Spatial scaling of discharge

-application for regional flood frequency analysis

Norges vassdrags- og energidirektorat
2003

Report no 2-03

Spatial scaling of discharge-application for regional flood frequency analysis

Utgitt av: Norges vassdrags- og energidirektorat

Forfatter: Thomas Skaugen og Thomas Væringstad

Trykk: NVEs hustrykkeri
Opplag: 50
ISSN-no 1502-3540
ISBN-no 82-410-0472-9

Sammendrag: From an assumption that the spatial distribution of specific discharge generation is a shifted exponential distribution, theoretical expressions of the maximum specific discharge values relative to catchment size are developed and a regional scaling equation is presented. Catchment heterogeneity due to physiographic features of catchments is incorporated in the model using multiple regression analysis. Flood quantiles for the Glomma regiona are thus estimated based on scaling properties and catchment characteristics.

Emneord: Discharge, flood, scaling, flood quantiles, regional analysis

Norges vassdrags- og energidirektorat
Middelthuns gate 29
Postboks 5091 Majorstua
0301 OSLO

Telefon: 22 95 95 95
Telefaks: 22 95 90 00
Internett: www.nve.no

March 2003

Contents

Preface 4

Summary 5

Introduction 6

Exponentially distributed runoff generation 7

Scaling under an exponential model 9

Scaling of flood quantiles 11

Application for regional flood frequency analysis 16

Case study and discussion 17

Comparison to previous regional flood frequency analysis studies in Norway 26

Conclusions 28

References 29

Preface

There is a persisting need for extreme value assessments of discharge for ungauged catchments for design purposes, flood protection and flood risk analysis. This study is aimed at improving the methods for regional frequency analysis and can serve as a point of departure for re-evaluating the operational methodology for estimating flood quantiles in ungauged catchments. This study has been financed trough the "Flood related project" at NVE.

Oslo, March 2003

Kjell Repp
head of department

Sverre Husebye
head of section

Summary

A methodology is put forward for estimating flood quantiles in ungauged catchments, taking into account the scaling behaviour of discharge in a homogeneous region. From an assumption that the spatial distribution of specific discharge generation is a shifted exponential distribution, theoretical expressions of the maximum specific discharge values relative to catchment size are developed and a regional scaling equation is presented. The homogeneity constraint refers to climatic conditions, whereas catchment heterogeneity due to physiographic features of catchments is incorporated in the models using multiple regression analysis. The methodology is tested for a region in Southern Norway with homogeneous climatic conditions but with varying physiographic catchment characteristics. It is investigated if the extreme discharge values estimated with traditional methods are adequately reproduced with the proposed methodology. For the tested region, the agreement between flood quantiles estimated with the proposed methodology and flood quantiles estimated from observed data is good. The effective lake percentage and the river gradient proved to be significant descriptors of the deviation from the scaling procedure.

Introduction

The classical problem of flood predictions for ungauged catchments (PUB) (see IAHS, 2002; Veitzer and Gupta, 2001; Gupta et al, 1996) still remains a challenge for operational and theoretical hydrology. These predictions often form the basis for civil engineering works and the formulation of land use plans. Scale issues in hydrology are closely linked to this problem in that operational hydrology is supposed to describe and forecast events in river basins on spatial scales extending from 10^{-1} to 10^{6} km^2. By the term scaling, we understand the effects on the statistical parameters by changing the scale of observation (or averaging) of hydrological quantities (Woods et al. 1995). However, these issues have not been taken sufficiently into consideration in practical applications in engineering hydrology (Blöschl and Sivapalan, 1995). The engineering practice for PUB has often been based on the index flood method (NERC, 1975), for which the principal assumption is that peak discharges within a homogeneous (geographical) region have a common probability distribution function when scaled by their mean or some other index discharge (Robinson and Sivapalan, 1997, Gupta et al. 1994). The scaling parameter (usually the mean annual peak flow) is derived for ungauged basins from relationships with physical parameters of the basin and climate where the drainage area is the most important or the only parameter used (Gupta et al. 1994). The assumption of a common probability distribution implies that the effects of changing scale of observations on the distribution of peak flow, are counted for by this scaling parameter, i.e. that the ratio of floods of a given return period to the mean annual flood is independent of drainage area. This is contrary to empirical observations that smaller catchments have “steeper frequency curves” than larger catchments, all other things being equal (Gupta et al. 1994). Whereas we can note that there has since long been recognised a connection between catchment size and quantile estimates of areal precipitation (see e.g. Bell, 1976 and Skaugen, 1997), these concepts have not been incorporated in operationally useful methods for PUB.

