ is the estimated random error for the three balances. All mass balances and errors are in m w.e. a^{-1}. Δ is the difference between geodetic and glaciological balance, corrected for internal balance.

glacier/ period	years	B glac	σ.glac. point	σ.glac. spatial	σ.glac. ref	B geod	σ.geod. DTM	σ.dc	B int	σ.B. int	Δ
Ålfotbreen/ 1968-1988	20	0.09	0.26	0.19	0.05	−0.12	0.09	0.01	−0.06	0.02	0.15
Ålfotbreen/ 1988-1997	9	0.99	0.26	0.19	0.05	0.87	0.08	0.05	−0.06	0.02	0.06
Ålfotbreen/ 1997-2010	13	−0.53	0.26	0.19	0.05	−1.05	0.04	0.06	−0.06	0.02	0.46
Hansebreen/ 1988-1997	9	0.07	0.26	0.19	0.05	0.61	0.05	0.04	−0.04	0.01	−0.58
Hansebreen/ 1997-2010	13	−1.01	0.26	0.19	0.05	−1.34	0.06	0.08	−0.04	0.01	0.29

The results show little to moderate discrepancies for Ålfotbreen 1988-1997 (0.06 m w.e. a^{-1}) and Ålfotbreen 1968-1988 (0.15 m w.e. a^{-1}).

However, large deviations were found for Ålfotbreen 1997-2010 (0.46 m w.e. a^{-1}), Hansebreen 1988-1997 (−0.58 m w.e. a^{-1}) and Hansebreen 1997-2010 (0.29 m w.e. a^{-1}). An opportune question is why these results differ so much from the two first periods for Ålfotbreen. Both the glaciological and the geodetic method have infirmity and inaccuracies in data surveying and calculation as described in chapter 2. The evaluations of the DTMs indicated some elevation errors, particularly in the 1968 and 1988 DTMs. These probable errors were taken into account and DTMs were adjusted. It was hence, supposed that the principal errors can be related to the glaciological mass balance record.

5.2 Calibration

The period 1997-2010 for Ålfotbreen, and the periods 1988-1997 and 1997-2010 for Hansebreen had a relative discrepancy (δ) above 1.96, which suggest the geodetic and glaciological series are different when uncertainties in both series are accounted for. Thus these series were calibrated. The other two series for Ålfotbreen (1968-1988 and 1988-1997) were not calibrated. Table 11 shows the results from all tested periods.

Table 11
Comparison of glaciological and geodetic mass balances. Δ (in m w.e. a^{-1}) is the difference over the period of record between cumulative glaciological balance and geodetic balance, corrected for internal ablation. δ (dimensionless) is the reduced discrepancy, where uncertainties are accounted. β is the probability of accepting H0 although the results of both methods are different at the 95 % confidence level, while ε (in m w.e. a^{-1}) is the limit for detection of bias. Bold is used to highlight periods with less than 10 years length, differences larger than 0.20 mm w.e. a^{-1} and reduced discrepancies larger than 1.96.

Glacier	Period	Δ	δ	H0	β	ε
Ålfotbreen	1968-1988	0.15	1.26	yes	76	0.43
Ålfotbreen	**1988-1997**	0.06	0.43	yes	93	0.51
Ålfotbreen	1997-2010	**0.46**	**3.84**	no	3	0.43
Hansebreen	**1988-1997**	**−0.58**	**−4.69**	no	0	0.44
Hansebreen	1997-2010	**0.29**	**2.14**	no	43	0.49

