MANUAL ON PROCEDURES IN OPERATIONAL HYDROLOGY

VOLUME 4

STAGE-DISCHARGE RELATIONS AT STREAM GAUGING STATIONS

SECOND EDITION

NORWEGIAN WATER RESOURCES AND ENERGY ADMINISTRATION
HYDROLOGY DEPARTMENT

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ØSTEN A. TILREM

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This manual consists of five volumes dealing with:

1. Establishment of Stream Gauging Stations
2. Operation of Stream Gauging Stations
3. Stream Discharge Measurement by Current Meter, by Dilution and by the Slope-Area Method
4. Stage-Discharge Relations at Stream Gauging Stations
5. Sediment Transport in Streams—Sampling, Analysis and Computation

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PREFACE

This Manual on Procedures in Operational Hydrology is a second, revised and enlarged edition of the original Manual, which was prepared jointly by Tanzanian and Norwegian authorities, when the author, Østen A. Tilrem, served as hydrologist in Western Tanzania. The first edition of the Manual was financially supported by the Norwegian Agency for International Development (NORAD). Mr. Tilrem has extensive experience in senior positions within hydrological co-operative programmes in both Latin America, Africa, and Asia.

The present edition of the Manual is produced by the Norwegian Water Resources and Energy Administration. It is intended for the use of field hydrologists and technicians, working under varying conditions in all parts of the world. The Manual should be useful as a reference book, which also provides textual depth for many topics.

Since the late seventies the micro-electronics field has developed rapidly, resulting in new possibilities for advanced capture and transmission of water-related data. Such advances are treated in the manual. The benefits of new technology are unfortunately often difficult to harvest, because of high costs, rapid changes in equipment design, or lack of supporting infrastructure. The field hydrologist should therefore also be familiar with traditional methods, which still have to be used in many parts of the world. Those traditional approaches may even have the additional benefit of providing a more direct insight in the hydrological processes taking place in the river or aquifer.

It is my sincere belief that users of the Manual will find it useful in their assessment of water resources.

Arne Tollan
Director

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ACKNOWLEDGEMENTS

The author has drawn on many sources for information contained in this Manual and is indebted to these. It is hoped that suitable acknowledgement is made in the form of references to these works. Special credit is due to the United States Geological Survey for permission to reproduce figures and use other material from the Water Supply Paper and the Techniques of Water-Resources Investigations series, and to the World Meteorological Organization (WMO) for permission to reproduce material from the Guide to Hydrological Practices and the Operational Hydrology Report series, and from other reports on hydrology.

Østen A. Tilrem

INTRODUCTION

1.1 General

This Volume describes methods and procedures for the determination of the stage-discharge relation by correlating water-level to discharge. Appendixes covering relevant statistical tests and definitions are included.

A stream-gauging station is a selected site on an open channel for making systematic observations for the purpose of determining records of the discharge and/or the stage of the stream. A gauging station can either be a recording or a nonrecording (manual) station. A nonrecording station consists usually of a staff gauge read regularly by an observer. At recording stations, the staff gauge is supplemented by a water-level sensor, usually a float-gauge or pressure transducer, attached to which is a recording device tracing the rise and fall of the water level analogically on a chart or digitally at small time intervals on an electronic memory.

The term *gauge height* is often used interchangeably both with water level and stage, the former being the more appropriate term when referring to stream gauging.

Stage and discharge of a stream both vary most of the time. In general, it is not practicable to measure the discharge continuously. However, to obtain a continuous record of the stage is relatively simple as explained above. Then, if a relation between stage and discharge can be established, an observed record of stage can easily be converted into a record of discharge. This two-step operation is the normal procedure for the determination of streamflow records.

The operations necessary to develop the stage-discharge relation include making a sufficient number of discharge measurements and establishing a discharge rating curve and is known as the calibration or rating of the station. The rating curve is developed by plotting measured discharges against the corresponding stages and drawing a smooth curve of relation between these two quantities.

When a new river gauging station has been established, the general practice is initially to carry out a series of discharge measurements well distributed over the range of discharge variation in order to establish quickly the discharge rating curve. Normally, there are no difficulties involved in measuring the lower and medium discharges. However, to obtain measurements at the higher stages is often a difficult task and may take time. Thus, at a majority of gauging stations, discharge measurements are not available for the high flood stages and the rating curve must therefore be extrapolated beyond the range of the available measurements.

Few rivers have absolutely stable characteristics. The calibration therefore, can not be carried out once and for all, but has to be repeated as frequently as required by the rate of change in the stage-discharge relation. Thus, it is the stability of the stage-discharge relation that governs the number of discharge measurements that are necessary to define the relation at any time and to follow the temporal changes in the stage-discharge relation. If the channel is stable, comparatively few measurements are required. On the other hand, in order to define the stage-discharge relation in unstable stream channels up to several discharge measurements a month may be required because of random shifts in the channel geometry.

Sound hydrological practice requires that the discharge rating curve be determined as rapidly as possible after the establishment of a gauging station. Unless the discharge rating curve is properly established and maintained, the record of stage can not be converted into a reliable record of discharge.

1.2 The Station Control

A prerequisite for an analysis of stage-discharge relations and the construction of the discharge rating-curve, is an insight into and appreciation of the functioning of stage-discharge controls on streams and rivers.

In order to have a permanent and stable stage-discharge relation, the stream channel must be capable of stabilising and regulating the flow so that for a given stage, the discharge will always be the same. The shape, reliability and stability of the stage-discharge relation are usually controlled by a section or a reach of channel at or downstream from the gauging station, known as the *station control*, the geometry of which eliminates the effects of all other downstream features on the velocity of flow at the station site. The channel characteristics forming the control include the cross-sectional area and shape of the stream channel, the channel sinuosity, the expansions and restrictions of the channel, the stability and roughness of the streambed and banks, and the aquatic vegetation in the channel, all of which collectively constitute the factors determining the channel conveyance.

In terms of open channel hydraulics, a control is a critical-depth control, generally termed a *section control,* if a critical-flow section exists a short distance downstream from the gauging station, or a *channel control* if the stage-discharge relation depends mainly on channel irregularities and channel friction over a reach downstream from the station. A control is *permanent* if the stage-discharge relation it defines does not change with time, otherwise it is impermanent and generally called a *shifting control*. From the standpoint of origin, a control is either *artificial* or *natural* depending on whether it is man-made or not.

Natural controls vary widely in geometry and stability. Some controls consist of a single topographic feature, such as a rock ledge crossing the channel at the crest of a rapid or waterfall, forming a *complete control* independent of all downstream conditions at all stages. Some are formed by a combination of two or more features, such as a rock ledge crossing the channel combined with a channel constriction. Some are V-shaped and thus sensitive to changes in discharge, some are U-shaped and thus less sensitive. Some consist of two or more interacting controls each effective in a particular range of stage. This is termed a *compound control*; a common situation is that section control is effective at low flow only and submerged by channel control at the higher discharges. Some controls consist of a long reach of stable bed extending downstream as the stage increases. In general, the distance covered by such controls vary inversely with the slope of the stream and increases as the stage of the stream rises. The tendency of a control to extend farther downstream as the stage rises, has a marked effect on the stage-discharge relation. As the stage increases, low-water and medium-water controlling elements are drowned out and new downstream elements are successively introduced causing a straightening out of the typical parabola curvature of the rating curve, and at times even causing a reversal of this curvature. In fact, in rivers with very flat slopes the station control may extend so far downstream that backwater complications, which do not exist at lower stages, are introduced at higher stages.

The simplest and most satisfactory type of control is formed by a rock ledge at the head of a rapid or at the crest of a waterfall. Firstly, it ensures permanency; secondly, it creates a pool or forebay in which a gauging station is often easily constructed; thirdly, favourable conditions for carrying out discharge measurements may be frequently found within the reach of such a pool; and fourthly, the point of zero flow (see later) is easily located and surveyed in this situation. Whenever practical this type of control should be utilized for a stream-gauging station.

It should be recognized that most natural controls are shifting more or less. However, a *shifting control* is considered to exist where the stage-discharge relation changes frequently, either gradually or abruptly because of changes in the physical features that form the control of the station. The controlling features may be modified by a number of factors. Principal among these are:

a) Scour and fill in an unstable channel,
b) Growth and decay of aquatic vegetation,
c) Formation of an ice cover,
d) Variable backwater in a uniform channel,
e) Variable backwater submerging a control section,
f) Rapidly changing discharge,
g) Overflow and ponding in areas adjoining the stream channel.

The corresponding stage-discharge relations are illustrated in Fig. 1. A short discussion of each case follows.

Permanent Control

Fig. 1A. If the control is permanent, occasional discharge measurements need to be made to verify the permanency. The stage-discharge relation for a permanent control can be expressed as a simple exponential function. (Section 2.2).

Sand-Bed Channel

Fig 1B. The movement of fluvial sediments, particularly in channels in alluvium, affects the conveyance, the hydraulic roughness, the channel sinuosity, and the energy slope. This makes the de-

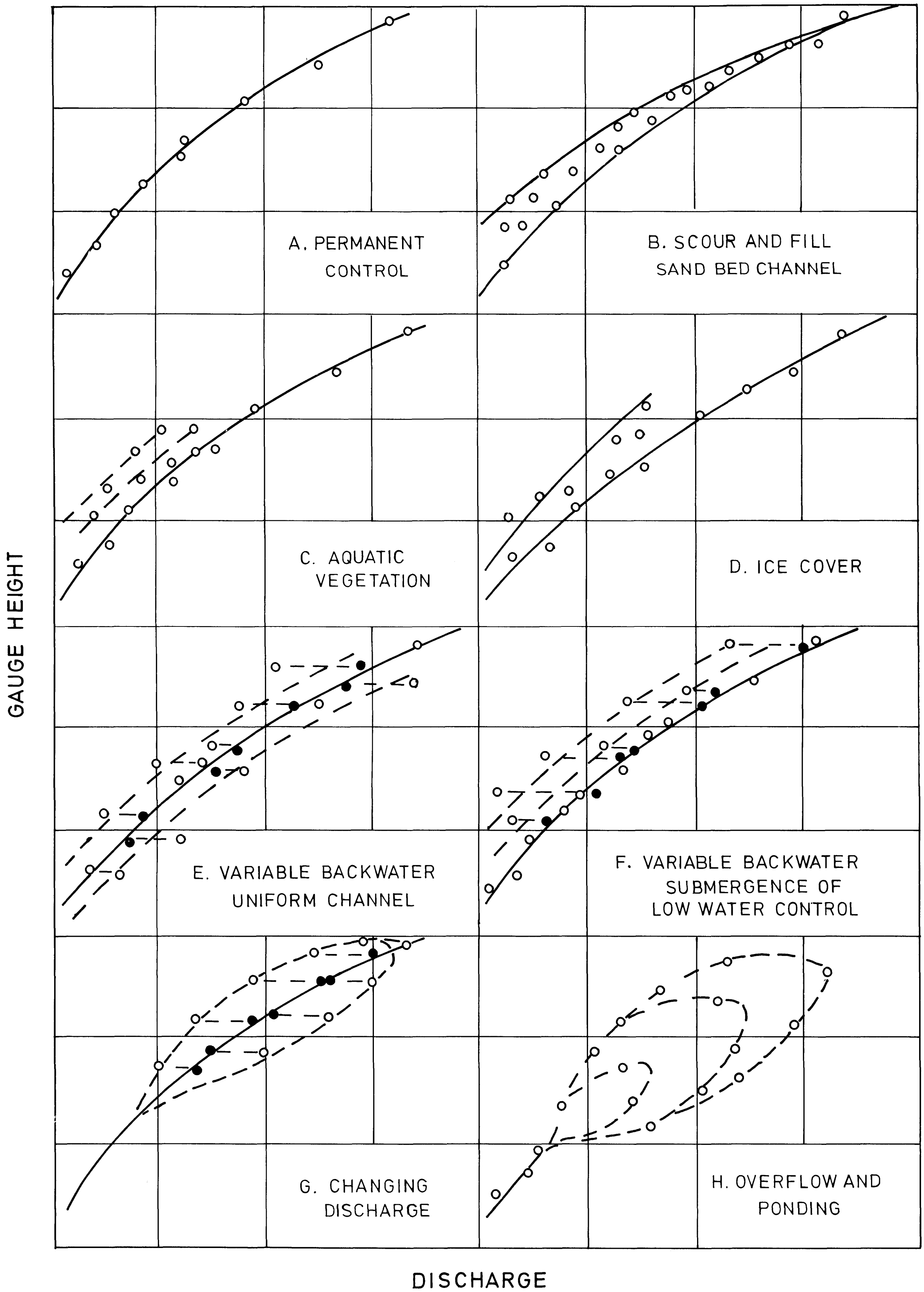


Figure 1. Rating curves for different hydraulic conditions (courtesy of the US Geological Survey).

termination of a stage-discharge relation difficult. In addition, since these movements are erratic, determination of the temporal variation of the stage-discharge relation is also complex (Section 2.3).

Aquatic Vegetation

Fig 1C. The growth of aquatic vegetation decreases the conveyance of the channel and changes the roughness with the result that the stage for a given discharge is increased. The converse is true when the vegetation dies and the stage-discharge relation gradually returns to its previous condition. The change in vegetational growth should be closely observed and determined by a series of discharge measurements.

Ice-cover

Fig 1D. Ice in a stream channel increases the hydraulic radius and the roughness and decreases the cross-sectional area. As with aquatic vegetation, the stage for a given discharge is increased. The effect of ice formation and thawing is complex and the temporal stage-discharge relation can only be determined by a series of discharge measurements using stage, temperature, and precipitation records as guide for interpolation between the measurements.

Variable Backwater—Uniform Channel

Fig. 1E. If the control reach has within it a weir or dam, a diversion or a confluent tributary which can increase or decrease the energy gradient for a given discharge, a *variable backwater* is introduced. That is, the slope in a reach is increased or decreased from the normal. In such cases, an auxiliary gauge is installed at some distance downstream from the main gauge in order to measure the fall for developing a *stage-fall-discharge* relation (Section 2.4).

Variable Backwater—Submergence

Fig 1F. Some channel reaches below gauging stations contain local control sections such as falls, rapids or a dam which determine the stage-discharge relation at low flow, but which may be submerged at times by inflow from a tributary downstream or by the operation of a dam. As in the case of rating a station with uniform channel and variable backwater, a second gauge is installed below this control section in order to measure the fall (Section 2.4).

Rapidly Changing Discharge

Fig. 1G. At some gauging stations, generally those of low energy slope, the stage-discharge relation is affected by the rate of change of discharge. If the discharge is increasing rapidly, it will be greater than that for zero rate of change, and conversely, if it is rapidly decreasing it will be less (Section 2.4.4).

Overflow and Ponding

Fig. 1H. At some gauging stations there are large overflow and ponding areas on the flood plains adjacent to the stream channel. During increasing discharge, a part of the flow goes into these areas increasing the slope and discharge relative to stage. Conversely, when the discharge decreases, water returning to the channel from the flooded areas causes backwater and the discharge for a given stage is decreased. Each flood produces its own loop rating. No satisfactory method has been found to develop a single rating under these conditions. A loop rating is required for each flood and must be determined by a series of discharge measurements.

The Point of Zero Flow

When constructing discharge rating curves, the *point of zero flow* (that is the gauge height of zero flow), is an important information especially helpful for shaping the lower part of the curve. The point of zero flow is the gauge height at which the water ceases to flow over the control. This gauge height should be determined by field surveys whenever the flow is sufficiently low to allow an accurate determination.

Stream gauges are usually established at an arbitrary datum. The elevation of the gauge zero is set below the lowest stage anticipated at the site. It is therefore only in few cases that the zero of the gauge will correspond by coincidence to the point of zero flow.

The control section is defined by surveying a close grid of spot-levels over a reach of the stream downstream from the station site or by surveying a sufficient number of cross sections. The point of zero flow will be the lowest point in the controlling section. In those cases the control is well defined by a rocky barrier over which the water flows, usually, it is easy to locate the point of zero flow and obtain its correct gauge height value.

Determination of the stage of zero flow from soundings taken during current-meter measurements is not possible. These soundings might have been taken in any cross section of the river in the vicinity of the gauge and will only give the correct stage if the soundings happened to be taken in that particular cross section containing the control.

Bibliography

Chapter 1: [6], [13].

CHAPTER 2

DEVELOPMENT OF THE DISCHARGE RATING CURVE

2.1 General

The discharge rating-curve is established from a graphical analysis of discharge measurements that are plotted on graph paper, either arithmetically or logarithmically ruled. A correct analysis of the proper shape and position of the rating curve requires a knowledge of the channel characteristics at the particular site in question, a knowledge of open channel hydraulics, and considerable experience and judgment.

In stream gauging, single-gauge stations and twin-gauge stations are employed. The use of single-gauge stations depends on the assumption that the stage in a cross section is a unique function of the discharge only. Where variable backwater is present, the stage is no longer a single-valued function of the discharge. In these cases a twin-gauge station has to be employed where the stage is observed at each end of a channel reach.

2.2 Simple Stage-Discharge Relations

The rating curve as developed for a single-gauge station, will give the discharge under steady or slowly changing discharge. Now, under conditions of rapidly changing discharge, the discharge at a given stage is significantly greater at rising stage and lower at falling stage than at steady flow conditions. However, it is still possible to approximately compute the true discharge under significant unsteady flow using the single-gauge rating curve, see Section 2.4.4.

2.2.1 Graphic Plot of Discharge Measurements

The general procedure in establishing the stage-discharge relation is as follows:

a) Traditionally, the discharge measurements are plotted on graph paper with discharge (the unknown variable) on the horizontal scale and the corresponding gauge height (the known variable) on the vertical scale. If a measurement was made under slowly changing discharge, the mean gauge height during the measurement is used.
b) The plotted data points are labelled in their chronologic order; rising and falling stage during the measurement should be indicated by distinguishing symbols.
c) The relation should be defined by a sufficient number of measurements suitably distributed throughout the whole range in stage, taking into account the shape of the stage-discharge relation. As a rule, the measurements should be spaced closer at the lower end of the range.
d) Ideally, the number and spacing of the measurements should conform to the relative frequency of flow at the various stages. That is, the number of measurements at various sub-ranges should be in proportion to the probable occurrence of discharge at these same ranges, covering the whole range of discharge for which the relation is plotted. Nevertheless, in actual practice it is desirable to have as many measurements as possible at the extreme ranges, both at the low flow and at the high flood stages.
e) The rating curve should be drawn evenly and smoothly through the scatter of plotted data points.

Although all discharge measurements have been checked and considered correct before plotting, measurements which plot more than 4 % in discharge off the curve should be checked again for possible errors. Look especially for the need to adjust or weigh the gauge height, for the use of the correct current-meter calibration table, and for errors in the computation of the discharge measurement. With respect to the latter, it is suggested that a plot be made of the cross-sectional areas and the corresponding mean velocities against the gauge height for the measurements. Such plots will reveal the presence of an error and where it is located in the computation, either in the velocity or in the cross-sectional area. If no apparent error is found,

Table 1. Smoothing and extrapolation of discharge rating curve.