This paper develops a methodology for estimating flood quantiles for catchments of different scales and derives an application for regional flood frequency analysis. We adopt, like Robinson and Sivapalan (1997), the definition of a homogenous region of Gupta et al. (1994), where the hydrological properties (such as flood and rainfall frequencies, stream length, lakes and slopes) can be related using a scale function that only involves drainage area. We consider an idealised situation where synoptic discharge generation per unit area is considered as a continuous spatial field and exponentially distributed. We further apply the Markov property of the exponential distribution (Feller, 1971, p.8-9) to develop theoretical inferences on the maximum (and minimum) average values of discharge as a function of scale. Deviations of flood quantiles from the derived scaling procedure within a homogeneous region is considered to be caused by catchment specific physiographical features and are modelled by a multiple regression analysis. The scaling procedure together with the modelling of the effects due to physiographical features provides an application for regional flood frequency analysis.

Exponentially distributed runoff generation

The theoretical derivations in the following sections that lead to expressions linking scale and quantiles of discharge assume that the spatial statistical distribution of generated discharge is of an exponential type with a location parameter (a minimum value). It is readily admitted that the exponential distribution might not be the most suitable to represent that physical process. However, the derived principles, so easily revealed due to the mathematical convenience of the exponential distribution, is thought to be of a general nature, useful for the understanding of the interactions between spatial scale and statistical parameters. Also, the case study, presented in a later section, indicates that the chosen statistical models are quite adequate. In the following we elaborate on the concept of a spatial statistical distribution of runoff generation, which is chosen to be of an exponential type.

Let catchment A be partitioned into a set of spatial elements $\varpi_1, \varpi_2, \ldots.$ from where, during a time interval Δt, runoff $q(\varpi, \Delta t)$ is generated. The spatial element ϖ represents a suitable spatial aggregation level of a continuous field. Wood et al. (1988) stated that a catchment can be treated as being composed of numerous (infinite) points where infiltration, evaporation and runoff form the local water balances fluxes, and we consider runoff generation measured at ϖ during a certain time interval as a (nearly) continuous field over a catchment. This is a consequence of regarding runoff as the result of interactions from the spatially continuous fields of hill-slopes, soil, vegetation, snow and snowmelt, temperature and precipitation. In the distribution function approach (Beven, 1991), hill-slope, soil, vegetation and rainfall characteristics are imagined drawn from stationary, spatially correlated distributions, and thereby define continuous spatial fields.

We assume that an exponential distribution serves as an adequate model of runoff generation $q(\varpi, \Delta t)$, with probability density function (PDF): $f(q) = \Lambda e^{-\Lambda q}, \ \ 0 < q < \infty$, expectation $E(q) = 1/\Lambda$ and variance $Var(q) = 1/\Lambda^2$. We can also think of discharge over a certain limited region with a minimum discharge b. The parameter b serves as a location parameter of the exponential distribution, and we have thus: $f(q) = \Lambda e^{-\Lambda(q-b)}, \ \ b < q < \infty$ with expectation

$$E(q) = b + 1/\Lambda \tag{1}$$

and variance

$$Var(q) = 1/\Lambda^2 \tag{2}$$

We cannot investigate the validity of the assumption of exponentially distributed discharge generation in a classical manner by plotting histograms of observed synoptic values of generated discharge. A procedure like this assumes point measurements, whereas measurements of discharge are inherently spatial averages, which will influence, also on specific discharge, the statistical parameters estimated from the sample.