The annual periodic glaciological mass balance for Ålfotbreen 1997-2010, Hansebreen 1988-1997 and Hansebreen 1997-2010 needed to be corrected with −0.46 m w.e. a^{-1}, +0.58 m w.e. a^{-1} and −0.29 m w.e. a^{1}, respectively. Whether the discrepancies are a result of a bias in winter or summer balance were not proved. Thus, corrections of both winter and summer balances were applied. The percentual distribution (winter vs. summer) of the annual corrections can be done in several ways. In this calibration, the winter and summer corrections were assessed according to the size of the balance values; the greater balance value, the greater part of the correction. For instance, for the year 1996 the homogenized B_w and B_s for Hansebreen were 1.73 and −3.54 m w.e., respectively. The annual correction for the period 1989-1997 (+0.58 m w.e.) was then distributed as 32 % ((1.73/(1.73+3.54))*100) to B_w, and 68 % ((3.54/(1.73+3.54))*100) to B_s, resulting in calibrated B_w as 1.92 m w.e. (1.73+(0.58*32 %)), and B_s as −3.15 m w.e. (−3.54+(0.58*68 %)). Winter, summer and annual balance curves for Hansebreen in 1996 before and after the calibration are shown in figure 33.

The original, homogenized and calibrated cumulative mass balance for Ålfotbreen over 1963-2010 were +6.3, +4.7 and −1.2 m w.e., respectively. The calibrated mean annual balance values for Ålfotbreen over 1963-2010 were 3.61 (B_w), −3.63 (B_s) and −0.03 m w.e. (B_a), respectively. The homogenized and original (in brackets) mean values for Ålfotbreen over 1963-2010 were 3.66 (3.70), −3.57 (−3.57) and 0.10 (0.13) m w.e., respectively.

The homogenized (1963-2010) and calibrated (1998-2010) mass balance series for Ålfotbreen were significant positive (>0.30 m w.e.) in 22 years, significant negative (<0.30 m w.e.) in 20 years and approximately in balance in six years.

The original, homogenized and calibrated cumulative mass balance for Hansebreen over 1986-2010 were −13.7, −15.1 and −13.7 m w.e., respectively. The calibrated mean annual balance values for Hansebreen over 1986-2010 were 3.40 (B_w), −3.95 (B_s) and −0.55 m w.e. (B_a), respectively. The homogenized and original (in brackets) mean values for Hansebreen over 1986-2010 were 3.36 (3.42), −3.96 (−3.97) and −0.60 (−0.55) m w.e., respectively.

The homogenized (1986-2010) and calibrated (1989-2010) mass balance series for Hansebreen were significant positive in 11 years, significant negative in 12 years and in balance in two years.