	Discharge, m³/s					
Gauge	From curve			Smoothed		
Heigh m	Q	ΔQ	Δ^2Q	ΔQ	Δ^2Q	Q
.20	0.800					0.800
		2.30		2.30		
.30	3.10		1.40		1.30	3.10
		3.70		3.60		
.40	6.80		1.30		1.20	6.70
		5.00		4.80		
.50	11.8		1.00		1.30	11.50
		6.00		6.10		
.60	17.8		1.20		1.20	17.60
		7.20		7.30		
.70	25.0		1.30		1.20	24.90
		8.50		8.50		
.80	33.5		1.50		1.10	33.40
		10.0		9.60		
.90	43.5		0.50		1.10	43.00
		10.5		10.7		
1.00	54.0		0.50		1.10	53.70
		11.0		11.8		
1.10	65.0		1.00		1.0	65.50
		13.0		12.8		
1.20	78.0		0.00		1.0	78.30
		13.0		13.8		
1.30	91.0		2.00		0.9	92.10
		15.0		14.7		
1.40	106.0		1.00		0.8	106.8
		16.0		15.5		
1.50	122.0		0.00		0.8	122.3
		16.0		16.3		
1.60	138.0		0.00		0.7	138.6
		16.0		17.0		
1.70	154.0		2.00		0.6	155.6
		18.0		17.6		
1.80	172.0				(0.6)	173.2
				(18.2)		
1.90					(0.5)	(191.4)
				(18.7)		
2.00					(0.5)	(210.1)
				(19.2)		
2.10						(229.3)

Figures in parentheses are extrapolated

then the measurement be discarded or shift correction applied if applicable.

There are several methods of fitting a curve to measured data. This may be done quite satisfactorily simply by *visual estimation* of the plot with the aid of ship drafting curves (see Appendix C). Ship drafting curves are useful in this respect as these curves are designed to conform to parabolic equations. Very often the trend of discharge measurements plotted on graph paper will closely follow a particular drafting curve due to the fact that the discharge of a stream tends to vary as some power of the depth of flow.

The criterion used when fitting a curve to plotted data points by visual estimation, is that there should be about the same number of plus and minus deviations (a deviation with respect to discharge being negative for a measurement lying to the left of the curve and positive when lying to the right). In other words, one is developing a median curve. In general, at least 10 discharge measurements well distributed over the range in stage are

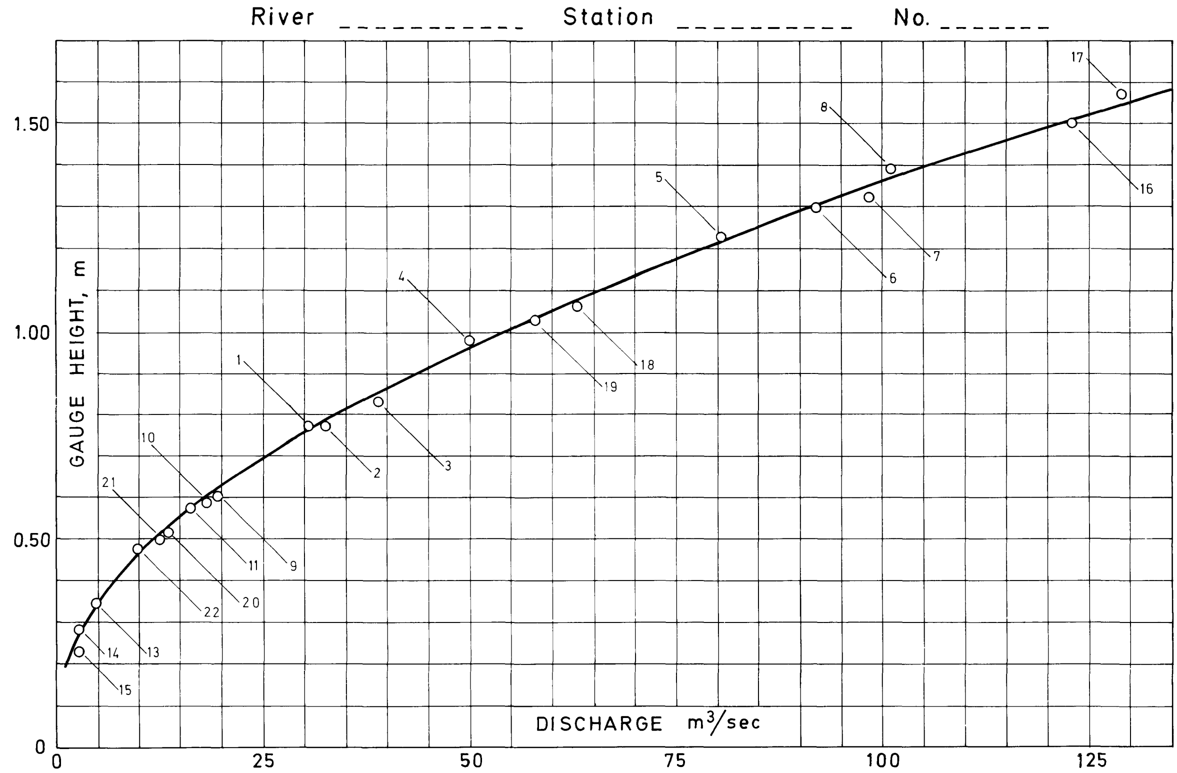


Figure 2. Stage-discharge curve, first plot.

considered desirable when constructing the initial rating curve. Assuming that these measurements were properly made under reasonably steady flow conditions, and that they apply to a stable stage-discharge relation, it is then reasonable to expect that it should be possible to fit a mean curve from which any individual discharge measurement will not deviate more than 4 %, which is about the maximum error of observation likely to occur under these conditions. Using group averages when fitting a curve is also quite useful; the method is illustrated in Appendix A.

2.2.2 *The Series of Differences Method*

A method sometimes used in fitting a smooth curve to measured data and of extrapolating the curve, is by use of the *series of differences*. The discharge measurements are plotted on ordinary graph paper and a mean curve is fitted to the data points by visual estimation (Fig. 2). At equal gauge height increments, say every 0.05 m or 0.10 m, the discharge is read from the curve and tabulated as illustrated in Table 1, columns 1 and 2:

a) The series of 1st differences, that is the differences between adjacent discharges, are computed and entered in the table (column 3). The 1st differences will either increase evenly or remain the same as the preceding difference. The series of differences is graphically smoothed as illustrated in Fig. 3, and each difference is plotted against its corresponding gauge height, the plotting position being the mid-interval of the gauge height.
b) A mean curve is fitted to the plotted points and the smoothed differences are read from this curve and entered in the table (column 5). An adjusted version of the original discharge series is thereafter calculated by successively adding the smoothed differences starting from the top (column 7). The first value, 0.800 m^3/s, is taken from column 2.

The method may be refined by introducing the series of 2nd differences (Table 1, column 4 and 6). The 2nd differences must also progress evenly between adjacent figures, but unlike the 1st differences, they may progress both upwards or downwards, or remain constant. When the 2nd differences change to a downward progression, this indicates a reversal in the rating curve, which is often the case when a section control is drowned out by a downstream control at the higher stages.

A rating curve established by this method may be extrapolated to a certain extent if the station control does not have any sharp breaks in the cross

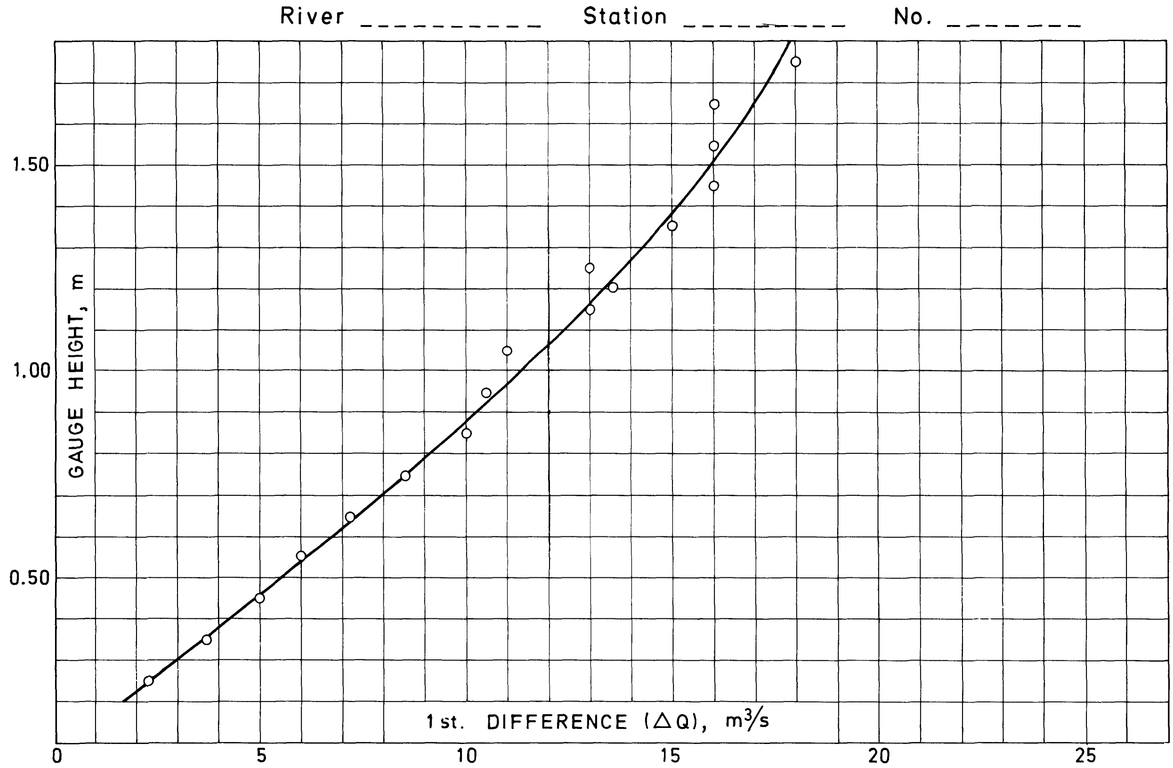


Figure 3. Smoothing graph for series of first differences.

section contour, or is not of a different character at the higher stages. For extrapolation purposes the series of differences is extended by following the apparent trend of the series and the rating curve is calculated accordingly. [4].

2.2.3 The Logarithmic Method

The logarithmic representation of the stage-discharge relation is commonly used because it produces the best graphical form of a standard rating curve and readily adapts to the use of drafting curves. Also, the logarithmic form of the rating curve can be developed into a straight line, or straight line segments, by adding or subtracting a constant value to the gauge height scale on the logarithmic graph paper. There are several other advantages that the logarithmic form has, such as

a) A percentage distance off the curve is always the same regardless of where it is located. Thus, a measurement that is 10 % off the curve at high stage will be the same distance away from the curve as a measurement that is 10 % off at low stage.
b) Halving, doubling or adding a percentage to the gauge height has no effect, the curve will merely shift position but retain the same shape.
c) It is easy to identify the range in stage for which different controls are effective.
d) The logarithmic form may be described by a simple mathematical equation that is easily handled by electronic computers.
e) The curve can easily be extrapolated.

Regarding extrapolations, however, one has to be careful. If the control does not change character at the higher stages, the same discharge equation will cover the whole range in stage and the rating curve can be extrapolated up to the highest observed water level. If the control changes either shape or character as the stage increases, the rating curve will consist of more than one segment. In these cases, an extrapolation of the first segment up to the higher stages will of course introduce serious errors.

2.2.3.1 Theory of the Logarithmic Rating Curve

According to the Chezy equation for uniform flow

$$V = C(RS)^{1/2} \tag{2.1}$$

in which V is the mean velocity, C is a factor of flow resistance, R is the hydraulic radius, and S is the slope of the energy line.

The discharge is given by

$$Q = AC(AS/P)^{1/2} \tag{2.2}$$

in which Q is the discharge, A the cross-sectional area, and P the wetted perimeter. In rectangular cross sections, width (W) times depth (D) can be substituted for A, and for P can be substituted (W + 2D), of which follows

$$Q = CWD(WDS/(W+2D))^{1/2} \tag{2.3}$$

or

$$Q = CWS^{1/2}\, D^{3/2}\, (W/(W+2D))^{1/2} \tag{2.4}$$

For very wide channels (W+2D) is approximately equal to W and therefore Eqn. (2.4) reduces to

$$Q = CWS^{1/2}\, D^{3/2} \tag{2.5}$$

Regarding $CWS^{1/2}$ as a constant, K, which is approximately correct in most cases, Eqn. (2.5) can be written as

$$Q = KD^{3/2} \tag{2.6}$$

For an approximate uniform and rectangular stream channel with stage equal to a local gauge height H, and the gauge height of the point of zero flow is H_0, (see Section 1.2), Eqn (2.6) can be written as

$$Q = K(H-H_0)^{3/2} \tag{2.7}$$

Similarly, it can be shown for sections of other shapes that

$$Q = K(H-H_0)^n \tag{2.8}$$

where

n = 3/2 for a rectangular cross section as above,
n = 2 for a concave section,
n = 5/2 for a triangular or semi-circular section.

The general equation of the relation between stage and discharge is therefore

$$Q = K(H-H_0)^n \tag{2.9}$$

Eqn. (2.9) is a parabolic equation where Q is the discharge, H is the gauge height, K and n are constants, and H_0 is the gauge height of the point of zero flow. Thus, H_0 will have a positive value when the zero of the gauge is situated below the point of zero flow, and a negative value when situated above.

The equation plots as a straight line on double logarithmic graph paper.

Eqn. (2.9) applies to cross sections of rectangular, triangular, trapezoidal, parabolic and other geometrically simple sections. Many natural streams approximate to these shapes making Eqn. (2.9) a general discharge equation.

The approximation that W equals (W+2D) in determining Eqn. (2.9) is valid only for wide streams. For deep, narrow streams, W is much smaller than (W+2D), which has the effect of increasing the exponent in Eqn. (2.9). Changes in the factor of flow resistance C and slope S with stage will also affect the exponent. The net result of all these factors is that the exponent in Eqn. (2.9) for relatively wide rivers with channel control will generally vary from 1.3 to 1.8 and rarely exceed 2.0. For relatively deep narrow rivers with section control the exponent n will almost always be greater than 2.0 and may often exceed a value of 3.0. [7].

However, for irregular channels and significantly nonuniform flow, Eqn. (2.9) can not be expected to apply throughout the entire range of the stage. Sometimes the curve changes from a parabolic to an odd curve or vice versa, and sometimes the constants and exponents vary throughout the range.

In fact, the logarithmic discharge equation is seldom a single straight line or a gentle curve through the entire range in stage at a gauging station. Even if the same channel cross section is the control for all stages, a sharp break in the contour of the cross section causes a break in the slope of the rating curve. Also, the other constants, K and n, in Eqn. (2.9) are related to the physical characteristics of the stage-discharge control.

If the control section changes location at various stages, it becomes necessary to fit two or more equations, each corresponding to the portion of the range over which a particular control is applicable. If, however, too many changes in the parameters are necessary in order to define the relation, it is possible that the logarithmic method may not be suitable, and a curve fitted by visual estimation should rather be employed.

The 1st derivative of Eqn. (2.9) is a measure of the change in discharge per unit change in stage, that is the 1st derivative gives the first-order differences of the discharge series.

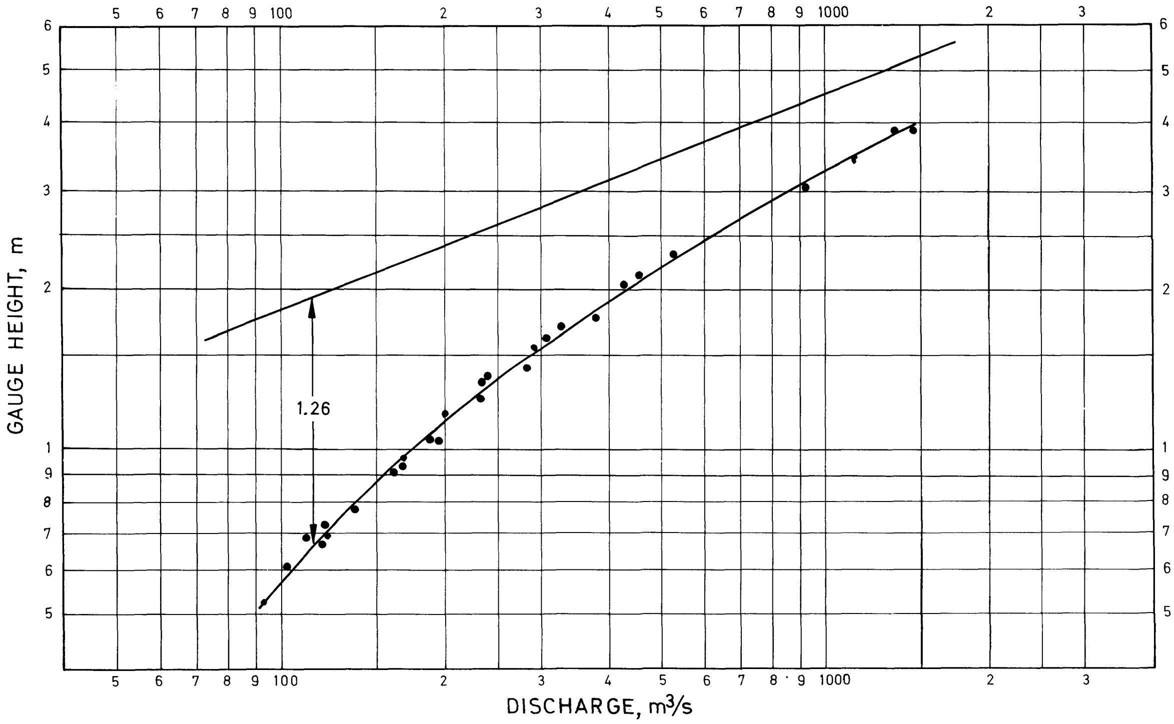


Figure 4. Trial and error method of determining H_0.

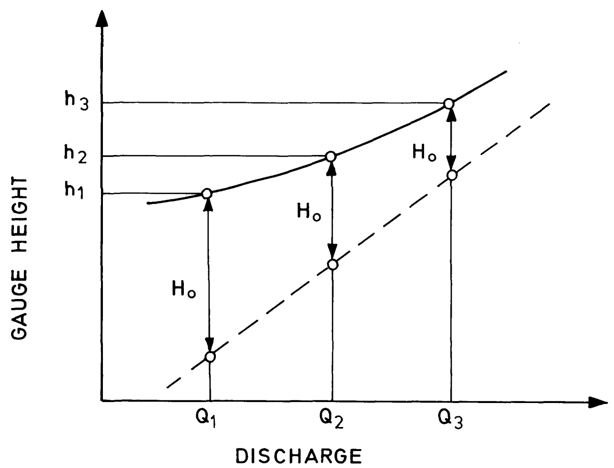
The 1st derivative is

$$dQ/dH = nK\,(H - H_0)^{(n-1)} \qquad (2.10)$$

The second-order differences are obtained by differentiating again. The 2nd derivative is

$$d^2Q/dH^2 = nK(n-1)(H - H_0)^{(n-2)} \qquad (2.11)$$

An examination of the 2nd derivative shows that the 2nd order differences increase with stage when n is greater than 2.0, that is for section control, and decrease with stage when n is less than 2.0, that is for channel control.