Scaling under an exponential model

Under an assumption that the spatial distribution of discharge is exponential, certain properties of the scaling behaviour can be determined. Feller (1971, p.8-9) discusses the *lack-of-memory-*, or the *Markov property* unique for the exponential distribution when considered as a waiting time- or lifetime distribution. Feller (1971) states "whatever the present age, the residual lifetime is unaffected by the past and has the same distribution as the lifetime itself." Translated into our framework of exponentially distributed (in space) discharge generation q, we can write: "given a certain threshold b of discharge intensity, the discharge intensities higher than b have the same distribution as q itself." If q is exponentially distributed with parameter Λ, for a certain fraction p of a catchment we find discharge intensities higher than b, distributed with parameter Λ.

To investigate this property in a scaling context, let us make a plot of an exponential complementary cumulative distribution function of q with a location parameter (minimum value) b for some event over a catchment A (see Fig.1).

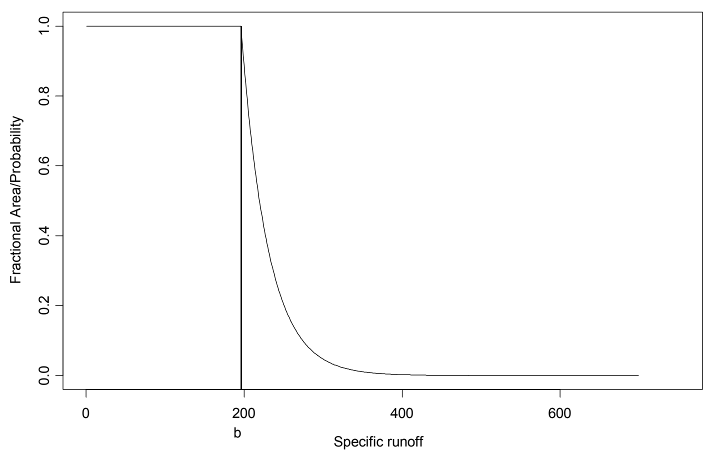

Figure 1. Complementary cumulative distribution function of generated runoff. *b* denotes the minimum value

If we sampled q with the scale of observation, $p = A_i / A$, everywhere within the catchment A, we would obtain a distribution of $q(p)$. This distribution remains unknown, but we can compute both the maximum values and the minimum values of the spatial averages. We see from Figure 1 that given a fraction p of A, we can determine a threshold $b_{\max}$, for which all the values within the area p are higher than the values outside of p. By setting the complementary CDF of q equal to p, $1 - F(q) = e^{-\Lambda(q-b)} = p$, b_{max} is determined for any p as:

$$b_{max} = \frac{-\log p}{\Lambda} + b \tag{3}$$

The maximum value for any scale p can, according to the Markov property, equation (1) and the relation in (3), be estimated as:

$$q(p)_{max} = b_{max} + E(q - b_{max} \mid q > b_{max}) = b + \frac{1}{\Lambda}(1 - \log p) \tag{4}$$

Also the minimum spatial mean given the fraction p can be determined. Let r be the complementary scale of p, $r = 1 - p$. The minimum value possible for r, $q(r)_{min}$, can then be computed if we note that the mean of q estimated over A consists of the weighted sum of the maximum average - and minimum average value with weights p and r respectively:

$$E(q)_A = pq(p)_{max} + rq(r)_{min}$$

which gives, according to (1) and (4):

$$q(r)_{min} = b + \frac{1}{\Lambda}\left(\frac{1-(1-r)(1-\log(1-r))}{r}\right) \tag{5}$$

Figure 2 shows theoretical maximum and minimum values for an event with $\Lambda = 1$ and $b = 0$. We can observe how the maximum and minimum values decrease and increase respectively as the scale of observation increases. It must be noted here that the validity of the above derivations depends on events being well correlated in space.