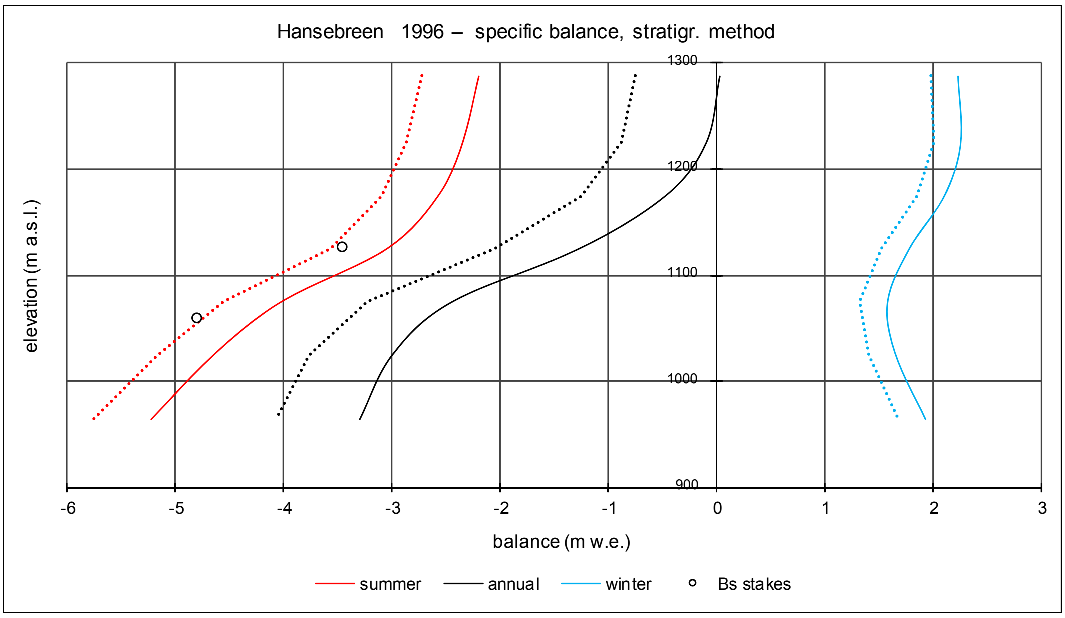


Figure 33
Winter, summer and annual balance curves for Hansebreen in 1996 before (dotted) and after (solid) the calibration. Summer balance at each stake is also shown (○).

The Equilibrium-line altitude (ELA) and the Accumulation-area ratio (AAR) were also influenced by the calibration as they were calculated from the mass balance curves. From the homogenized to the calibrated mass balance series, the ELA for Ålfotbreen over 1998-2010 was elevated between 30 and 135 m. For Hansebreen the ELA was lowered 25-45 m and elevated 10-40 m over the two periods 1989-1997 and 1998-2010, respectively. Accordingly, the mean AAR over the same years was decreased from 44 % to 36 % for Ålfotbreen. Correspondingly, the AAR for Hansebreen was increased from 53 % to 65 % (1989-1997) and decreased from 32 % to 27 % (1998-2010), respectively.

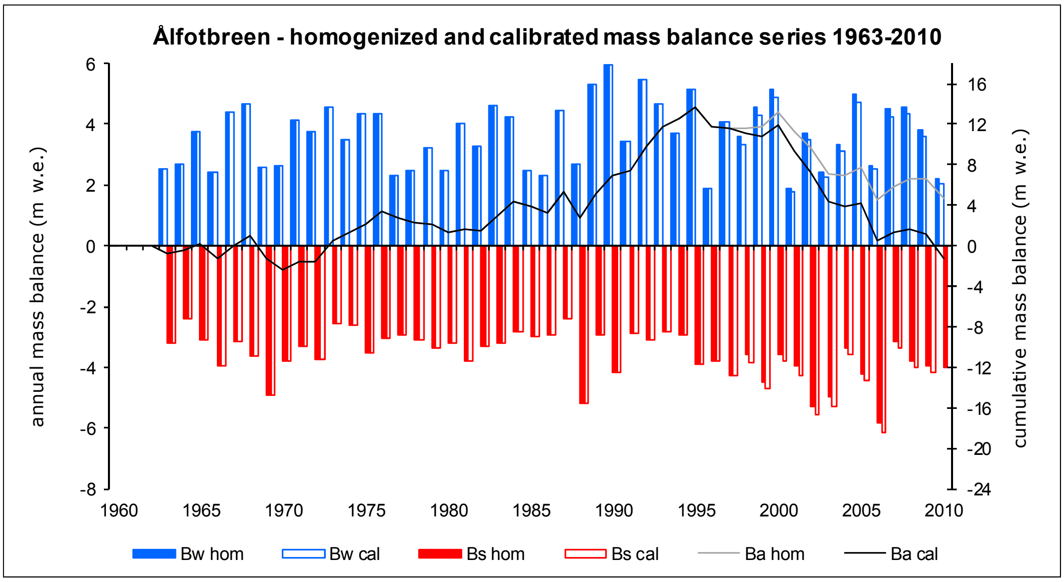
The homogenized and calibrated mass balance series for Ålfotbreen 1963-2010 and Hansebreen 1986-2010 are shown in table 12, Table 13 and figure 34.

Table 12
Homogenized and calibrated mass balance series for Ålfotbreen over 1963-2010.