Figur 5. Transforming a curved line into a straight line on double logarithmic graph paper.

In Table 1 an inspection of the 2nd differences in column 6 reveals that the illustrated rating is for a compound control. This rating represents the condition of section control at the lower stages drowned out by channel control at the higher stages. Inspection of the 2nd differences column shows the 2nd differences to be increasing at the low-water end, that is section control and n > 2, and decreasing at the high-water end, that is channel control and n < 2. [7].

2.2.3.2 Estimating H_0

There are three methods of estimating the gauge height level of the point of zero flow (Section 1.2.1), apart from making a field survey. However, if at all possible these estimates should always be sought verified by field investigations. The three methods are:

a) The Trial and Error Procedure,
b) The Arithmetical Procedure,
c) The Graphical Procedure.

2.2.3.3 Trial and Error Procedure of Estimating the Point of Zero Flow

All discharge measurements available are plotted on double logarithmic graph paper (log-log paper) and a median line is balanced through the scatter of data points. Usually, this line will be a curved line. Various trial values, one value for each trial, are added or subtracted to the gauge heights of the measurements until the plot obtained forms a straight line. The trial value forming the straight line is the value of H_0 (Fig. 4).

All the plotted data points may be used in the trial operation. However, it is better to use only a few points selected from the median line first fitted to the points.

2.2.3.4 Arithmetical Procedure of Estimating the Point of Zero Flow

All discharge measurements are plotted on log-log paper (Fig. 5). An average line drawn through the scatter of points has resulted in the solid curved line. Three values of discharge, Q_1, Q_2, and Q_3, are selected in geometric progression, that is, two values Q_1 and Q_3 are chosen from the curve, the third value Q_2 is then computed according to

$$Q_2^2 = Q_1 \times Q_3 \qquad (2.12)$$

The corresponding gauge heights read from the plot are H_1, H_2, and H_3. It is now possible to verify that [16]

$$H_0 = \frac{H_1 H_3 - H_2^{\,2}}{H_1 + H_3 - 2H_2} \qquad (2.13)$$

The solid curved line may now be transformed into a straight line by subtracting H_0 from each value of the gauge height H and replotting the new values. (Fig. 5).

2.2.3.5 Graphical Procedure of Estimating the Point of Zero Flow

As above, three values of discharge in geometric progression are selected, but this time from a plot on arithmetical graph paper. The points are A, B, and C as illustrated in Fig. 6.

Vertical lines are drawn through A and B and horizontal lines are drawn through B and C to intersect the verticals at D and E respectively. Let DE and AB meet at F, then the ordinate of F is the value of H_0. [15].

The last two methods are based on the assumption that the lower part of the stage-discharge relation, including the selected points, is a part of a parabola. In most cases this assumption holds and

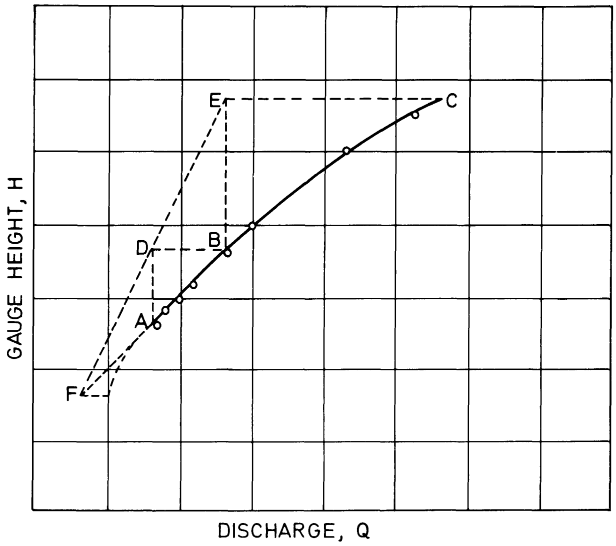


Figure 6. Graphical determination of H_0.

the method will give acceptable results on the condition that there are enough discharge measurements available to satisfactorily define the curvature of the lower part of the rating curve.

2.2.3.6 Estimating the Constants K and n

After a straight line plot of the discharge measurements has been obtained on log-log graph paper, the plot is first analyzed in order to establish whether the rating curve is composed of one or more straight line segments, each having its own constants K and n. The constants for each separate segment are calculated separately. There are three different procedures by which the two constants K and n of Eqn. (2.9) can be worked out:

a) The Arithmetical Procedure,
b) The Least Squares Procedure,
c) The Graphical Procedure.

A series of discharge measurements at a gauging station are given below in Table 2. By a field survey, the point of zero flow has been found to have a level of H_0 = -1.26 m on the gauge height scale. The constants K and n of flow Eqn. (2.9) shall be determined. The data are given in metres and m^3/s.

2.2.3.7 Arithmetical Procedure of Estimating K and n

To the gauge heights of the measurements given in Table 2, 1.26 m is added. The measurements are

Table 2. Table of Discharge Measurements

No.	H	Q	No.	H	Q
1	1.55	300	15	3.87	1374
2	1.44	287	16	2.33	540
3	1.26	235	17	3.49	1152
4	1.05	193	18	3.93	1452
5	0.73	125	19	2.03	440
6	0.69	113	20	1.61	306
7	0.70	124	21	2.13	469
8	1.70	340	22	1.37	246
9	0.96	169	23	1.05	189
10	0.94	168	24	0.91	163
11	1.35	240	25	0.79	139
12	1.17	202	26	0.68	120
13	1.79	387	27	0.61	104
14	3.09	930	28	0.53	94.6

thereafter plotted on log-log graph paper, the gauge heights on the vertical scale and the discharges on the horizontal scale. The plot defines a straight line (Fig. 7).

Select two points on this line as far from each other as possible but within the range of measured discharges. Let the two selected points, (Q, H) be given in m^3/s and metres as (97, 1.80) and (1300, 5.00).

In general, the equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) in a rectangular coordinate system is written as

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1} \tag{2.14}$$

Similarly, a logarithmic linear function can be drawn as a straight line on log-log paper, of which follows

$$\frac{\log y - \log y_1}{\log x - \log x_1} = \frac{\log y_2 - \log y_1}{\log x_2 - \log x_1} \tag{2.15}$$

In the present case, after changing the notations, it follows that

$$\frac{\log Q - \log Q_1}{\log (H-H_0) - \log H_1} = \frac{\log Q_2 - \log Q_1}{\log H_2 - \log H_1} \tag{2.16}$$

Substituting the values of the two points (97, 1.80) and (1300, 5.00), and $H_0 = -1.26$ into Eqn. (2.16) gives

$$\frac{\log Q - \log 97}{\log (H+1.26) - \log 1.80} = \frac{\log 1300 - \log 97}{\log 5.00 - \log 1.80}$$

$$\frac{\log Q - 1.9868}{\log (H+1.26) - 0.2553} = \frac{3.1139 - 1.9868}{0.6990 - 0.2553} = 2.54$$

$$\log Q = 2.54 \log (H+1.26) + 1.3383$$

$$Q = 21.79\,(H+1.26)^{2.54} \tag{2.17}$$

Eqn. (2.17) is the rating equation for the stage-discharge relation curve as shown in Fig. 7.

As already discussed, it is often found that the discharge measurements do not plot as one straight line all through the range of stage, but will diverge at a certain stage. In such cases, the rating curve will be composed of two, or even more, straight line segments differing in slope and each segment having its own particular equation as illustrated in Fig. 8. Here it appears that the logarithmic plot has a curvature above 3.50 m on the gauge, and that the upper part of the curve has to be moved down 1.70 m in order to plot as a straight line.

At this station the zero of the gauge seems to be set at the point of zero flow, since the lower part of the curve is a straight line on log-log graph paper. At about 3.50 m a high-water control downstream is taking effect decreasing the rate of increase in channel conveyance with stage. [4].

2.2.3.8 The Least Squares Procedure of Estimating K and n

The values of K and n may be computed statistically by the Method of least squares. That is, the sum of the squares of the deviations between the logarithms of the measured discharge and the estimated discharge from the curve should be a minimum.

According to this, the values of K and n are obtained from the following equations:

$$\sum(Y) - m \log K - n\sum(X) = 0 \tag{2.18}$$

$$\sum(XY) - \sum(X) \log K - n\sum(X^2) = 0 \tag{2.19}$$

where

$\sum(Y)$ = the sum of all values of log Q,
$\sum(X)$ = the sum of all values of log $(H-H_0)$,
$\sum(X^2)$ = the sum of all values of the square of (X),

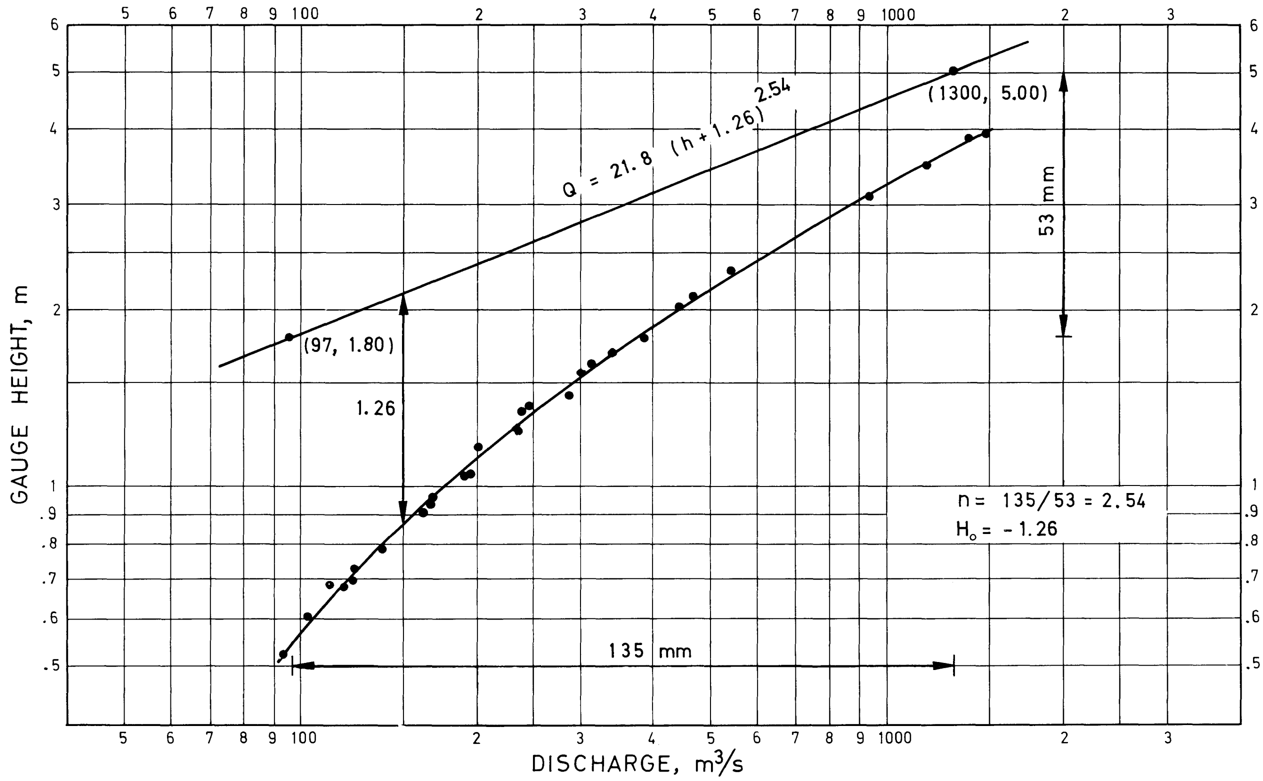


Figure 7. Stage-discharge curve established by the logarithmic method.

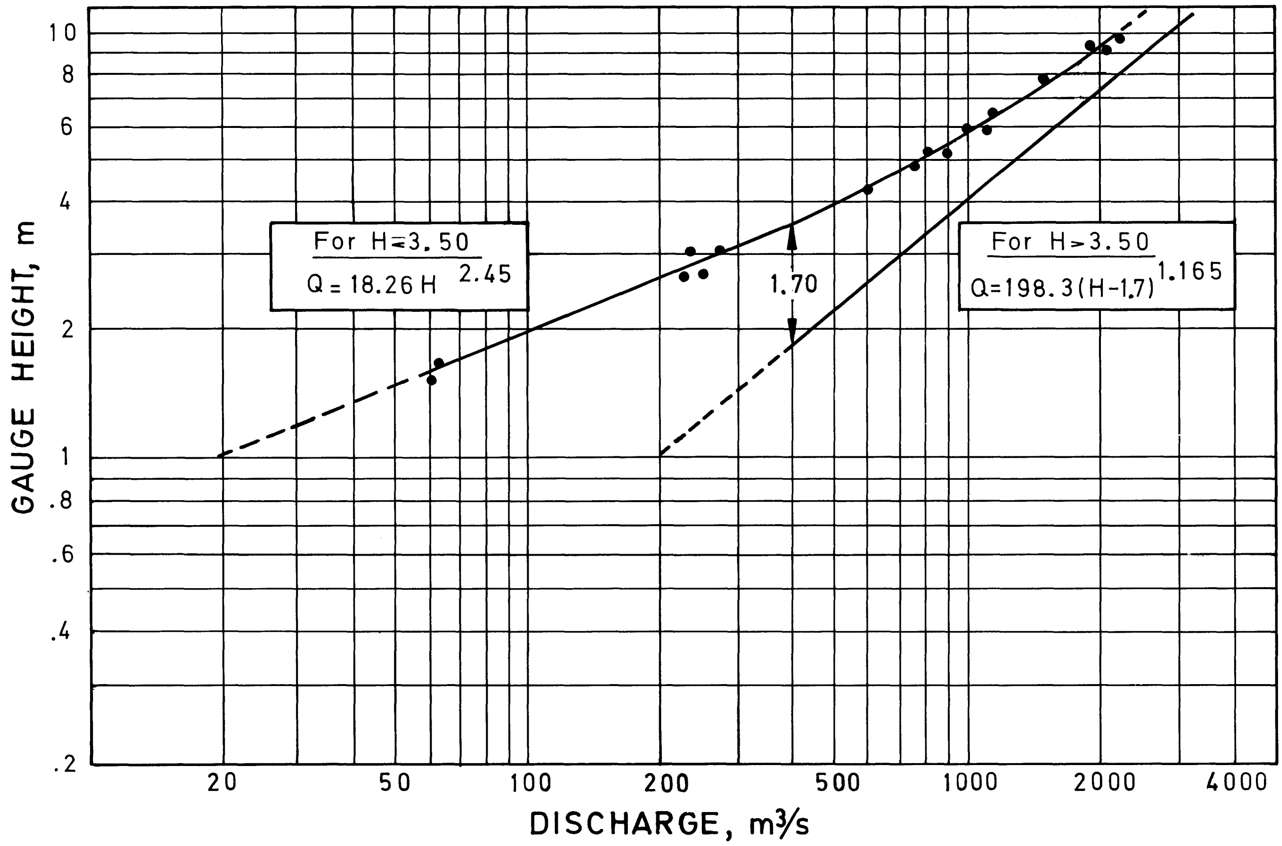


Figure 8. Stage-discharge curve consisting of two segments of different slope.

Table 3. Tabulation of data for determination of the constants K and n.

No.	H	Q	$H-H_o$	log Q = (Y)	log $(H-H_o)$ = (X)	(XY)	(X^2)
1	1.55	300	2.81	2.4771	0.4487	1.1115	0.2013
2	1.44	287	2.70	2.4579	0.4314	1.0603	0.1861
3	1.26	235	2.52	2.3711	0.4014	0.9518	0.1611
4	1.05	193	2.31	2.2856	0.3636	0.8310	0.1322
5	0.73	125	1.99	2.0969	0.2989	0.6268	0.0893
6	0.69	113	1.9S	2.0531	0.2900	0.5954	0.0841
7	0.70	124	1.96	2.0934	0.2923	0.6119	0.0854
8	1.70	340	2.96	2.5315	0.4713	1.1931	0.2221
9	0.96	169	2.22	2.2279	0.3464	0.7717	0.1200
10	0.94	168	2.20	2.2253	0.3424	0.7619	0.1172
11	1.35	240	2.61	2.3802	0.4166	0.9916	0.1736
12	1.17	202	2.43	2.3054	0.3856	0.8890	0.1487
13	1.79	387	3.05	2.5877	0.4843	1.2532	0.2345
14	3.09	930	4.35	2.9685	0.6385	1.8954	0.4077
15	3.87	1374	5.13	3.1380	0.7101	2.2283	0.5042
16	2.33	540	3.59	2.7324	0.5551	1.5168	0.3081
17	3.49	1152	4.75	3.0615	0.6767	2.0717	0.4579
18	3.93	1452	5.19	3.1620	0.7152	2.2615	0.5115
19	2.03	440	3.29	2.6435	0.5172	1.3672	0.2675
20	1.61	306	2.87	2.4857	0.4579	1.1382	0.2097
21	2.13	469	3.39	2.6712	0.5302	1.4163	0.2811
22	1.37	246	2.63	2.3909	0.4200	1.0039	0.1764
23	1.05	189	2.31	2.2765	0.3636	0.8277	0.1322
24	0.91	163	2.17	2.2122	0.3365	0.7444	0.1132
25	0.79	139	2.05	2.1430	0.3118	0.6682	0.0972
26	0.68	120	1.94	2.0792	0.2828	0.5984	0.0828
27	0.61	104	1.87	2.0170	0.2718	0.5482	0.0739
28	0.53	94.6	1.79	1.9759	0.2529	0.4997	0.0640
			Sum	68.0506	12.0182	30.4351	5.6430

m = 28, Ho = –1.26

$\sum(XY)$ = the sum of all values of the product of (X) and (Y),
m = the number of observations.

In order to illustrate the method the data of Table 2 are prepared as shown in Table 3.

Substituting the calculated values of Table 3 in Eqn. (2.18) and (2.19) obtains

$$68.0506-28 \log K-n\ 12.0182=0$$
$$30.4351-12.0182 \log K-n\ 5.6430=0$$

From these two equations it follows that n = 2.53 and K = 22.10, which is in close agreement with Eqn. (2.17) of the arithmetical procedure (Fig. 7).

A word of caution. In developing the stage-discharge rating by the Least squares method, it is common practice to give all the discharge measurements equal statistical weight in spite of the fact that most of the measurements available for defining the relation will always be located at the low and medium stages. Thus, an extrapolation of the discharge equation to the higher stages, where at best very few and usually no data points are available, will be biased because of the greater number of low-lying data points. It follows that extrapolation of discharge equations developed by use of the Least squares method, should be done carefully and always checked against other methods of extrapolation.

2.2.3.9 The Graphical Procedure of Estimating K and n

The dependent variable Q in Eqn. (2.9) is conventionally plotted as the abscissa and the independent variable H as the ordinate. Then, from a straight line plot on log-log graph paper of Eqn. (2.9), the slope n of the line is calculated as the ratio of the horizontal projection of the line to the vertical projection (Fig. 7).

The factor K in Eqn. (2.9) equals the numerical value of the discharge Q when the head $(H-H_0)$ above the point of zero flow is 1.00. K is constant for a given control condition.