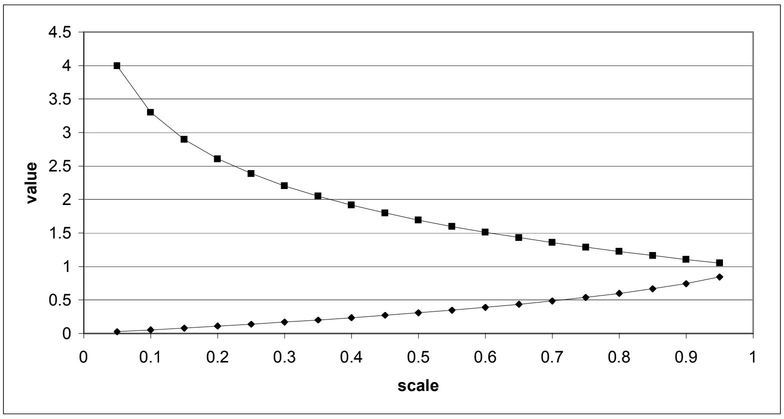

Figure 2. Theoretical maximum and minimum average values versus scale for an exponentially distributed event with $\lambda = 1$.

In accordance with the application of the Markov property used in this study, we can also increase the area *A*, to comprise all positive values of *q* and thus equal *b* to zero. *q* will still be exponentially distributed with parameter Λ, and the area, termed A_0, will vary in spatial extent for different events.

Scaling of flood quantiles

The proposed approach will provide us with a methodology for estimating flood quantiles for catchments of different sizes over a homogeneous region as defined by Gupta et al. (1994). Two arguments must be taken into consideration to establish the link between scaling and quantiles, i) the parameter of the spatial distribution (Λ) varies according to return period, and ii) catchments from a homogeneous region with identical area and catchment characteristics have identical extreme value distributions. In relation to the first argument, we have demonstrated, from the previous section, that for a single event with spatial standard deviation $1/\Lambda$, we have, by (4), maximum areal average values for all scales *p*. Let us say that, over an area, we have an event where we measure a certain spatial standard deviation, $1/\Lambda$ that gives us a $q(p)_{max}$ of return period *T*. If we increase the scale by Δp, $q(p+\Delta p)_{max}$ is also of return period *T* because there is an unambiguous relationship between $q(p)_{max}$ and $q(p+\Delta p)_{max}$. For $q(p)_{max}$ to be of return period *T*, $q(p+\Delta p)_{max}$ can only take on one, and only one, value which therefore must be of return period *T*. From this argument we find that there is associated a specific value of the spatial standard deviation $1/\Lambda_T$ to the return period *T*. We thus propose a general scaling equation for flood quantiles as:

$$q(p)_T = \frac{1}{\Lambda_T}(1 - \log p) \tag{6}$$

We see that *b* is zero in (6) compared to (4) because we define A_0 in $p = \frac{A_i}{A_0}$ to be the spatial extent of the exponentially distributed event corresponding to the return period T. This implicates that the minimum positive discharge value (*b*) equals zero. We further note that as A_0 is a function of the return period T, also the scale *p* is a function of T. The scale *p* is thus seen relative to the spatial extent of the event A_0, and not as a fixed quantity relative to an arbitrarily chosen reference area.

The second argument is derived from the definition of regional homogeneity used in this paper. This definition states that a set of catchments are homogeneous if we can relate the flood frequencies using a scale function that only involves catchment size and not location within the region (Robinson and Sivapalan, 1997). Effectively this definition tells us that if two catchments in a homogeneous region have the same size, the probability distributions of the peak flows are the same. The extreme value statistics associated with a certain scale can thus be treated as spatially stationary. Let us say that for a certain area ϖ , we have that the extreme values of generated discharge can be approximated by the exponential distribution with parameters b_ϖ and λ_ϖ . By the definition of homogeneity above, we have a stationary field so that we find the same extreme value distribution for every ϖ within the region. For the exponential distribution, the quantile function can be written as:

$$q(\frac{1}{T})_\varpi = b_\varpi - \frac{1}{\lambda_\varpi}(log(1/T)) \tag{7}$$