	Homogenized mass balance series								Calibrated mass balance series					
Year	B_w	B_s	B_a	ΣB_a	ELA	AAR	DEM	Area	B_w	B_s	B_a	ΣB_a	ELA	AAR
1963	2.52	-3.21	-0.70	-0.70	1270	37	1968	4.49	2.52	-3.21	-0.70	-0.70	1270	37
1964	2.66	-2.38	0.29	-0.41	1165	69	1968	4.49	2.66	-2.38	0.29	-0.41	1165	69
1965	3.75	-3.07	0.68	0.27	1105	83	1968	4.49	3.75	-3.07	0.68	0.27	1105	83
1966	2.40	-3.93	-1.53	-1.26	>1380	0	1968	4.49	2.40	-3.93	-1.53	-1.26	>1380	0
1967	4.43	-3.12	1.30	0.04	1020	94	1968	4.49	4.43	-3.12	1.30	0.04	1020	94
1968	4.65	-3.64	1.01	1.06	1090	86	1968	4.49	4.65	-3.64	1.01	1.06	1090	86
1969	2.58	-4.92	-2.34	-1.28	>1380	0	1968	4.49	2.58	-4.92	-2.34	-1.28	>1380	0
1970	2.65	-3.78	-1.13	-2.41	>1380	0	1968	4.49	2.65	-3.78	-1.13	-2.41	>1380	0
1971	4.14	-3.31	0.83	-1.58	1130	77	1968	4.49	4.14	-3.31	0.83	-1.58	1130	77
1972	3.78	-3.74	0.04	-1.54	1205	58	1968	4.49	3.78	-3.74	0.04	-1.54	1205	58
1973	4.57	-2.56	2.01	0.47	<869	100	1968	4.49	4.57	-2.56	2.01	0.47	<869	100
1974	3.49	-2.63	0.85	1.32	1080	87	1968	4.49	3.49	-2.63	0.85	1.32	1080	87
1975	4.32	-3.50	0.82	2.14	undef.		1968	4.49	4.32	-3.50	0.82	2.14	undef.	0
1976	4.37	-3.06	1.31	3.45	<869	100	1968	4.49	4.37	-3.06	1.31	3.45	<869	100
1977	2.31	-2.94	-0.63	2.82	undef.		1968	4.49	2.31	-2.94	-0.63	2.82	undef.	0
1978	2.46	-3.08	-0.62	2.20	1340	8	1988	4.17	2.46	-3.08	-0.62	2.20	1340	8
1979	3.20	-3.36	-0.15	2.05	1250	44	1988	4.17	3.20	-3.36	-0.15	2.05	1250	44
1980	2.48	-3.21	-0.73	1.32	1295	28	1988	4.17	2.48	-3.21	-0.73	1.32	1295	28
1981	4.04	-3.81	0.24	1.56	1195	62	1988	4.17	4.04	-3.81	0.24	1.56	1195	62
1982	3.30	-3.31	-0.01	1.54	undef.		1988	4.17	3.30	-3.31	-0.01	1.54	undef.	0
1983	4.61	-3.23	1.38	2.93	1005	97	1988	4.17	4.61	-3.23	1.38	2.93	1005	97
1984	4.22	-2.85	1.38	4.30	undef.		1988	4.17	4.22	-2.85	1.38	4.30	undef.	0
1985	2.48	-2.98	-0.50	3.81	1295	28	1988	4.17	2.48	-2.98	-0.50	3.81	1295	28