2.2.4 Procedures for Establishing the Discharge Rating Curve

In actual practice, the two techniques discussed in Section 2.2.2 and 2.2.3, the *Series of differences method* and the *Logarithmic method*, are not regarded as two separate methods but rather worked into a single procedure. The following steps have been found to be practicable:

a) The discharge measurements are plotted on ordinary arithmetical graph paper, gauge height on the vertical scale and discharge on the horizontal scale. If the point of zero flow has been obtained by an actual field survey, this point is included in the plot. The scales should be selected so that the direction of the plot approximately follows the diagonal of the graph sheet from left to right. Uncommon odd scales should not be used. Suggested scales for the gauge height are 1:5, 1:10, and 1:20, preferably 1:10. A curve is fitted to the data points by visual estimation (Section 2.2.1).
b) At equal gauge height increments, the discharge is selected from the curve and tabulated together with its gauge height (Section 2.2.1). Usually, increments in gauge height of 0.10 m are practical, however, at the lower part of the curve where the curvature is greatest, it may sometimes be better to use increments of 0.05 m. At the upper part of the curve increments of 0.20 m may often be preferable.
c) The 1st and 2nd series of differences of the discharges are calculated and smoothed. From the smoothed series of the 1st differences, adjusted values of the discharge are calculated (Section 2.2.1). Re-plot adjusted discharge values on arithmetical graph paper. Inspect the plot, adjust if necessary.

   When the rating curve is of a fairly regular shape, it is not considered necessary to use the 2nd differences in order to smooth the 1st differences.
d) Plot the final adjusted discharges against their corresponding gauge height on double logarithmic graph paper; draw a smooth curve through the data points by means of drafting curves.
e) Estimate H_0 by trial and error (Section 2.2.3.3). That is, add or subtract trial values for H_0 to the gauge height until the curve drawn on log-log graph paper becomes transformed into a straight line, or into two or more straight line segments. Usually, the following instances will occur:
   i) One single straight line; produced by a complete section control of regular shape, often the crest of a rapid or a waterfall.
   ii) One broken line with two segments, each segment with a different slope but the same H_0; produced by a complete section control having a sharp break in the cross-sectional contour but otherwise of regular shape.
   iii) Two or more disconnected straight line segments each with its own slope n and H_0; produced by a compound control of various combinations, usually section control at low stage. This case is the most common.
   iv) Sometimes it happens that the plotted curve can not be transformed into straight line segments, or rather, the segments will be so short and numerous that the logarithmic representation of the curve would not be practical. Such cases are produced by very irregular controls.

   H_0 as obtained from a field survey or by the arithmetical and graphical techniques (Section 2.2.3.2.), is valid for the lowest segment only. The *trial and error technique* has to be used for the upper segment or segments. The trial and error technique is not too time-consuming, after some practice it will be found that only a few trials are necessary in order to find the correct H_0. It is not necessary to plot all the incremental data points of the table during the trials, only a few. For a final check of the H_0 value selected, however, all the points should be used.
f) Inspect the straight line plot. One last adjustment of the tabulated discharges may prove necessary.
g) When the curve has been found acceptable, the mathematical equation for each segment is calculated.
h) Finally, each segment is tested for bias and goodness of fit as illustrated in Appendix B.

ILLUSTRATION

At a gauging station, a series of discharge measurements has been obtained and chronologically arranged as in Table 4 below. It is desired to es-

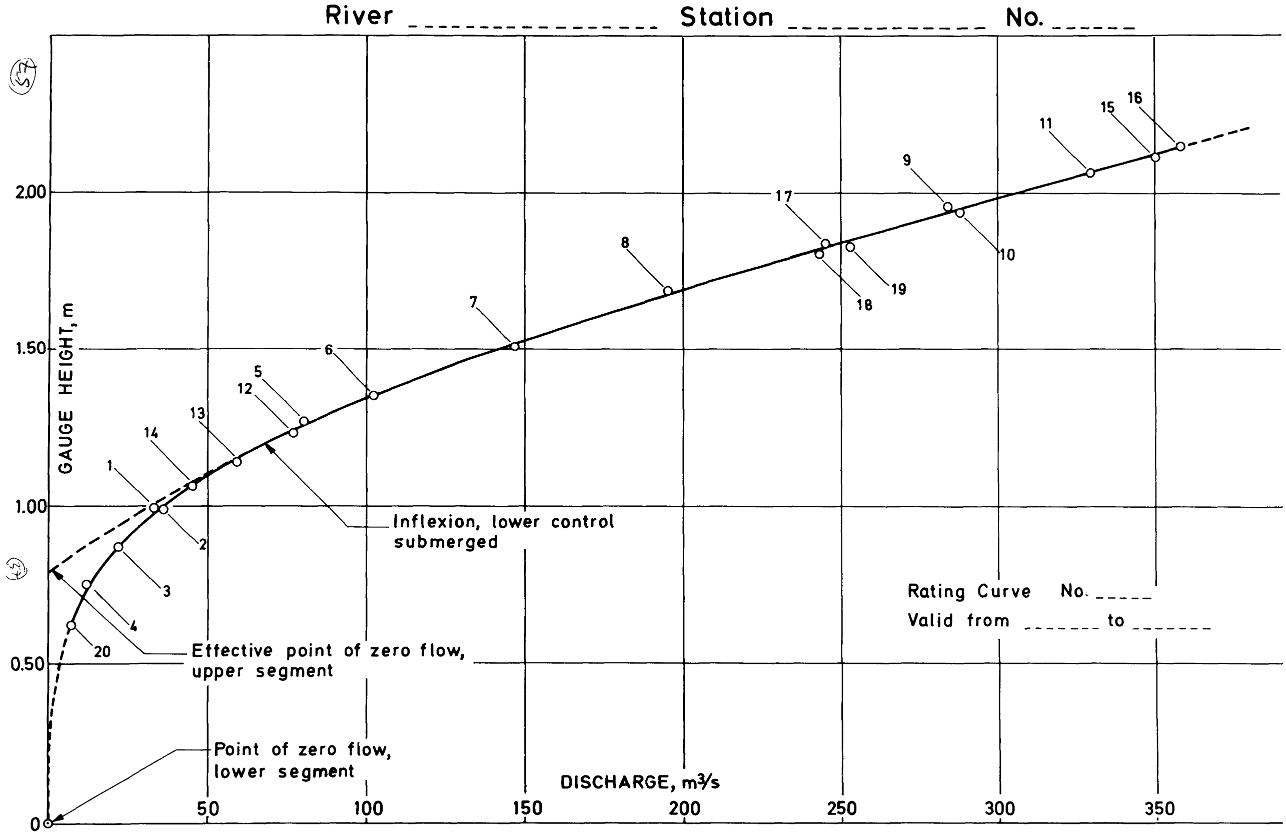


Figure 9. First plot of discharge measurements and visual estimation of the rating curve.

tablish a rating curve for the station. The point of zero flow is not known.

a) Plot the measurements on ordinary graph paper and fit a curve to the data points by visual estimation as illustrated in Fig. 9.
b) For equal gauge height increments of 0.10 m select from the curve corresponding discharges and tabulate on calculation form as illustrated in Fig. 10, columns 1 and 2.

Table 4. Table of Discharge Measurements

No.	H	Q	No.	H	Q
1	0.99	33.0	11	2.07	330.0
2	0.99	36.0	12	1.23	77.0
3	0.87	22.0	13	1.15	57.0
4	0.75	12.5	14	1.10	46.0
5	1.27	80.0	15	2.12	350.0
6	1.35	102.5	16	2.15	358.0
7	1.51	147.0	17	1.84	245.0
8	1.69	195.0	18	1.81	343.0
9	1.96	284.0	19	1.83	253.0
10	1.94	288.0			

c) Calculate the 1st differences from the discharges in column 2 and enter in column 3 of the calculation form. Plot and smooth the 1st differences as illustrated in Fig. 11. Enter smoothed 1st differences in column 4 of calculation form.
d) Calculate an adjusted discharge series by successively adding the 1st differences in column 4 from the top, start with first value in column 2. The adjusted discharges are entered in column 6.
e) Plot the adjusted discharge values of column 6 against their corresponding gauge height on log-log graph paper, gauge height on the vertical scale and discharge on the horizontal scale. Draw a smooth curve through the data points by means of drafting curves (Fig. 12).
f) Determine the point of zero flow (H_0) by trial and error. If the trial value of H_0 is less than the actual value, the curve will bend upwards; if it is greater than the actual value, the curve will bend downwards.

The lower part of the plotted curve up to a gauge height of 1.30 m appears to be a straight line, therefore, nothing should be added or subtracted to the gauge height for this range. This means that the point of zero flow has an eleva-

Gauge height	Discharge, m³/s				
	Visual estimate		Smoothed		Adjusted
m	Q	Δ Q	Δ Q	Δ^2 Q	Q
0.50	3.0				3.0
		3.0	2.8		
0.60	6.0				5.8
		4.0	4.2		
0.70	10.0				10.0
		6.1	6.0		
0.80	16.1				16.0
		8.4	8.2		
0.90	24.5				24.2
		11.0	11.0		
1.00	35.5				35.2
		15.0	14.3		
1.10	50.5				49.5
		17.5	18.0		
1.20	68.0				67.5
		22.0	22.0		
1.30	90.0				89.5
		25.0	25.0		
1.40	115.0				114.5
		27.0	27.0		
1.50	142.0				141.5
		29.0	29.0		
1.60	171.0				170.5
		31.0	31.0		
1.70	202.0				201.5
		33.0	33.0		
1.80	235.0				234.5
		35.0	35.0		
1.90	270.0				269.5
		35.0	35.0		
2.00	305.0				304.5
		35.0	35.0		
2.10	340.0				339.5
		35.0	35.0		
2.20	375.0				374.5

Figure 10. Calculation form for a stage-discharge relation.

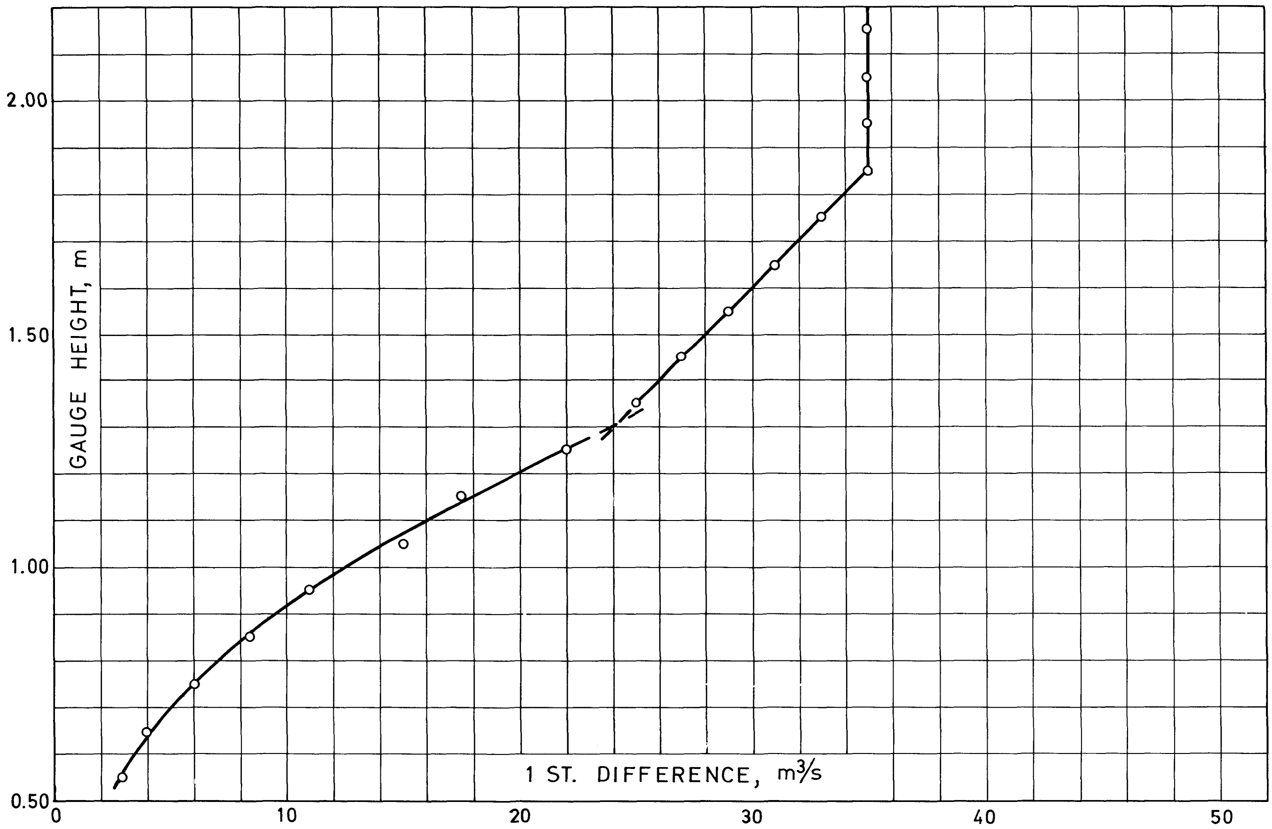


Figure 11. Smoothing graph for series of 1st differences.

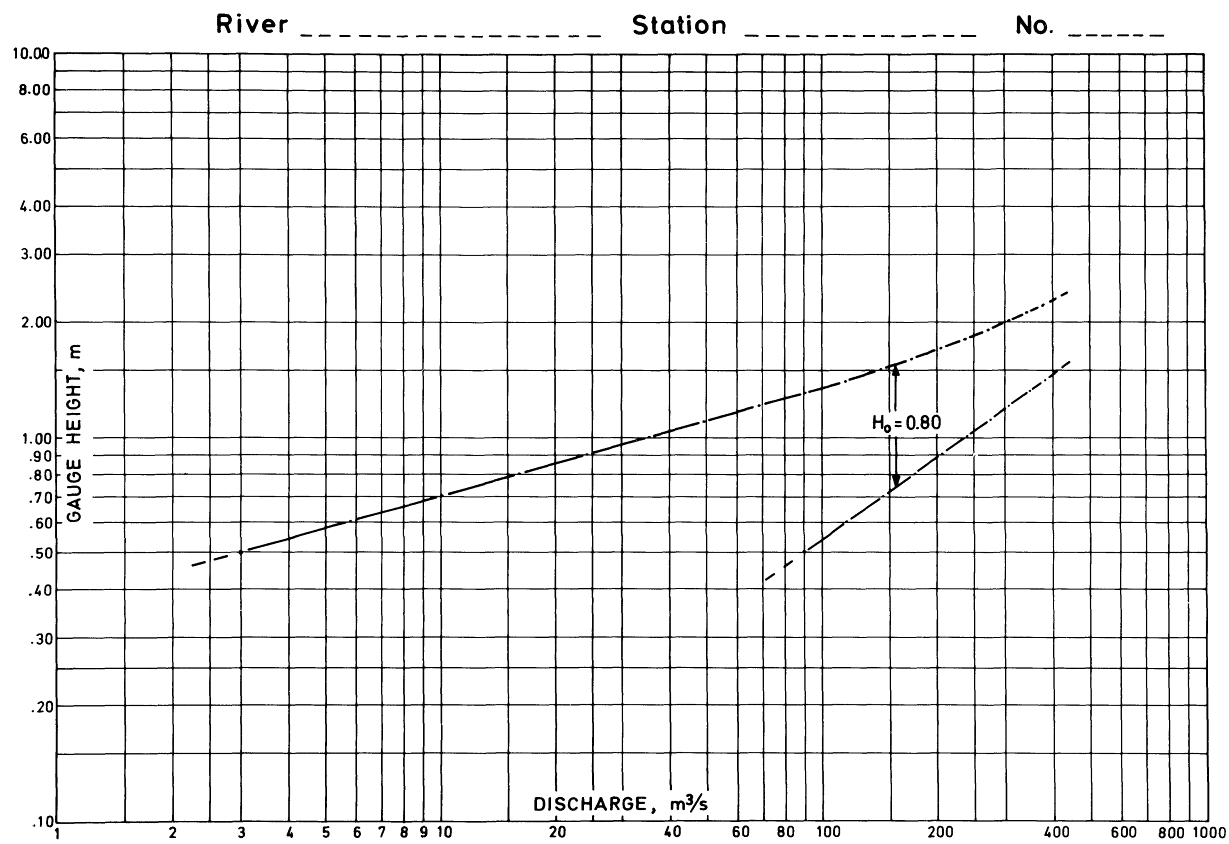


Figure 12. Second plot and determination of the stage-discharge relation.

tion equal to the zero of the gauge, and that around 1.30 m on the gauge the low water control is drowned out by a downstream high water control becoming effective.

The upper part of the rating curve, above approximately 1.30 m, is bending upward. By successive trials, it is found that the curve will approach a straight line for a value of H_0 equal to 0.80 m.

The rating curve consists of two straight line segments, the one below and the other above a stage of approximate 1.30 m. H_0 for the lower segment is equal to 0.00, and for the upper segment equal to 0.80 m.

g) The flow equations for the two segments are developed as follows.

Lower Segment

Select two points on the lower straight line segment, the points should be as far as possible from each other and within the range of measured discharges. Let the coordinates (Q, H) of the points be (3.0, 0.50) and (89.5, 1.30). Using Eqn. (2.16) obtains

$$\frac{\log Q - \log Q_1}{\log (H - H_0) - \log H_1} = \frac{\log Q_2 - \log Q_1}{\log H_2 - \log H_1}$$

$$\frac{\log Q - \log 3.0}{\log (H - 0) - \log 0.50} = \frac{\log 89.5 - \log 3.0}{\log 1.30 - \log 0.50}$$

$$\frac{\log Q - 0.4771}{\log H + 0.3010} = \frac{1.9518 - 0.4771}{0.1139 + 0.3010} = 3.5543$$

$$\log Q = (\log H + 0.3010)\, 3.5543 + 0.4771$$
$$\log Q = 3.5543 \log H + 1.5469$$
$$Q \quad = 35.2\, H^{3.554}$$

which is the discharge equation for the lower segment, i.e. for H equal to 1.30 m or less.

Upper Segment

H_0 has been determined by trial and error to equal 0.80 m. Select two points on the upper straight line segment, let the coordinates be (89.5, 0.50) and (339.5, 1.30). Substituting in Eqn. (2.16) obtains

$$\frac{\log Q - \log 89.5}{\log (H - 0.80) - \log 0.50} = \frac{\log 339.5 - \log 89.5}{\log 1.30 - \log 0.50}$$

$$\frac{\log Q - 1.9518}{\log (H - 0.80)} = \frac{2.5308 - 1.9518}{0.1139 + 0.3010} = 1.3956$$

$$\log Q = (\log (H - 0.80) + 0.3010)\, 1.3956 + 1.9518$$
$$\log Q = 1.3956 \log (H - 0.80) + 2.3718$$
$$Q \quad = 235.4\, (H - 0.80)^{1.40}$$

which is the discharge equation for the upper segment, i.e. for H equal to 1.30 m or larger.

h) Check the flow equations if they give satisfactory results as illustrated next.