We want to develop a link between λ and Λ_T . We can fit the quantile function of the type (7) to the extreme values for two catchments, A_i and A_j , located in a homogeneous region:

$$q(\frac{1}{T})_{A_i} = b_{A_i} - \frac{1}{\lambda_{A_i}}(log(1/T)) \tag{8}$$

and

$$q(\frac{1}{T})_{A_j} = b_{A_j} - \frac{1}{\lambda_{A_j}}(log(1/T)) \tag{9}$$

Now, for return period T , the quantiles scale according to (6) with specific Λ_T and $A_{0,T}$. From hereafter, $A_{0,T}$ is denoted A_0 . By inserting $p = \frac{A_i}{A_0}$ into (6) for the catchments A_i and A_j , we get:

$$q(\frac{A_i}{A_0})_T = \frac{1}{\Lambda_T}(1 - log(\frac{A_i}{A_0})) \tag{10}$$

and

$$q(\frac{A_j}{A_0})_T = \frac{1}{\Lambda_T}(1 - log(\frac{A_j}{A_0})) \tag{11}$$

From (10) and (11) we can solve for Λ_T by eliminating $log(A_0)$, and we get:

$$\Lambda_T = \frac{log(A_i / A_j)}{q(\frac{A_j}{A_0})_T - q(\frac{A_i}{A_0})_T}, \tag{12}$$

or, since (8) equals (10) and (9) equals (11), by inserting (8) and (9) into (12) we get:

$$\Lambda_T = \frac{log(A_i / A_j)}{b_{A_j} - b_{A_i} + log(1/T)(\frac{1}{\lambda_{A_i}} - \frac{1}{\lambda_{A_j}})} \tag{13}$$

Equation (13) tells us that if we know the quantiles for two catchments, then Λ_T is known for all quantiles, and by (6), we can determine how the quantiles scale for all scales p.

We can further insert equation (13) into (6) and obtain an expression for quantiles of discharge as a function of both return period T, and scale p, as:

$$q(p, \frac{1}{T}) = (\frac{b_{A_j} - b_{A_i} + log(1/T)(\frac{1}{\lambda_{A_i}} - \frac{1}{\lambda_{A_j}})}{log(A_i / A_j)})(1 - log(p)) \tag{14}$$

Figure 3 displays the function (14) as a three dimensional image with the *x*- and *y*-axis being scale and probability of occurrence respectively. The parameters of (14) used to make the figure are obtained from fitting (7) to the extreme specific discharge values of two arbitrarily chosen catchments in south eastern Norway.