1986	2.34	-2.95	-0.61	3.19	undef.		1988	4.17	2.34	-2.95	-0.61	3.19	undef.	0
1987	4.45	-2.38	2.07	5.27	995	98	1988	4.17	4.45	-2.38	2.07	5.27	995	98
1988	2.69	-5.18	-2.50	2.77	>1376	0	1988	4.17	2.69	-5.18	-2.50	2.77	>1376	0
1989	5.29	-2.95	2.34	5.11	1035	95	1988	4.17	5.29	-2.95	2.34	5.11	1035	95
1990	5.96	-4.19	1.76	6.88	995	98	1988	4.17	5.96	-4.19	1.76	6.88	995	98
1991	3.44	-2.87	0.57	7.44	1085	89	1988	4.17	3.44	-2.87	0.57	7.44	1085	89
1992	5.48	-3.08	2.39	9.84	1020	96	1988	4.17	5.48	-3.08	2.39	9.84	1020	96
1993	4.69	-2.82	1.87	11.71	<903	100	1997	4.48	4.69	-2.82	1.87	11.71	<903	100
1994	3.72	-2.92	0.80	12.51	975	98	1997	4.48	3.72	-2.92	0.80	12.51	975	98
1995	5.14	-3.91	1.23	13.74	1105	84	1997	4.48	5.14	-3.91	1.23	13.74	1105	84
1996	1.87	-3.82	-1.94	11.80	>1383	0	1997	4.48	1.87	-3.82	-1.94	11.80	>1383	0
1997	4.09	-4.25	-0.16	11.64	1220	55	1997	4.48	4.09	-4.25	-0.16	11.64	1220	55
1998	3.58	-3.60	-0.03	11.61	1225	53	1997	4.48	3.35	-3.83	-0.48	11.15	1255	43
1999	4.55	-4.47	0.08	11.69	1220	55	1997	4.48	4.32	-4.69	-0.37	10.78	1265	40
2000	5.17	-3.58	1.59	13.28	1055	91	1997	4.48	4.90	-3.77	1.14	11.92	1105	84
2001	1.90	-3.97	-2.07	11.21	>1383	0	1997	4.48	1.76	-4.28	-2.52	9.39	>1383	0
2002	3.69	-5.30	-1.62	9.59	>1383	0	1997	4.48	3.50	-5.57	-2.07	7.32	>1383	0
2003	2.41	-4.98	-2.57	7.02	>1383	0	1997	4.48	2.26	-5.29	-3.03	4.29	>1383	0
2004	3.32	-3.35	-0.03	6.99	1225	51	2010	3.97	3.09	-3.58	-0.48	3.81	1265	36
2005	4.99	-4.21	0.77	7.77	1050	93	2010	3.97	4.74	-4.42	0.32	4.13	1185	65
2006	2.65	-5.85	-3.19	4.57	>1368	0	2010	3.97	2.51	-6.16	-3.65	0.48	>1368	0
2007	4.49	-3.17	1.32	5.89	990	98	2010	3.97	4.23	-3.36	0.86	1.34	1055	92
2008	4.57	-3.78	0.79	6.68	1120	82	2010	3.97	4.32	-3.99	0.33	1.67	1160	72
2009	3.83	-3.95	-0.13	6.55	1235	48	2010	3.97	3.60	-4.19	-0.58	1.09	1315	18
2010	2.19	-4.03	-1.84	4.71	>1368	0	2010	3.97	2.03	-4.33	-2.30	-1.21	>1368	0