Test of lower segment using data point (24.2, 0.90):

$$Q = 35.2\; 0.90^{3.55} = 24.2$$

Test of upper segment using data point (269.5. 1.90):

$$Q = 235.4\; 1.10^{1.40} = 269.0$$

It is seen that both equations give good results as compared to the data in Fig. 10.

Each segment should be checked using three data points, one data point each for low stage, medium stage, and high stage. The reason for checking with three points is the possibility that only a part of the curve is represented by the equation within acceptable limits of accuracy. If the result of the check is not satisfactorily, adjustments of the straight line plot are made and the procedure repeated.

i) Check the rating curve for bias and goodness of fit as illustrated in Appendix B.

2.2.5 Rating Tables

The rating table is a tabular representation of the rating curve and is a useful tool for converting gauge height readings into discharges when this is done manually (Fig. 13).

The discharges entered in the .00-column are copied from the final adjusted values of Fig. 10, column 6, and give the discharge for every 0.10 m increments in gauge height. Intermediate values are obtained by interpolating between the values of the .00-column. The difference between adjacent discharges should increase smoothly or be the same as the preceding difference.

RATING TABLE

River _______________ Station _____________ No. _______

From ________ to ____________ from __________ to _____________

G.H. m	Discharge, m^3/s										ΔQ
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	
.0											
.1											
.2											
.3											
.4											
.5											
.6											
.7											
.8											
.9											
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.9											
.0											
.1											
.2											
.3											
.4											
.5											
.6											
.7											
.8											
.9											
.0											

Figure 13. Calculation form making rating tables.

2.2.6 *Verification of the Rating Curve*

The stage-discharge relation is checked from time to time by discharge measurements at low stage, and at medium and high stage, and always after major floods. If a significant departure from the established rating curve is found, further checks are made. If the difference is confirmed, sufficient discharge measurements are made to redefine the curve in the range in which the relation has altered and a new rating curve is made (Appendix B.

If a particular change of the rating curve can be attributed to a definable incident in the history of the station, the new curve should apply from the time of that incident.

2.2.7 *Extrapolation of Rating Curves*

Extrapolation of the rating curve in both directions is often necessary. If the point of zero flow has been determined, the curve may be interpolated between this point and the lowest discharge meas-

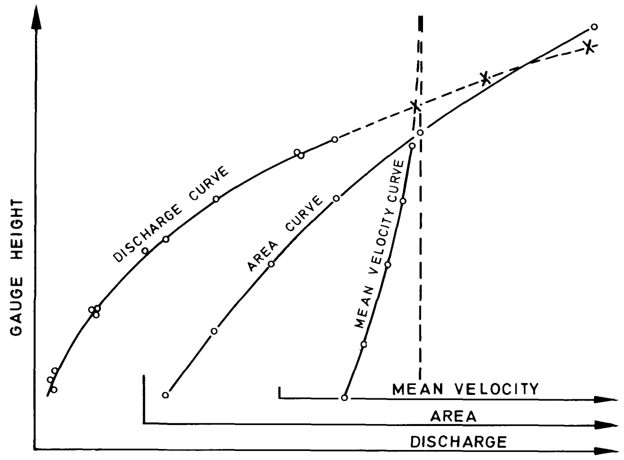


Figure 14. Discharge-rating curve extrapolated by the stage-area/ stage-velocity method.

urements without much error. But, if the point of zero flow is not available, it is not advisable to extrapolate far in this direction.

In the upper part of the curve extrapolation is almost always necessary. Only in very few cases have discharge measurements been obtained at about the highest flood peak observed.

Two methods of extrapolation have already been discussed. The series of differences method can be used if the control does not change at the higher stages. This also applies to the logarithmic extrapolation which has proved to be a reliable method for shorter extensions. If, however, extended extrapolations have to be made, special methods are used, some of which will be described in the following. [4].

2.2.7.1 The Stage-Velocity-Area Method

The best method to use is the extension of the stage-velocity curve. A plot with stage as the ordinate and the mean velocity as the abscissa gives a curve which, if the cross section is fairly regular and no bank overflow occurs, tends to become asymptotic to a vertical line at higher stages. That is, the rate of increase in the velocity at the higher stages diminishes rapidly and this curve can therefore be extended without much error.

Furthermore, by plotting the stage-area curve (stage as ordinate, area as abscissa) for the same cross section as that from which the mean velocity was obtained, the area can be read off at any stage desired. Multiplication of the area by the mean velocity gives the discharge (Fig. 14).

The area is obtained by a field survey up to the highest stage required and is therefore a known quantity.

2.2.7.2 The Manning-equation Method

The uniform flow equation as developed by Manning may be used for extrapolation of rating curves, it is expressed as

$$Q = (1/n)\ A\ R^{2/3}\ S^{1/2} \qquad (2.20)$$

where

n = Manning's roughness constant,
A = area of cross section (m^2),
R = hydraulic radius (m),
S = slope of water surface,
Q = discharge (m^3/s).

In terms of mean velocity the equation may be written

$$\bar{v} = (1/n)\ R^{2/3}\ S^{1/2} \qquad (2.21)$$

For the higher stages, the factor $(1/n)\ S^{1/2}$ becomes approximately constant, that is

$$K = (1/n)\ S^{1/2} \qquad (2.22)$$

Eqn. (2.20) and (2.21) can therefore be rewritten as

$$Q = KAR^{2/3} \qquad (2.23)$$

and

$$v = KR^{2/3} \qquad (2.24)$$

By using various values from the known portion of the stage-velocity curve and the corresponding values of R, the values of K can be computed by Eqn. (2.24) for the range in stage for which the velocity is known. By plotting these values of K against the corresponding gauge height, a curve is obtained that asymptotically approaches a vertical line for the higher stages (Fig. 15). This K-curve may be extended without much error and values of K obtained from it for the higher stages. These high-stage values of K combined with their respective values of A and $R^{2/3}$ using Eqn. (2.23), will give the discharge Q which may be used to extrapolate the rating curve. A and R are obtained by field surveys and are thus known for any stage required.

2.2.7.3 The Chezy-equation Method

This method is based on the Chezy equation for uniform flow

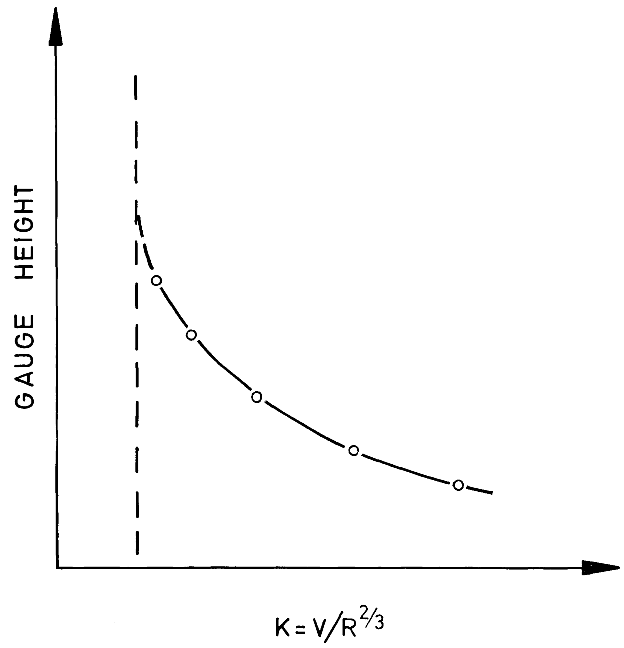


Figure 15. Extrapolation of K by use of the Manning equation.

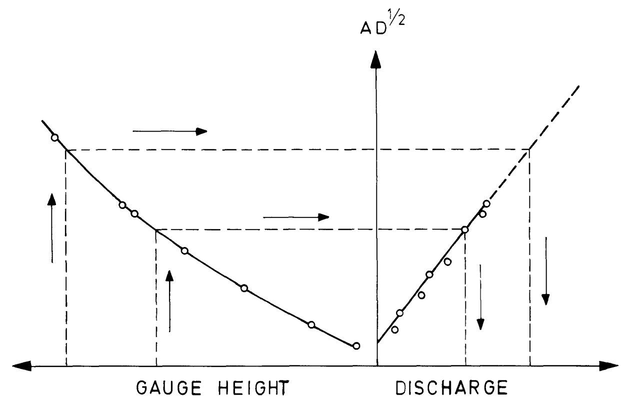


Figure 16. Discharge rating curve extrapolated by use of the Chezy equation.

$$Q = AC\,(RS)^{1/2} \tag{2.25}$$

For shallow streams with a relatively small depth-width ratio, the mean depth D does not differ much from the hydraulic radius R. Then, by substituting D for R, Eqn. (2.25) may be rewritten

$$Q = CS^{1/2}AD^{1/2} \tag{2.26}$$

At higher stages, the slope S in most cases may be considered a constant. Then, by plotting $AD^{1/2}$ against Q in Eqn. (2.26), an approximate straight line is obtained which is readily extended.

As illustrated in Fig. 16, values of $AD^{1/2}$ are plotted both against gauge height H and discharge Q, and the latter curve extended up to the higher stages. Both A and D are obtained by field surveys and are therefore known factors. [4].

2.2.7.4 The Hydraulic River-Model Analysis Method

A method of extrapolating the stage-discharge relation that has proved very useful is the technique of hydraulic model analysis carried out in a hydraulic laboratory. A hydraulic model is in principle an exact replica of the prototype in all significant details at a reduced scale.

Based on accurate field data, a model is built of the stream at the gauging station including all the controlling features. Some field measurements of the discharge at low and medium stage must be available in order to adjust the model to conform exactly to the prototype at the lower/medium discharges. By observing the model when its discharge is further increased, the stage-discharge relation for the prototype is derived for the flood stages.

2.2.8 Computerized Logarithmic Method

When facilities for electronic data processing by computer are available, it is recommended that the logarithmic procedure be computerized. The constants n, K and H_0 would then be conveniently fitted and matched by mathematical-statistical programs.

The discharge equation is developed by use of double logarithmic regression analysis and iterative procedures on the corresponding gauge heights and discharges that are measured. If H_0 is known, standard linear regression analysis is used and for unknown H_0 nonlinear regression analysis. The points of intersection of the different segments are also easily computed.

The observed stages and the corresponding measured discharges are normally plotted first on log-log graph paper for visual inspection. Also, it is important that the result be confirmed by site investigations.

2.3 Discharge Rating of Shifting Controls

Shifts in the control features occur often in sand-bed streams. However, even in solid stable stream channels shifts will occur, particularly at low flow because of aquatic and vegetal growth in the channel or due to debris caught in the control section.

In sand-bed streams, the stage-discharge relation usually changes with time, either gradually or abruptly, due to scour and silting in the river channel and because of moving sand dunes and bars. These variations will cause the rating curve to vary both with time and the magnitude of flow. Nevertheless, runoff records at a particular location may be of great importance and observations and measurements have to be carried out the best way possible.

2.3.1 Characteristics of Sand-Bed Channels

In sand-bed channels, the configuration of the bed varies with the magnitude of the flow. The bed configurations occurring with increasing discharge are ripples, dunes, plane bed, standing waves, antidunes, and chutes and pools (Fig. 17). The bed forms are associated with a particular mode of sand movement and with a particular range of resistance to the flow of water. The resistance to the flow is greatest in the dunes range. When the dunes are washed out and the sand is rearranged to form a plane bed, there is a marked decrease in bed roughness and resistance to the flow causing an abrupt discontinuity in the stage-discharge relation.

The sequence of bed configurations shown in Fig. 17 is arranged as developed by increasing discharge. The bed configurations are grouped into two regimes. The lower regime (A - C) occurs with lower discharges, the upper regime (E - H) with higher discharges. An unstable discontinuity (D) in the depth-discharge relation appears between these more stable regimes.

Fine sediments present in the water influences the configuration of the sand-bed and thus the resistance to flow. It has been demonstrated that a concentration of fine sediments in the order of 40,000 ppm may reduce the resistance to flow in the dune regime as much as 40 %. Thus, the stage-discharge relation in a stream varies with the sediment concentration when the water is heavily loaded with fine sediments.

The viscosity of the water increases with lower temperature and thereby the mobility of the sand will increase. Changes in water temperature may therefore alter the bed form, and hence roughness and resistance to flow in sand-bed channels. [9].

2.3.2 Discharge Rating of Sand-Bed Channels

For sand-bed channels where neither bottom nor sides are stable, a plot of stage against discharge will as a rule scatter widely and thus be indeterminate (Fig. 18). By changing variables however, a hydraulic relationship will become apparent. The effect of variation in bottom elevation is eliminated by replacing stage by the hydraulic radius (or mean depth). The effect of variation in width is eliminated by using mean velocity instead of discharge. [9].

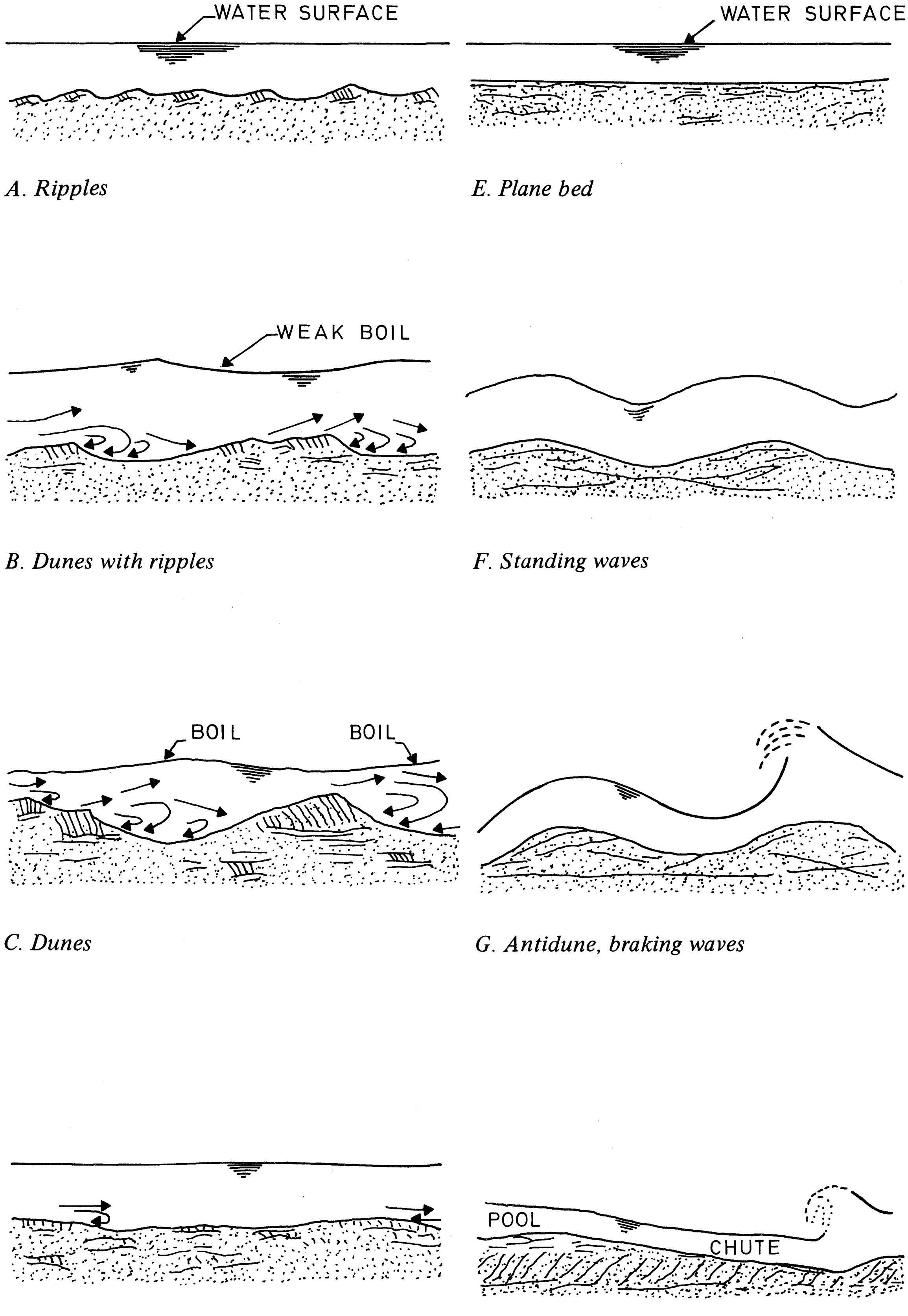
Plots of hydraulic radius against mean velocity are very useful in the analysis of stage-discharge relations, provided the measurements are referred to one and the same cross section. These plots will identify the bed-form regime associated with each individual discharge measurement (Fig. 19). Thus, only the measurements associated with the upper flow regime are used to define the upper part of the rating curve. And similarly, only measurements identified with the lower flow regime are used to define the lower part. Measurements made in the transition zone will scatter widely and should not be taken as representing shifts in the more stable parts of the rating.

Knowledge of the bed-form which existed at the time of the individual discharge measurements is helpful in developing discharge ratings. Indication of bed-forms may be obtained by visual observation of the water surface.

A very smooth surface indicates a plane bed, large boils and eddies indicate dunes, standing waves indicate smooth bed waves in phase with surface waves, and breaking waves indicate antidunes. The visual observation of the water surface should be recorded on the note sheet when making discharge measurements in sand-bed channels.

The upper part of the stage-discharge relation associated with the upper flow regime for a sand-bed channel is usually comparatively stable. The middle part of the stage-discharge relation associated with the transition zone between the upper and lower regime varies almost randomly with time, and frequent discharge measurements are necessary in order to define this part of the relation.

At low flow when the water is not covering the whole width of a sand-bed channel, the flow tends to meander in the course of time. Under this condition, it is not possible to observe a systematic gauge height and a record of the discharge can therefore not be obtained by the velocity-area method. If feasible, an artificial control structure for the low flow range should be considered in such cases.



Figur 17. Stream bed and water surface configurations found in sand-bed channels (courtesy of the US Geological Survey). [9].

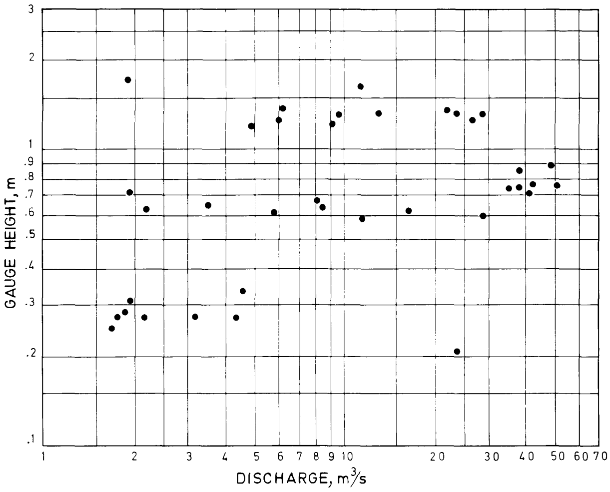


Figure 18. Plot of discharge against gauge height for a sand-bed channel with indeterminate stage-discharge-relation (courtesy of the US Geological Survey). [9].

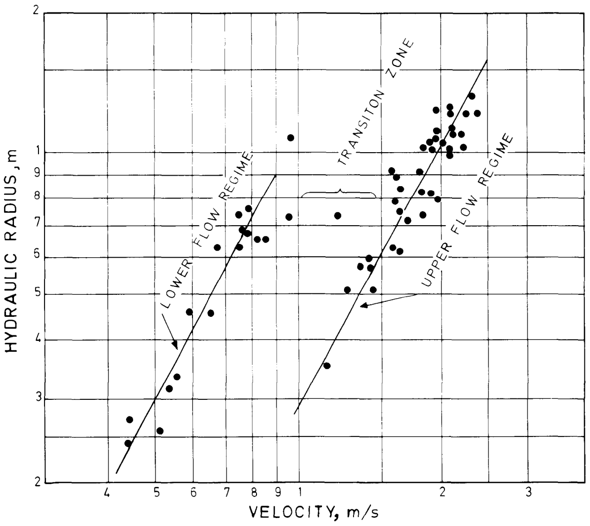


Figure 19. Relation of hydraulic radius to mean velocity for same channel as in Fig. 18 (courtesy of the US Geological Survey). [9].

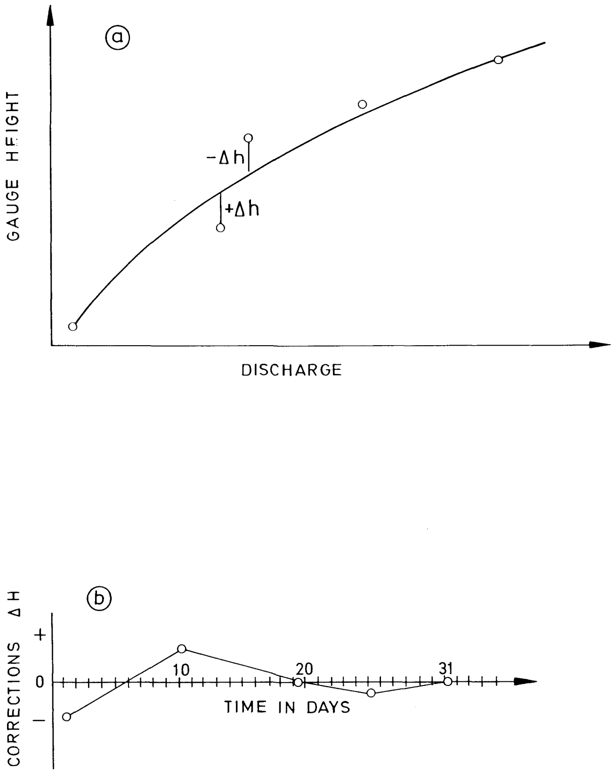


Figure 20. The Stout method of correcting gauge height readings when control is shifting. [4].

2.3.3 The Stout Method

For making adjustment for shifting control the Stout method is commonly used. In this method, the gauge height corresponding to discharge measurements taken at intervals are corrected so that the discharge values obtained from the established rating curve may be the same as the measured values. From the plot of these corrections against the chronological dates of measurements, a gauge height correction curve is made. Corrections from this curve are applied to the recorded gauge heights for the intervening days between the discharge measurements. [4].

An ordinary staff gauge is established at the best available site on the river and readings taken at appropriate intervals, say once a day. Discharge measurements are made as often as found necessary, and may be required as often as once or twice a week. How often discharge measurements need to be taken depends on several factors, such as the hydraulic conditions in the river, the accuracy and the feasibility based on economy and other factors.

The measurements are plotted against observed gauge height on ordinary graph paper and a median curve is fitted to the points. Most of the subsequent discharge measurements will deviate from the established curve. For points lying above the curve, a small height, Δh, is subtracted from the observed gauge height in order to make these points lie on the curve. That is, minus corrections in gauge height are applied to points lying above the curve, plus corrections are applied to points lying below the curve (Fig. 20a).

Next, a correction graph is made as shown in Fig. 20b. The plus and minus corrections are plotted on the date of measurement and the points connected by straight lines or a smooth curve. Gauge height corrections for each day are now obtained directly from the correction graph, remembering that the parts of the correction graph below the abscissa axis give minus corrections and the parts above give plus corrections.

When discharge measurements plot within 4 %

CALCULATION FORM FOR THE STOUT METHOD

River ____________ Station __________ No. _____

Month ________ 19___

DATE.	Observed Gauge Height	Measured Discharge	Shift Correction	Corrected Gauge Height	Corrected Discharge	Remarks
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
TOTAL	—	—	—		—	
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
TOTAL	—	—	—		—	
21						
22						
23						
24						
25						
26						
27						
28						
29						
30						
31						
TOTAL	—	—	—		—	
MONTHLY TOTAL		—	—		—	

Figure 21. Calculation form for the Stout method. [4].

of the rating curve, with some plus and some minus deviations, it is acceptable to use the curve directly without adjustment for shifting control.

For computation purposes special forms may be made. Forms are made for each month as shown i Fig. 21.

It is not too important how the median curve is drawn between the measurements. Different curves will give different corrections and the final result will be approximately the same. Extrapolation of the curve, however, has to be done with care.

A rating of this type requires much work in order to obtain good results. The accuracy depends on the hydraulic conditions in the river and on the number and accuracy of the discharge measurements and the gauge height readings. The reliability is much less than for stations with permanent control.

The Stout method presupposes that the deviations of the measured discharges from the established stage-discharge curve are due to a change or shift in the station control only, and that the corrections applied to the observed gauge heights vary gradually and systematically between the days on which the check measurements are taken.

In fact, the deviation of a discharge measurement from an established rating curve may be due to (a) gradual and systematic shifts in the control, (b) abrupt random shifts in the control, and (c) error of observation and systematic errors due both to instrumental and human factors.

The Stout method is strictly appropriate for making adjustments for the 1st type of errors only. If the check measurements are taken frequently enough, fair adjustments may be made for the second type of error also. However, the drawback of the Stout method is that the error of observation and the systematic errors are disregarded as such and simply mixed with the errors due to shift in control, although at times the former errors may be of a higher magnitude than the latter. This means that "corrections" may be applied to a discharge record when in reality the rating is correct. The apparent error is not due to a shift in the control but to faulty equipment or careless measuring procedures. [4].

2.4 Complex Stage-Discharge Relations

2.4.1 General

If variable backwater or highly unsteady flow occurs at a gauging station, a single-valued stage-discharge relation does not exist. A third variable, fall or rate of change of stage (also slope or rate of change of discharge may be used) will have to be included in order to define the discharge rating.

Backwater is caused by constrictions such as narrow reaches of a stream channel or artificial structures such as dams or bridges. Backwater from fixed obstructions is always the same at a given stage, in which case the discharge rating is a function of stage only.

If the reach downstream from a gauging station has within it a dam, a diversion, or a confluent stream, which can increase or decrease the energy gradient for a given discharge, *variable backwater* is produced. That is, the slope in a reach is increased or decreased from the normal. Under this condition, a third parameter, fall over a channel reach, is introduced in order to develop a stage-fall-discharge relation.

Stage-fall-discharge ratings are established from observation of (a) stage at a base gauge, (b) the fall of the water surface between the base gauge and an auxiliary gauge downstream, and (c) the discharge.

The plot of the discharge measurements with the fall marked against each measurement, will reveal whether the stage-discharge relation is affected by variable slopes at all stages or is affected only when the stage rises above a particular level. If the stage-discharges relation is affected by variable backwater at all stages, a correction is applied by the *constant-fall* method. On the other hand, however, when the relation is affected when the stage rises above a particular level only, the *normal-fall* method is applied.

In order to observe the fall, an auxiliary gauge is established a distance L downstream from the base gauge at the station and set to the same datum as the base gauge. With a difference in gauge reading of F, the surface slope is approximated by

$$S \sim F/L \qquad (2.26)$$

2.4.2 The Constant-Fall Method

The Manning uniform flow equation may be written in the form

$$Q = (1/n)\, AR^a S^b \qquad (2.27)$$

where

Q = the discharge (m^3/s),
n = bed roughness factor,
A = area of cross section (m^2),
R = hydraulic radius (m),

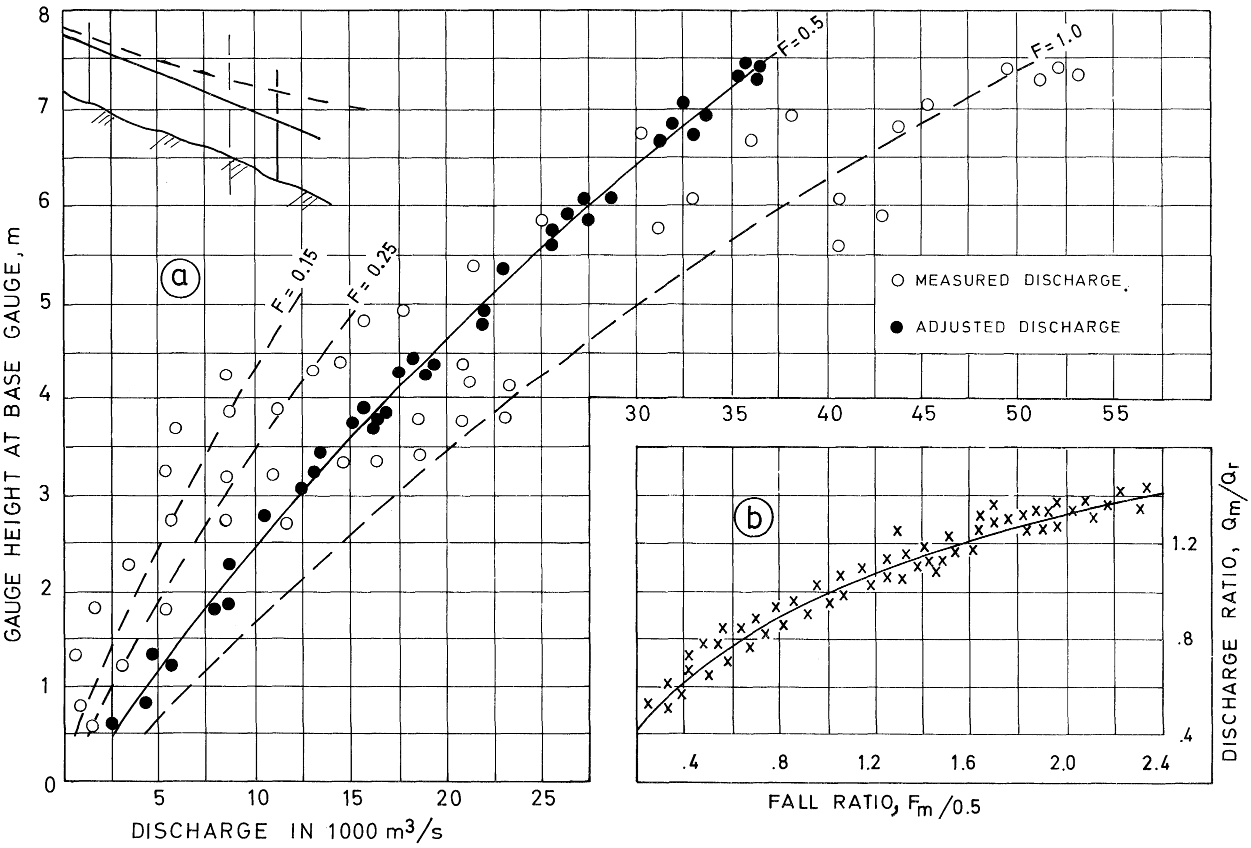


Figure 22. Stage-fall-discharge rating for variable backwater, nearly uniform channel (courtesy of US Geological Survey). [9].

S = slope of the water surface,
a, b = exponents.

Substituting for S, Eqn. (2.27) will read

$$Q = (1/n)\ AR^{a}\ (F/L)^{b} \qquad (2.28)$$

The roughness factor *n* and the conveyance AR^a are functions of stage, then for a given stage the relation between discharge and fall can be developed as

$$(Q_m/Q_r) = (F_m/F_c)^b \qquad (2.29)$$

where

Q_m = measured discharge (m^3/s),
F_m = measured fall (m),
Q_r = discharge from the rating curve which corresponds to the base gauge stage (m^3/s),
F_c = a selected constant fall on which the rating curve is based (m).

In Manning's equation the exponent b in Eqn. (2.28) would be expected to equal 0.5. However, the slope S is approximated only by F/L, the exponent b would then not necessarily equal 0.5 and must be determined empirically. This is done by a graphical plot of Q_m/Q_r against F_m/F_c as explained below.

The procedure of the constant-fall method in developing a stage-fall-discharge relation is as follows:

a) Plot the discharge measurements against stage in the usual manner and indicate the observed fall beside each point. Select a constant fall for which a rating curve can be drawn through the measurements and draw the curve by visual estimation. For curve a in Fig. 22 a constant fall of 0.50 m is selected.
b) Read the discharge Q from the rating curve for each measurement and compute the ratios Q_m/Q_r and F_m/F_c. Plot these ratios and draw a smooth curve through them, that is curve b in Fig. 22. Note that curve b expresses the exponent b in Eqn. (2.29).
c) From the curve of relation between the discharge ratios and the fall ratios, curve b in Fig. 22, select the smoothed discharge ratio for each measurement.

Adjust each measurement by dividing it by its smoothed discharge ratio, then re-plot the adjust-

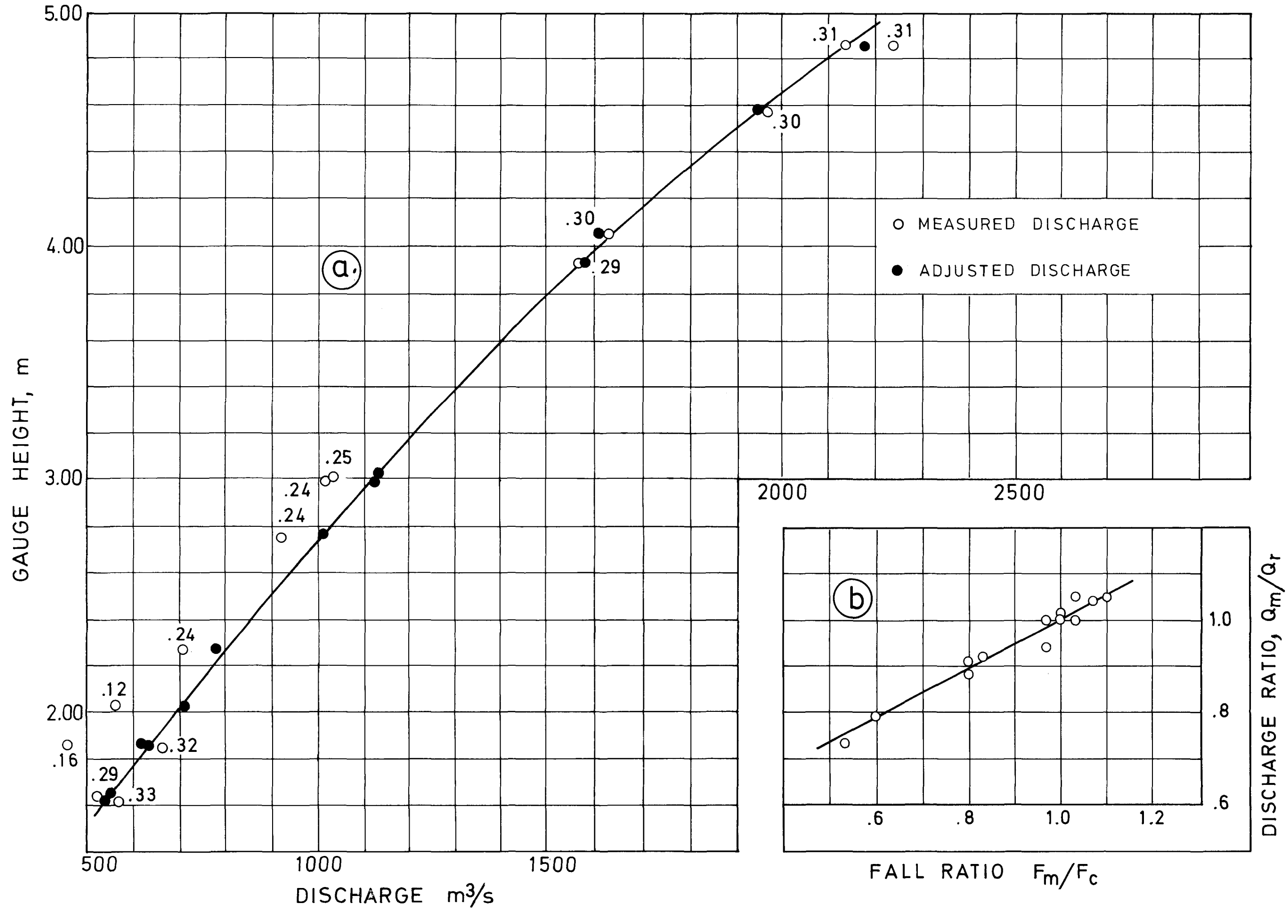


Figure 23. Developing a stage-fall-discharge rating (courtesy of US Geological Survey. [9].

ed discharge measurements. Examine the plot. If the first approximation of the rating curve does not appear to be well balanced among the adjusted discharge measurements, then adjust the rating curve and repeat the procedure. Usually, not more than one repeat is necessary.

A constant-fall rating is not the usual case in natural streams. However, if discharge measurements cover the whole range of flow and if the measurements conform to a constant-fall rating, the constant-fall method is sufficiently accurate and there is no need to use a more complicated technique. If the stream geometry is not too far from uniform and the velocity head increments are negligible, the relation between the discharge ratio and the fall ratio should approach a single curve. [6].

ILLUSTRATION

Refer Table 5 and Fig. 23.

14 discharge measurements are available at a twin-gauge station. In columns 2, 3, and 5 of Table 5, values of observed gauge height (H), measured discharge (Q_m), and measured fall (F_m) are shown. It is desired to develop a stage-fall-discharge relation for this station.

a) Plot the discharge measurements in the usual manner and indicate measured fall beside each point. Select a constant fall for which a rating curve can be drawn. Here, a suitable fall is F_c = 0.30 m. Draw the curve (Fig. 23a).
b) Compute the fall ratios F_m/F_c and enter in Table 5, column 6.
c) Read the discharge Q_r from the rating curve in Fig. 23 for each measurement and enter it in Table 5, column 4. Compute the discharge ratio Q_m/Q_r and enter in column 7.
d) Plot the fall ratios F_m/F_c against the discharge ratios Q_m/Q_r and draw a smooth mean curve of relation (Fig. 23b). Here it appears that the curve of relation approaches a straight line. Usually this curve bends downwards.
e) Entering the curve of relation with the fall ratios, adjusted values of the discharge ratios are obtained and entered in column 8.
f) Now, divide each value in column 3 by its cor-

Table 5. Developing a stage-fall-discharge relation.