Table 13
Homogenized and calibrated mass balance series for Hansebreen over 1986-2010

	Homogenized mass balance series								Calibrated mass balance series					
Year	B_w	B_s	B_a	ΣB_a	ELA	AAR	DEM	Area	B_w	B_s	B_a	ΣB_a	ELA	AAR
1986	2.16	-2.86	-0.70	-0.70	undef.		1988	3.07	2.16	-2.86	-0.70	-0.70	undef.	0
1987	3.50	-2.62	0.88	0.18	1110	69	1988	3.07	3.50	-2.62	0.88	0.18	1110	69
1988	2.48	-5.23	-2.75	-2.57	>1318	0	1988	3.07	2.48	-5.23	-2.75	-2.57	>1318	0
1989	4.06	-3.68	0.39	-2.19	1130	60	1988	3.07	4.36	-3.40	0.96	-1.61	1095	74
1990	4.08	-4.16	-0.09	-2.27	1155	50	1988	3.07	4.36	-3.87	0.49	-1.12	1130	60
1991	2.93	-3.02	-0.10	-2.37	1145	54	1988	3.07	3.21	-2.73	0.48	-0.64	1105	71
1992	4.24	-3.57	0.67	-1.69	1130	60	1988	3.07	4.55	-3.31	1.25	0.61	1095	74
1993	4.18	-3.27	0.91	-0.78	1075	82	1997	3.18	4.50	-3.02	1.48	2.09	<929	100
1994	3.23	-2.86	0.36	-0.42	1115	69	1997	3.18	3.53	-2.59	0.94	3.03	1085	79
1995	4.32	-3.80	0.52	0.09	1150	55	1997	3.18	4.62	-3.53	1.09	4.12	undef.	
1996	1.73	-3.54	-1.81	-1.72	>1325	0	1997	3.18	1.92	-3.16	-1.24	2.88	>1325	0
1997	3.58	-3.80	-0.22	-1.94	1175	47	1997	3.18	3.86	-3.50	0.36	3.24	1130	63
1998	3.17	-3.48	-0.32	-2.25	1185	44	1997	3.18	3.03	-3.64	-0.60	2.64	1195	41
1999	4.31	-4.17	0.14	-2.11	1150	55	1997	3.18	4.16	-4.31	-0.14	2.49	1165	50
2000	4.63	-3.79	0.84	-1.27	1065	84	1997	3.18	4.47	-3.92	0.55	3.04	1105	73
2001	1.71	-4.46	-2.75	-4.02	>1325	0	1997	3.18	1.63	-4.67	-3.04	0.00	>1325	0
2002	3.54	-5.39	-1.85	-5.87	>1325	0	1997	3.18	3.43	-5.56	-2.14	-2.14	>1325	0
2003	2.38	-4.99	-2.61	-8.48	>1325	0	1997	3.18	2.28	-5.19	-2.90	-5.04	>1325	0
2004	2.96	-3.60	-0.64	-9.12	1230	24	2010	2.75	2.83	-3.75	-0.93	-5.97	>1310	0
2005	4.53	-4.62	-0.09	-9.22	1145	52	2010	2.75	4.39	-4.77	-0.38	-6.35	1170	43
2006	2.41	-6.50	-4.08	-13.30	>1310	0	2010	2.75	2.33	-6.71	-4.37	-10.72	>1310	0
2007	4.07	-3.17	0.90	-12.40	1025	93	2010	2.75	3.91	-3.30	0.61	-10.11	1060	85
2008	4.25	-3.80	0.46	-11.95	1105	68	2010	2.75	4.10	-3.93	0.17	-9.95	1120	62
2009	3.48	-4.39	-0.91	-12.86	>1310	0	2010	2.75	3.35	-4.55	-1.20	-11.15	>1310	0
2010	2.10	-4.31	-2.22	-15.08	>1310	0	2010	2.75	2.00	-4.51	-2.50	-13.66	>1310	0