No.	H	Q_m m^3/s	Q_r m^3/s	F_m m	F_m/F_c	Q_m/Q_r	Adjusted Q_m/Q_r	Adjusted Q_r m^3/s
I	1.62	580	550	0.33	1.10	1.05	1.05	552
2	1.64	526	560	0.29	0.97	0.94	0.99	531
3	1.85	668	642	0.32	1.07	1.04	1.04	642
4	1.86	497	645	0.16	0.53	0.77	0.75	662
5	2.02	568	715	0.18	0.60	0.79	0.79	719
6	2.26	710	810	0.24	0.80	0.88	0.90	789
7	2.75	922	1015	0.24	0.80	0.91	0.90	1024
8	2.98	1018	1115	0.24	0.80	0.91	0.90	1131
9	3.00	1035	1125	0.25	0.83	0.92	0.91	1140
10	3.92	1562	1560	0.29	0.97	1.00	0.99	1578
11	4.05	1620	1625	0.30	1.00	1.00	1.00	1620
12	4.56	1960	1935	0.30	1.00	1.01	1.00	1960
13	4.85	2130	2125	0.31	1.03	1.00	1.02	2088
14	4.85	2230	2125	0.31	1.03	1.05	1.02	2186

$F_c = 0.30$ m

responding value in column 8 obtaining adjusted values for Q_r which are entered in column 9.

g) Re-plot the adjusted values of Q_r. It appears that in this case the original rating curve has a well-balanced fit to the new adjusted values of Q_r, therefore, no repeat is necessary.

The procedure of converting observed gauge height and fall to discharge by means of the stage-fall-discharge relation as developed in Fig. 23 is as follows:

Observed gauge height:	H	= 3.40 m
Observed Fall:	F_m	= 0.22 m
Selected constant fall:	F_c	= 0.30 m
Computed fall ratio: 0.22/0.30		= 0.73
Discharge ratio corresponding to a fall ratio of 0.73 (curve b):		= 0.86

Discharge of rating curve corresponding to a gauge height of 3.40 is 1300 m^3/s.

The true discharge is

1300 $m^3/s \times 0.86 =$ 1118 m^3/s

2.4.3 *The Normal-Fall Method*

At some stations, a simple single-gauge rating is applicable at low discharge when the surface slope is comparatively steep, while at higher discharges when the slope becomes more flat the discharge is affected by variable backwater caused by downstream features, refer Section 1.2, Fig. 1F.

Critical values of the fall (or slope) dividing these two regions are termed the normal-fall. The value of the normal-fall at any discharge, can be defined by studying the plot of stage against discharge (Fig. 24).

The points at which backwater has no effect will group to the extreme right. This is the simple single-gauge rating with no variable backwater effects. A plot of the normal-fall values from this curve is made against the corresponding stage and a curve of normal-fall is obtained. This permits drawing a curve of discharge ratios against fall ratios when normal-fall is used in place of constant-fall. The rest of the procedure is similar to that of the constant-fall method (Fig. 24).

The normal-fall procedure in developing a stage-fall-discharge rating is as follows:

a) Plot the discharge measurements and write the fall beside each as indicated in Fig. 24a (the fall values are not shown in the figure). A smooth curve is fitted to the measurements grouped to the extreme right. This curve is the stage-discharge relation for a condition of no variable backwater at the gauge.

b) The fall for the measurements used to fit the rating curve are plotted against stage as illustrated in Fig. 24b, a curve is fitted to the points. This curve, the normal-fall curve, shows the fall when backwater takes effect at the different stages. That is, there are no backwater effects at a fall value to the right of the normal-fall curve. For fall values to the left of the curve there is variable backwater present.

c) Each fall ratio F_m/F_n is plotted against its corresponding discharge ratio Q_m/Q_r and a smooth curve of relation is drawn (Fig. 24c).

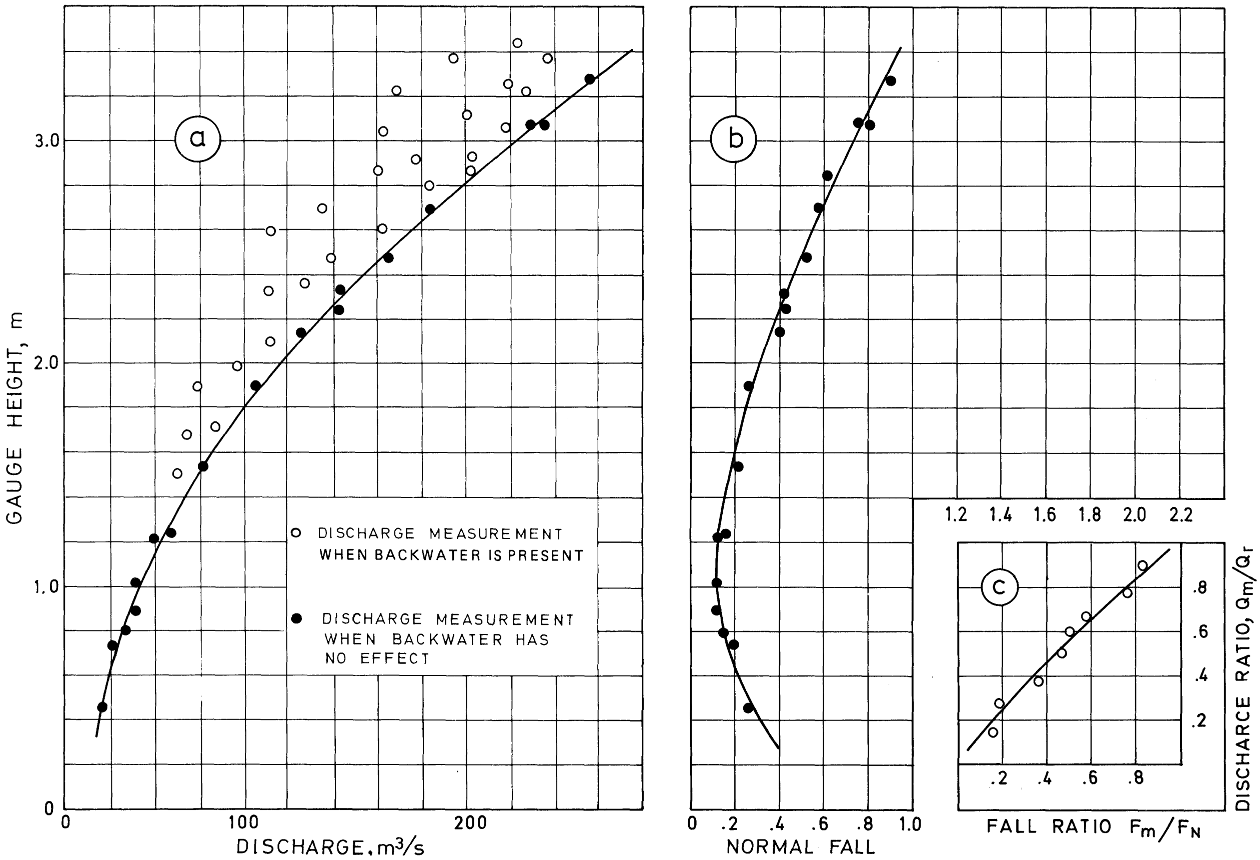


Figure 24. Stage-fall-discharge rating, submergence of low flow control (courtesy of US Geological Survey). [9].

The rest of the procedure is identical to that of the constant-fall method. The only difference is that a normal-fall value varying with stage is used instead of a constant-fall value. [6].

2.4.4 *Rapidly Changing Discharge—Unsteady Flow*

At gauging stations located in a reach where the slope is very flat, the stage-discharge relation is frequently affected by the superimposed slope of the rising and falling limb of a passing flood wave. During the rising stage, the velocity and discharge are greater than they would be for the same stage at steady flow conditions. Similarly, during the falling stage, the discharge is less for any given gauge height than it is when the discharge is constant.

The method used in developing rating curves at single-gauge stations is, as discussed in Section 2.2.1, to draw a median curve through a scatter of plotted discharge measurements. This procedure gives a correct result when all discharge measurements have been made at steady or nearly steady flow conditions. In fact, if each plotted measurement had been tagged as to whether it had been measured on a rising or falling stage, the curve would have taken the shape of a loop (Fig. 25). This effect is especially noticeable for large rivers having flat slopes with channel control extending far downstream. For smaller rivers having section control or steeper slopes and the measuring site is not too far from the site where the stage is observed, the looping effect is rarely of such a magnitude as to have any practical significance. The looping effect is due to several causes. The first of them is channel storage. If a discharge measurement is made at some distance from the station control during a period of rising or falling stage, the discharge passing the measuring section will not be the same as the discharge at the control. A correction for the channel storage has to be applied to the measured discharge by adding or subtracting from the measured discharge a quantity equal to the product obtained by multiplying the water surface area, between the measuring section and the control, by the average rate of change in stage in the same river reach. If the measurement is made above the control, the correction will be plus for falling stages and minus for rising stages. If made below the control, it will be minus for falling and plus for rising stages.

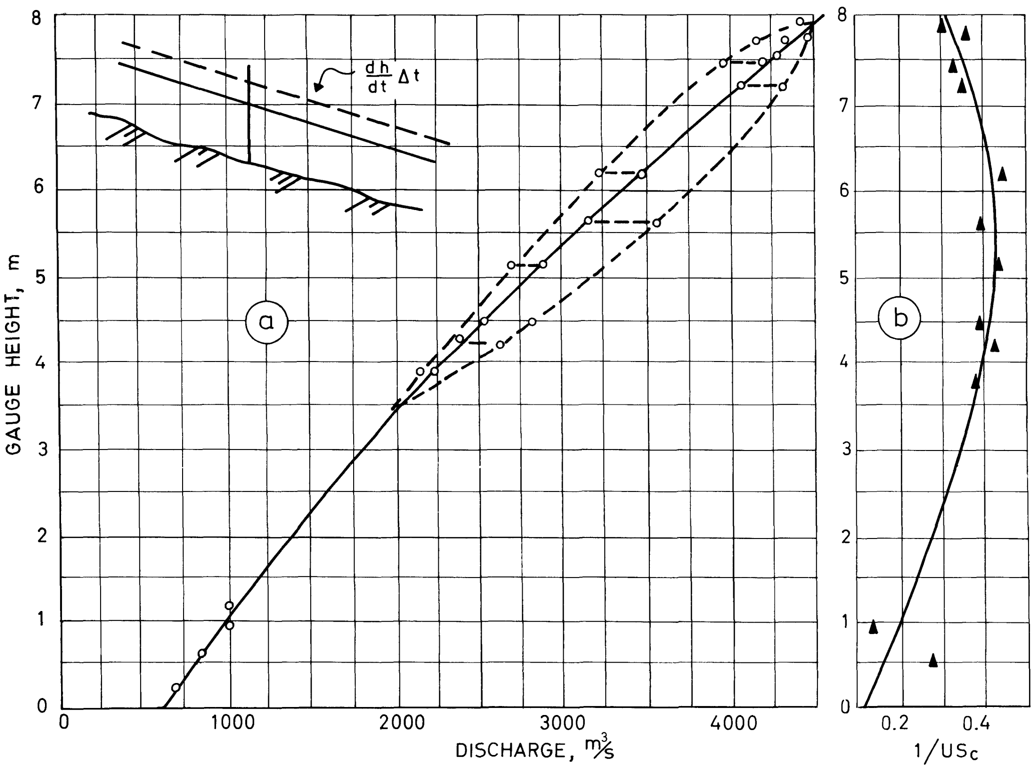


Figure 25. Stage-discharge relation. Rapidly changing flow (courtesy of US Geological Survey). [9].

ILLUSTRATION

A measurement is made 1000 m upstream of the control. The average width of the channel is 100 m, the average rate of rise of the water surface in the reach during the measurement is 0.15 m/hr. Measured discharge is 120 m^3/s.

The rate of change of storage in the reach is given by

$$ds = 1000 \times 100 \times 0.15 = 15000 \ m^3/hr$$
$$ds = 4.2 \ m^3/s$$

The discharge measurement is plotted as 120 $m^3/s - 4$ m^3/s = 116 m^3/s (rounded), which is the discharge passing the control and to which the mean gauge height during the measurement corresponds.

The second reason for the looping of rating curves is the variation in the surface slope which occurs as a flood wave passes a river gauging station. Discharge measurements taken on either side of a flood wave may be corrected to the theoretical steady state condition by application of the following equation

$$Q_m/Q_r = (1 + 1/US_c \ dh/dt)^{1/2} \qquad (2.30)$$

where

- Q_m = measured discharge (m^3/s),
- Q_r = estimated steady state discharge from rating curve (m^3/s),
- U = wave velocity (celerity) (m/s),
- S_c = energy slope at steady state flow,
- dh/dt = rate of change of stage, positive for rising stage and negative for falling stage (m/t).

Rearranging Eqn. (2.30) obtains

$$1/US_c = ((Q_m/Q_r)^2 - 1)/(dh/dt) \qquad (2.31)$$

If a sufficient number of measurements have been made during both rising and falling stage and at steady state conditions, Eqn. (2.31) may be solved by a graphical method, known as the Boyer method, without having to compute the energy slope and the velocity of the flood wave.

The discharge measurements are plotted in the usual manner and a rating curve is drawn as a median curve through the uncorrected values (Fig. 25a. The steady state discharge Q_r is estimated from this median curve. Q_m and dh/dt have been measured and are therefore known quantities. Then, by substituting in Eqn. (2.31), the term 1/ US_c is obtained for each discharge measurement.

The term $1/US_c$ is plotted against stage and a mean curve fitted to the plotted points (Fig. 25b). From this curve, new smoothed values of $1/US_c$ are obtained and inserted in Eqn. (2.31) in order to obtain the steady state Q_r. The new values of Q_r are then plotted against stage to obtain a corrected steady state rating curve. [6].

For gauging stations situated in tidal reaches with significant unsteady flow, calculation of the discharge is generally carried out by special methods, e.g. *the method of cubature* and *unsteady flow mathematical modelling.*

The method of cubature is based on the law of continuity. The rate of rise and fall of the water surface is used to determine the rate of gain and loss of channel storage in a reach. The discharge at the downstream end of the channel reach is calculated from the known inflow to the reach and the computed gain or loss in channel storage during the time required for the water surface to rise and fall. [17].

Unsteady flow mathematical models are based on assumptions of moderately unsteady, homogeneous, and one-dimensional flow and prismatic channel geometry. On these assumptions, a system of unsteady flow equations can readily be set up to describe the tidal flow. Initial and boundary conditions are determined by field measurements. The actual computation of discharge is performed by digital computer. [17].

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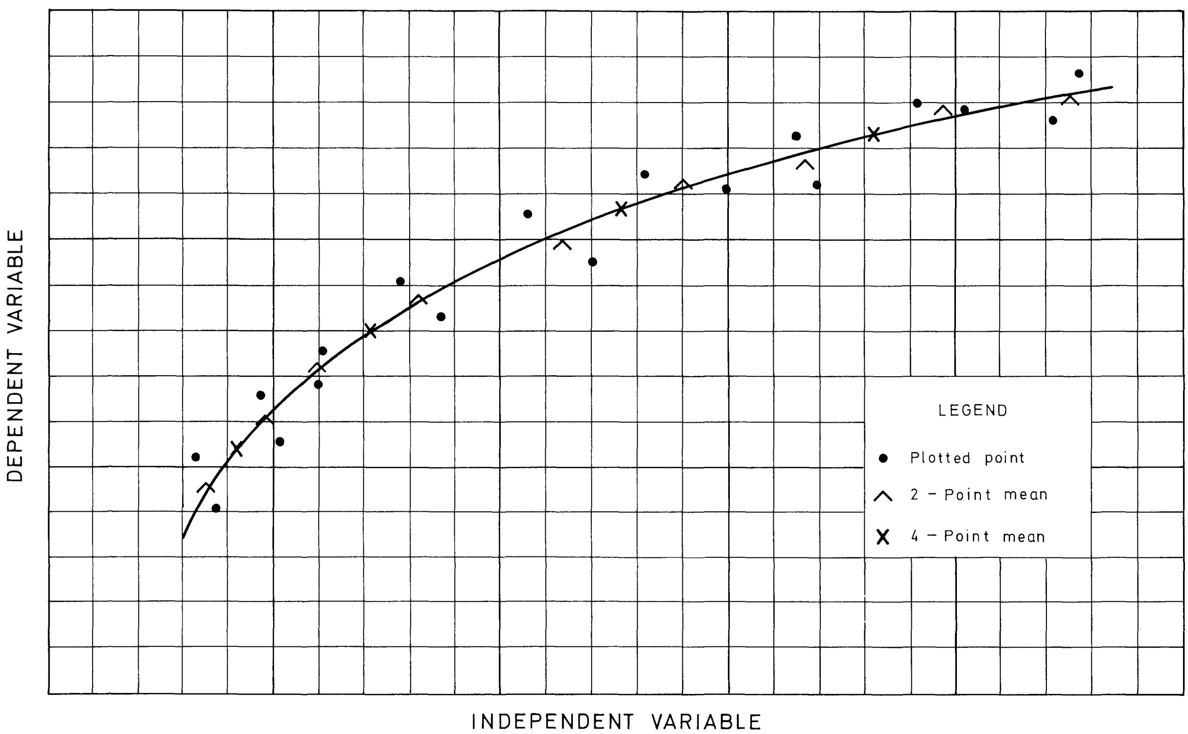
Appendix A

GRAPHICAL CURVE FITTING

GRAPHICAL CURVE FITTING

The simplest way of fitting a curve to plotted data points would be to draw a continuous and smooth median curve through the scatter of the points. Where the nature of the relation is indicated and the points are not too scattered, this might give quite a satisfactory result. In other cases, however, the points might be more widely scattered and the underlying relation might be more difficult to determine so that different persons drawing in the curve by free hand might draw rather different curves. Some method is therefore needed to ensure a greater degree of precision and stability to the result. This is obtained by determining average coordinates for groups of points. The general nature of the relation is then expressed by an irregular line connecting the several group averages, and a continuous and smooth curve is derived from this irregular line.

The group averages are estimated graphically by first estimating two-point averages, that is halfway between two and two of the plotted points. The four-point averages are halfway between each pairs of the two-point averages, and so on (Fig. A.1). The points are grouped with respect to the independent variable. If upon inspection of the group averages, it is decided that the relation is curvilinear, then a curve can be fitted to the average points with the aid of drafting curves. If the curve is a straight line, this line must pass through the centre of gravity of all the points. [14].



Figur A.1. Graphical curve fitting by use of group averages. [14]

Appendix B

METHODS OF TESTING THE STAGE-DISCHARGE CURVE

METHODS OF TESTING THE STAGE-DISCHARGE CURVE

B.1 Error of Observation of Discharge Measurements

As discussed before, a rating curve based on discharge measurements is developed by balancing it through a scatter-plot of the measurements. If the measurements were made without error and if the station control remained constant, all points would fall on a smooth curve. Such ideal conditions are not attainable in practice, there will always be deviations from a smooth curve.

In fact, it is well known that measurements and observations of all kind are invariably subject to *error of observation.* The error of observation is generally composed of several independent errors associated with the operational procedure of making the observation, such as the measurement of depth, measurement of width, number of verticals in the cross section, number of points in the vertical, time of observation, drift of the instrument, obliquity of the current, determination of the gauge height, sensitivity of the current meter, the precision of the timer, and others. All of these operations may be considered independent of each other and the associated errors may therefore be regarded as random variables. Thus, the composite error, i.e. the error of observation, may be regarded as normally distributed, however only about the mean for measurements made at one and the same gauge height. Which is confirmed by investigations.

Further, investigations show that the error of measurement is not independent, but depends on the magnitude of the measured discharge. It has been demonstrated that the absolute error of observation is proportional to the magnitude of the measured discharge over each range having the same station control.