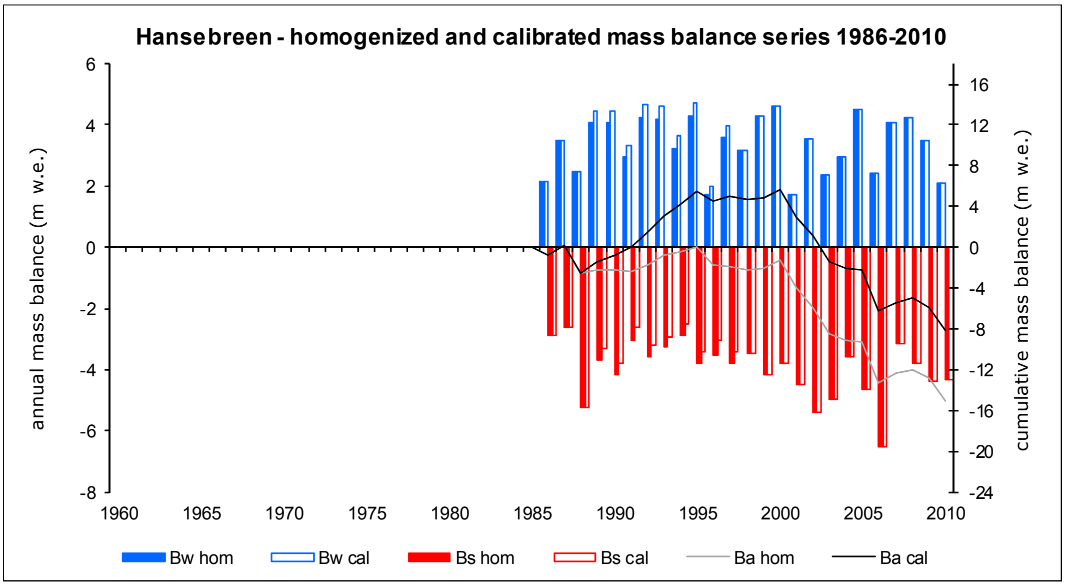


Figure 34
Homogenized and calibrated mass balance series for Ålfotbreen 1963-2010 (upper) and Hansebreen 1986-2010 (lower).

6 Conclusions

The aim of this report was to re-analyse the glaciological mass balance series at Ålfotbreen and Hansebreen based on comparison with geodetic mass balance. The analysed glaciological mass balance series cover the periods from 1963 to 2010 for Ålfotbreen and from 1986 to 2010 for Hansebreen. Within this period, usable Digital Terrain Models (DTMs) from 1968, 1988, 1997 and 2010 were available. Glaciological and geodetic mass balance were compared for the periods 1969-88, 1989-97 and 1998-2010 at Ålfotbreen and for the periods 1989-97 and 1998-2010 at Hansebreen.

In order to obtain comparable values the glaciological and the geodetic mass balances were first reviewed, adjusted and homogenized. The homogenized glaciological cumulative mass balance for Ålfotbreen over the years 1969-88, 1989-97 and 1998-2010 was +1.71, +8.87 and −6.93 m w.e., respectively. The corresponding geodetic mass balance was −2.43, +7.81 and −13.59 m w.e., respectively. The homogenized glaciological cumulative mass balance for Hansebreen over the years 1989-97 and 1998-2010 was +0.64 and −13.14 m w.e., respectively. The corresponding geodetic mass balance was +5.45 and −17.42 m w.e., respectively. The internal mass balance was quantified as −0.06 m w.e. a^{-1} at Ålfotbreen and −0.04 m w.e. a^{-1} at Hansebreen. Accordingly the mean annual differences ($\Delta_a = B_{a\ glac.} - B_{a\ geod.} + B_{a\ int.}$) over 1969-88, 1989-97 and 1998-2010 for Ålfotbreen were 0.15, 0.06 and 0.46 m w.e., respectively. The mean annual differences over 1989-97 and 1998-2010 for Hansebreen were −0.58 and +0.29 m w.e., respectively. Hence, a calibration of the period 1998-2010 for Ålfotbreen and for the periods 1989-1997 and 1998-2010 for Hansebreen were required.

The periodic annual corrections were spread over both winter and summer balances. The percentual distribution between winter and summer balance corrections was assessed according to the size of the balance values.

The homogenized (1963-2010) and calibrated (1998-2010) glaciological cumulative mass balance for Ålfotbreen over 1963-2010 was −1.2 m w.e., while the original mass balance over the same period was +6.3 m w.e. The homogenized (1986-2010) and calibrated (1989-2010) glaciological cumulative mass balance for Hansebreen was −13.7 m w.e. The original mass balance over the same period was also −13.7 m w.e.

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Norges
vassdrags- og
energidirektorat

NVE

Norges vassdrags- og energidirektorat

Middelthunsgate 29
Postboks 5091 Majorstuen
0301 Oslo

Telefon: 09575
Internett: www.nve.no