Investigations show that with proper measuring procedures it should be possible to keep the error of observation in the order of 3-5 % of the measured discharge.

There may also be systematic errors in a discharge measurement caused by faulty measuring instruments, or by human factors. Such errors can not be eliminated by repeated measurements, but only by checks against proven instruments by different operators. With today's current meters, the systematic error due to meter performance is not expected to exceed 1 %, when the meter is properly functioning and calibrated.

B.1.1 The Standard Error of Estimate of the Rating Curve

The basic assumptions for a valid estimation of the standard deviation of the error of observation, are that the error of observation be normally distributed and independent. For a regression (e.g. the stage-discharge curve) this is achieved by a simple transformation to relative values. The percentage deviation of each discharge measurement from the estimated discharge by the rating curve, is first worked out, after which is calculated the standard deviation and the standard error of the percentages (refer Table B.1). Thus,

$$p = ((Q_m - Q_r)/Q_r) \times 100\ \% \qquad \text{(B.1)}$$

The percentage standard deviation then is

$$s_D = (\sum(p-\bar{p})^2/(n-1))^{1/2} \qquad \text{(B.2)}$$

and the percentage standard error is

$$s_E = s_D/n^{1/2} \qquad \text{(B.3)}$$

where

Q_m = measured discharge (m^3/s),
Q_r = discharge estimated by the rating curve (m^3/s),
p = percentage deviation,
$\bar{p}$ = mean percentage deviation,
s_D = standard deviation of the percentage deviation p,
s_E = standard error of the percentage deviation p,
n = number of discharge measurements.

B.2 Reliability of Discharge Rating Curves

B.2.1 Required Number of Discharge Measurements for Establishing a Reliable Rating Curve

The number of discharge measurements required in order to obtain a reliable rating curve, may be calculated from the following equation

$$n \geq ((2s_D)/E)^2 \qquad \text{(B.4)}$$

where

n = number of measurements,
s_D = standard deviation in per cent ($2s_D$ is allowable width of scatter band),
E = a specified precision expressed as a percentage, usually 5 %.

s_D is calculated separately according to Eqn. (B.2), refer Table B.1, for each range of stage having a separate station control, and the test is applied separately to each range.

The following table indicates the variation in the required number of measurements with variation in width of scatter band, E being taken as 5 %.

Width of scatter band $2s_D$ %	Minimum number of measurements, n
10	6
15	9
20	16
25	25
30	36

It is recommended that n should never be less than six for any one interval of the range. [15].

B.2.2 Acceptance Limits for Discharge Measurements

The reliability of the estimated rating curve may generally be assessed by the concept of the *acceptance region* for the observations.

A pair of curves drawn one on each side of the rating curve and each at a distance of two standard deviations from the rating curve, are called control curves and define the 95 % acceptance region. That is, 95 out of every hundred (or nineteen out of every twenty) measurements should lie between the control curves. A single measurement lying far outside (say beyond three standard deviations) is most probably the result of faulty gauging equipment or of poor measuring practices.

However, in those cases where two or more consecutive points, either chronologically or over a range in stage, appear to be well on one side of a control curve, a change of the stage-discharge relation has probably occurred. This means that owing to shift in the station control, a new rating curve is required and the calibration of the station must be repeated.

B.2.3 Statistical Tests Applied to Rating Curves for Absence of Bias and Goodness of Fit

The following two criteria are commonly used to test a rating curve for *absence of bias:*

a) The average of the percentage differences between the measured discharges and the discharges estimated by means of the rating curve, should not be significantly different from zero. This is tested by the *Student's paired t-test.*
b) The number of positive and negative deviations of the measured discharges from the rating curve, should be evenly distributed, i.e. the difference in number of pluses and minuses should not be more than can be explained by chance fluctuations. This is tested by the *sign test.*

These two tests are only applicable for stations where the stage-discharge relation is permanent and not effected by variable backwater or highly unsteady flow conditions.

After the rating curve has been checked for absence of bias, it should next be checked for *shifts in control*. The errors due to shifting control would be of a systematic nature. The difference between Q_m and Q_r, the measured discharge and the estimated discharge by the rating curve, is checked for the same sign in long runs. This is tested by the *run of sign test*. The *goodness of fit* is also checked by this test.

B.2.3.1 The Student's Paired t-test

The Student's paired t-test of the differences between the discharge measured and the discharge estimated by the rating curve is used to check wether a rating curve, on an average, gives significant overestimates or underestimates as compared with the discharge measurements on which the curve is based.

The mean deviation, $\bar{p}$, in per cent, is tested against its standard error to see if it is significantly different from zero.

The basic assumption underlying this test are that the percentage differences between the measured values and the values estimated by the curve are independent of the magnitude of the discharge and normally distributed about a mean value of zero.

ILLUSTRATION

In Table B.1 is tabulated 28 discharge measurements. The *test statistic t* has been calculated and is equal to 0.23. Tables of the *t-distribution* giving values of *t* for different levels of significance are available (Table B.2). Values of the calculated *t* exceeding these tabulated values would indicate bias.

The tabulated values of *t* at the 5 % significance level and for a sample size of 28 is found to equal 2.052 (two-tailed test, (n-1) = 27 degrees of freedom).

Conclusion: Since the calculated value of t is less than 2.052, the rating curve is free of bias as judged by this method.

(Strictly speaking, this test would be more appropriate if the number of discharge measurements at the various ranges of discharges were in proportion to the probable occurrences of these discharges, covering the whole range of discharges for which the curve is estimated).

B.2.3.2 The Sign Test

The test is used to check if the curve has been drawn in a sufficiently balanced manner so that the two sets of discharge values, those measured and those estimated from the curve, may be reasonably supposed to represent the same population.

The number of positive and negative deviations of the measurements from the estimated rating curve shall be evenly distributed. That is, the difference in number between the two shall not be more than can be explained by chance fluctuations.

This is a simple test and is performed by counting the observed points falling on either side of the curve. If Q_m is the measured value and Q_r the estimated value, then $(Q_m - Q_r)$ should have an equal chance of being positive or negative, that is the probability of $(Q_m - Q_r)$ being positive is 0.5. Then, assuming the successive signs to be independent of each other, the sequence of the differences may be considered distributed according to the binomial law $(p+q)^n$ where n is the number of observations, and p and q, the probabilities of occurrence of positive and negative values, are 0.5 each.

The statistics for the sign test applied to the curve of Table B.1 are:

1. Number of observations: $n = 28$
2. Number of positive signs: $n_1 = 15$
3. Probability of sign being positive: $p = 0.5$
4. Probability of sign being negative: $q = 0.5$
5. Expected number of positive signs: $np = 14$
6. Standard deviation of np: $s_D = (npq)^{1/2} = 2.65$
7. Test criterion:
   $t = (|n_1 - np - 0.5^{*)}|)/s_D$
   $t = (|15 - 14 - 0.5|)/2.65 = 0.189$

*): Continuity correction

The tabulated value of *t* at the 5 % significance level and for a sample size of 28 is equal to 2.052 (Table B.2, two-tailed test 27 degrees of freedom).

Conclusion: Since the calculated value of *t* is less than the tabulated value, the rating curve is free of bias as judged by this method.

B.2.3.3 The Run of Sign Test

This test is used to check if shifts in the station control have occurred. It consists of detecting the presence of possible abnormally long runs of positive or negative deviations.

The test is based on the number of changes of sign in the series of deviations of measured discharge from the established rating curve.

From Table B.l is written down the signs of deviations in chronological order. Starting from the second number of the series, we write under each a "0" if the sign is the same or "1" if the sign is not the same as the immediate preceding sign. If there are n observations in the original series, there will be (n - 1) number of signs in the derived series, Thus,

- + + + - + - + + - - + + - - + + - - - - + + + + - +
1 0 0 1 1 1 1 0 1 0 1 0 1 0 1 0 1 0 0 0 1 0 0 0 1 1

Assuming that the deviations can be regarded as arising from random fluctuations about the estimated values from the curve, the probability for a change in sign may be taken to be 0.5. If n is fairly large, say 25 or more, this will be a reasonable assumption and the derived series may be assumed to follow the binomial distribution. The test is carried out as follows:

1. Number of deviations: N $= 27$
2. Number of change in sign: n $= 13$
3. Probability of change in sign: p $= 0.5$
4. Probability of no change in sign: q $= 0.5$
5. Expected number of changes in sign: $(N-1)p = 13$
6. Test criterion: $t = \dfrac{|n-(N-1)\,p-0.5|}{((N-1)\,pq)^{1/2}}$

$t = (|13-13-0.5|)/2.55 = 0.196$

The tabulated value of t at the 5 % significance level and for a sample size of 27 is equal to 2.056 (Table B.2, two-tailed test, 26 degrees of freedom).

Conclusion: As the calculated value of t is less than the tabulated value, the assumption of random fluctuations has not been disproved.

The test shows that there is no systematic trend in the deviations with time, indicating that the curve as drawn does not need any adjustment for shift in control.

The Run of sign test is used also to test for goodness of fit. In this case the discharge measurements will have to be arranged in an ascending order of stage before counting up the plus and minus deviations.

Bibliography

Appendix B: [15], [16].

Table B.1. Table of Discharge Measurements and Computation of Statistical Parameters

No .	Measurements H	Measurements Q_m	Rating Q_r	Percentage deviation, P: P +	P −	$P-\bar{P}$ +	$P-\bar{P}$ −	$(P-\bar{P})^2$
1	1.55	300	303	–	0.99	–	1.13	1.28
2	1.44	287	274	4.74	–	4.60	–	21.16
3	1.26	235	231	1.73	–	1.59	–	2.53
4	1.05	193	185	4.32	–	4.18	–	17.47
5	0.73	125	125	–	–	–	0.14	0.10
6	0.69	113	118	–	4.24	–	4.38	18.49
7	0.70	124	120	3.33	–	3.19	–	10.18
8	1.70	340	344	–	1.16	–	1.30	1.69
9	0.96	169	166	1.81	–	1.67	–	2.79
10	0.94	168	162	3.70	–	3.56	–	12.67
11	1.35	240	252	–	4.76	–	4.90	24.01
12	1.17	202	211	–	4.27	–	4.41	19.45
13	1.79	387	372	4.03	–	3.89	–	15.13
14	3.09	930	916	1.53	–	1.39	–	1.93
15	3.87	1374	1395	–	1.51	–	1.65	2.72
16	2.33	540	558	–	3.23	–	3.37	12.74
17	3.49	1152	1135	1.50	–	1.36	–	1.85
18	3.93	1452	1430	1.54	–	1.40	–	1.96
19	2.03	440	452	–	2.65	–	2.79	7.78
20	1.61	306	319	–	4.08	–	4.22	19.54
21	2.13	469	488	–	3.89	–	4.03	16.24
22	1.37	246	257	–	4.28	–	4.42	19.54
23	1.05	189	185	2.16	–	2.02	–	4.08
24	0.91	163	156	4.49	–	4.35	–	18.92
25	0.79	139	135	2.96	–	2.82	–	7.95
26	0.68	120	117	2.56	–	2.42	–	5.86
27	0.61	104	106	–	1.89	–	2.03	4.12
28	0.53	94.6	95.2	0.60	–	0.46	–	0.21
			Sum	41.00	36.95	38.90	38.77	272.39

Percentage deviation, p $= \dfrac{Q_m - Q_r}{Q_r} 100$

Mean percentage deviation of p, $\bar{p}$ $= \dfrac{P_1+P_2+..+P_n}{n} = \dfrac{41.00-36.95}{28} = 0.14\ \%$

Standard percentage deviation of p, s_D $= \left(\dfrac{\sum(p-\bar{p})^2}{(n-1)}\right)^{1/2} = \left(\dfrac{272.39}{27}\right)^{1/2} = 3.17\ \%$

Standard percentage error of p, s_E $= \dfrac{s_D}{n^{1/2}} = \dfrac{3.17}{28^{1/2}} = 0.60\ \%$

Test statistic, $t = \dfrac{\bar{p}}{s_E} = \dfrac{0.14}{0.60} = 0.23$

Tabell B.2 Table of Student's t-distribution

	Probability of a larger value of t, sign ignored (two-tail test)											
d.f.	.9	.8	.7	.6	.5	.4	.3	.2	.1	.05	.02	.01
1	.158	.325	.510	.757	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657
2	.142	.289	.445	.617	.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925
3	.137	.277	.424	.584	.765	.978	1.250	1.638	2.353	3.182	4.541	5.841
4	.134	.271	.414	.569	.741	.941	1.190	1.533	2.132	2.776	3.747	4.604
5	.132	.267	.408	.559	.727	.920	1.156	1.476	2.015	2.571	3.365	4.032
6	.131	.265	.404	.553	.718	.906	1.134	1.440	1.943	2.447	3.143	3.707
7	.130	.263	.402	.549	.711	.896	1.119	1.415	1.895	2.365	2.998	3.499
8	.130	.262	.399	.546	.706	.889	1.108	1.397	1.860	2.306	2.896	3.355
9	.129	.261	.398	.543	.703	.883	1.100	1.383	1.833	2.262	2.821	3.250
10	.129	.260	.397	.542	.700	.879	1.093	1.372	1.812	2.228	2.764	3.169
11	.129	.260	.396	.540	.697	.876	1.088	1.363	1.796	2.201	2.718	3.106
12	.128	.259	.395	.539	.695	.873	1.083	1.356	1.782	2.179	2.681	3.055
13	.128	.259	.394	.538	.694	.870	1.079	1.350	1.771	2.160	2.650	3.012
14	.128	.258	.393	.537	.692	.868	1.076	1.345	1.761	2.145	2.624	2.977
15	.128	.258	.393	.536	.691	.866	1.074	1.341	1.753	2.131	2.602	2.947
16	.128	.258	.392	.535	.690	.865	1.071	1.337	1.746	2.120	2.583	2.921
17	.128	.257	.392	.534	.689	.863	1.069	1.333	1.740	2.110	2.567	2.898
18	.127	.257	.392	.534	.688	.862	1.067	1.330	1.734	2.101	2.552	2.878
19	.127	.257	.391	.533	.688	.861	1.066	1.328	1.729	2.093	2.539	2.861
20	.127	.257	.391	.533	.687	.860	1.064	1.325	1.725	2.086	2.528	2.845
21	.127	.257	.391	.532	.686	.859	1.063	1.323	1.721	2.080	2.518	2.831
22	.127	.256	.390	.532	.686	.858	1.061	1.321	1.717	2.074	2.508	2.819
23	.127	.256	.390	.532	.685	.858	1.060	1.319	1.714	2.069	2.500	2.807
24	.127	.256	.390	.531	.685	.857	1.059	1.318	1.711	2.064	2.492	2.797
25	.127	.256	.390	.531	.684	.856	1.058	1.316	1.708	2.060	2.485	2.787
26	.127	.256	.390	.531	.684	.856	1.058	1.315	1.706	2.056	2.479	2.779
27	.127	.256	.389	.531	.684	.855	1.057	1.314	1.703	2.052	2.473	2,771
28	.127	.256	.389	.530	.683	.855	1.056	1.313	1.701	2.048	2.467	2.763
29	.127	.256	.389	.530	.683	.854	1.055	1.311	1.699	2.045	2.462	2.756
30	.127	.256	.389	.530	.683	.854	1.055	1.310	1.697	2.042	2.457	2.750
∞	.12566	.25335	.38532	.52440	.67449	.84162	1.03643	1.28155	1.64485	1.95996	2.32634	2.57582
d.f.	.45	.40	.35	.3	.25	.2	.15	.1	.05	.025	0.01	.005
	Probability of a larger value of t, sign considered (one -tail test)											

Appendix C

SHIP DRAFTING CURVES

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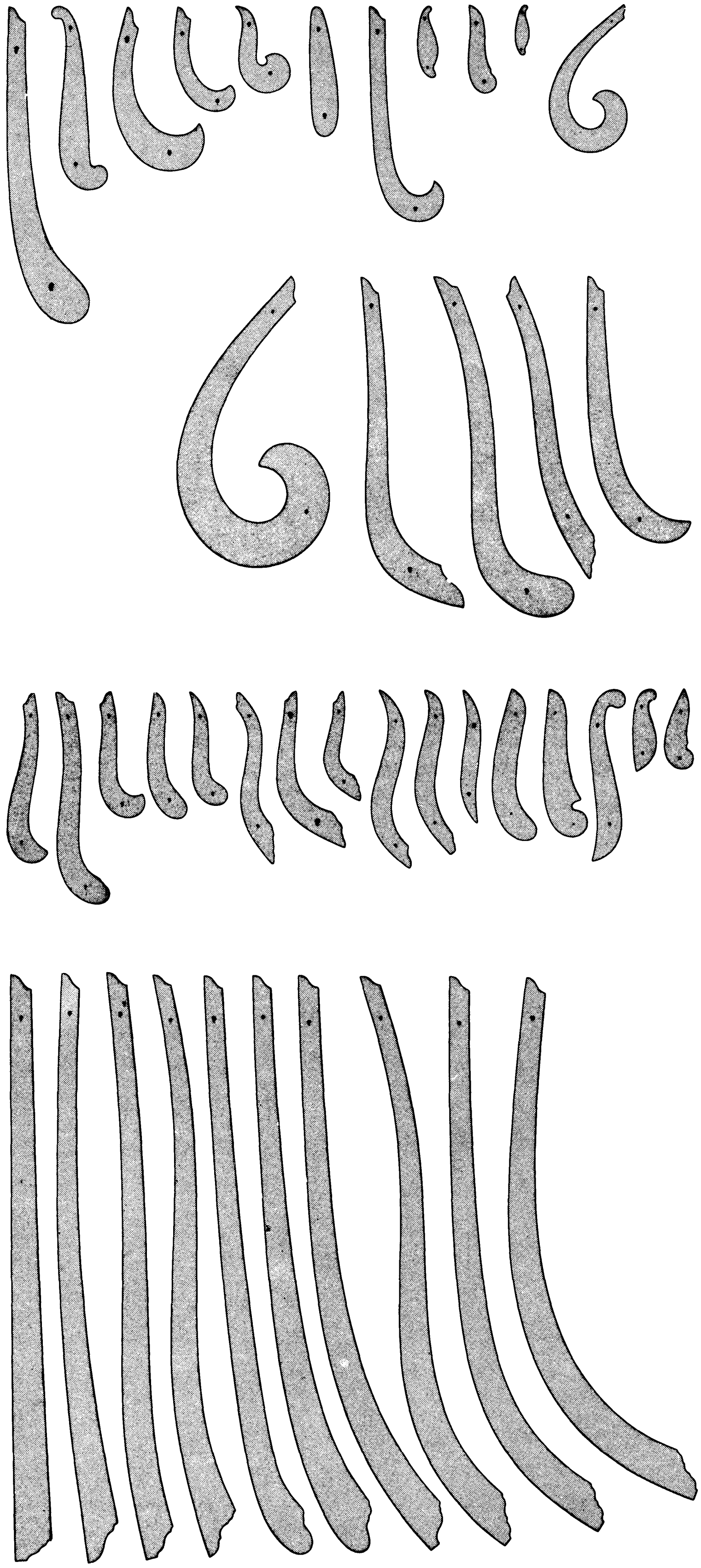
Approximate scale 1:7



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