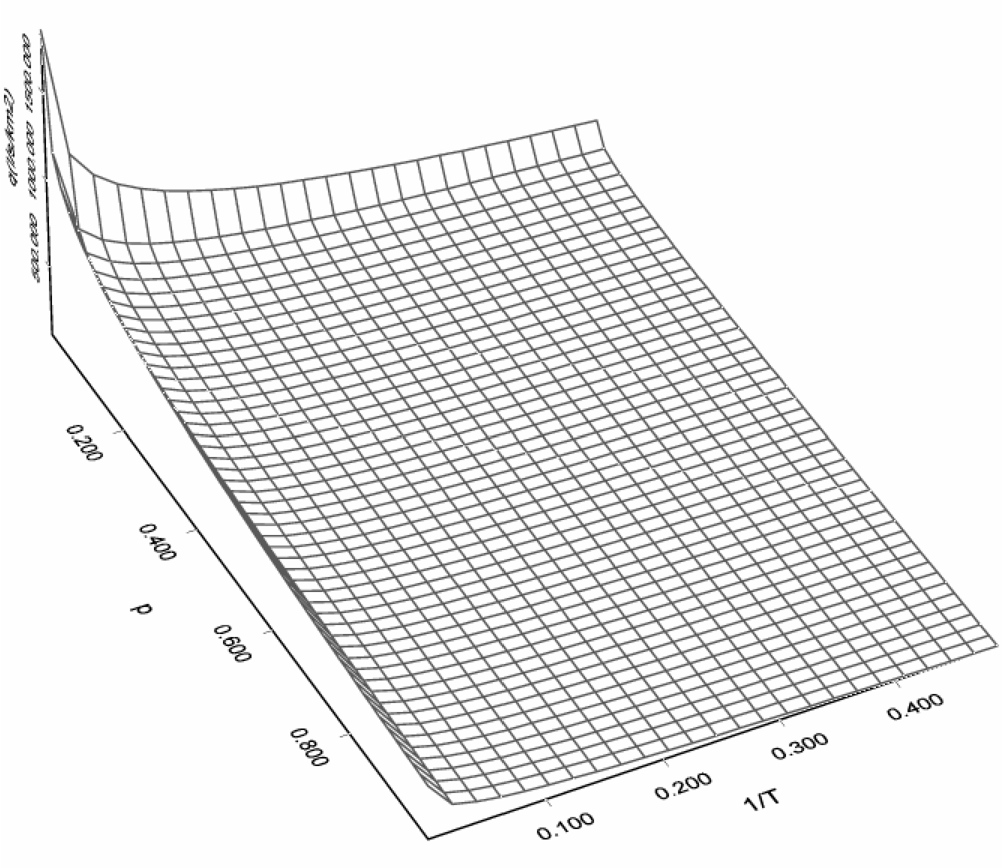

Figure 3. Theoretical function of a 3-dimensional scaling surface, where the X- and Y-axes are scale and return period (T) respectively and the Z-axis displays specific discharge.

To relate an actual catchment size, A_i , to *p*, we can, for each return period *T*, estimate A_0 from equation (10) or (11). The value of Λ_T is previously known from equation (12), and by (10) we have:

$$A_0 = exp[\,q(\frac{A_i}{A_0})\Lambda_T - 1\,]\,A_i \tag{15}$$

Application for regional flood frequency analysis

In the preceding sections, we have described how differences in catchment area, in a homogeneous region defined according to Gupta et al. (1994), will influence the flood quantiles. Obviously, there are also physiographical features of the catchment, like lakes, slopes, bogs vegetation, soils, topography and geology, which influence flood peaks and the shape of the hydrograph. The definition of homogeneity (Gupta et al. (1994)) necessary for the applied scaling procedure will, in our view, mainly address the homogeneity of the regional climate and we assume that the mean annual specific discharge (MAD) can serve as a descriptor of the regional climate. The scaling properties form a basis from which deviations from this pattern are linked to physiographic factors. This link is modelled by multiple regression analysis.

Multiple regression using catchment characteristics

The proposed model is on the form:

$$log(q(p_i ,1/T)_{obs}) = log(q(p_i ,1/T)_{scl}) + e_T , \tag{16}$$

where e_T is the deviation from the scaling procedure, and a function of return period, $q(p_i ,1/T)_{obs}$ is flood quantile of return period *T*, estimated from observed runoff series of catchment A_i ($p_i = A_i / A_0$) and $q(p_i ,1/T)_{scl}$ is the flood quantile of return period *T* according to the scaling procedure. Exploratory analysis shows that the variability of e_T increases as the catchment area decreases. When, however, logaritmic values of the quantiles are used the variability of e_T is approximately the same for the different catchment areas. The deviation e_T is a function of catchment characteristics, and formulated as:

$$e_T = a + b \cdot \log(B) + c \cdot \log(C) + d \cdot \log(D) + ..., \quad (17)$$

where capital letters denote different catchment features and lower case letters are constants which will be determined by linear multiple regression. By using this model together with the scaling procedure, we get an expression of estimated specific flood quantile as:

$$q(p_i, 1/T)_{scl\_corr} = q(p_i, 1/T)_{scl} \, exp[e_T] \quad (18)$$

The first term on the right hand side is the estimate from the scaling procedure whereas the second term is determined by multiple regression using catchment features.

Case study and discussion

The data set used is extreme specific discharge values estimated from observed series from unregulated rivers in Norway. Some of the larger catchments are regulated to a modest degree, but the effect of the regulation is considered negligible. We will take a closer look at the Glomma region, which is located in the southeast of Norway (see Fig.4) with a typical continental climate. The differences in catchment physiographic features within the region suggest a non-homogeneous region. This heterogeneity has to be quantified and incorporated into the scaling procedure.

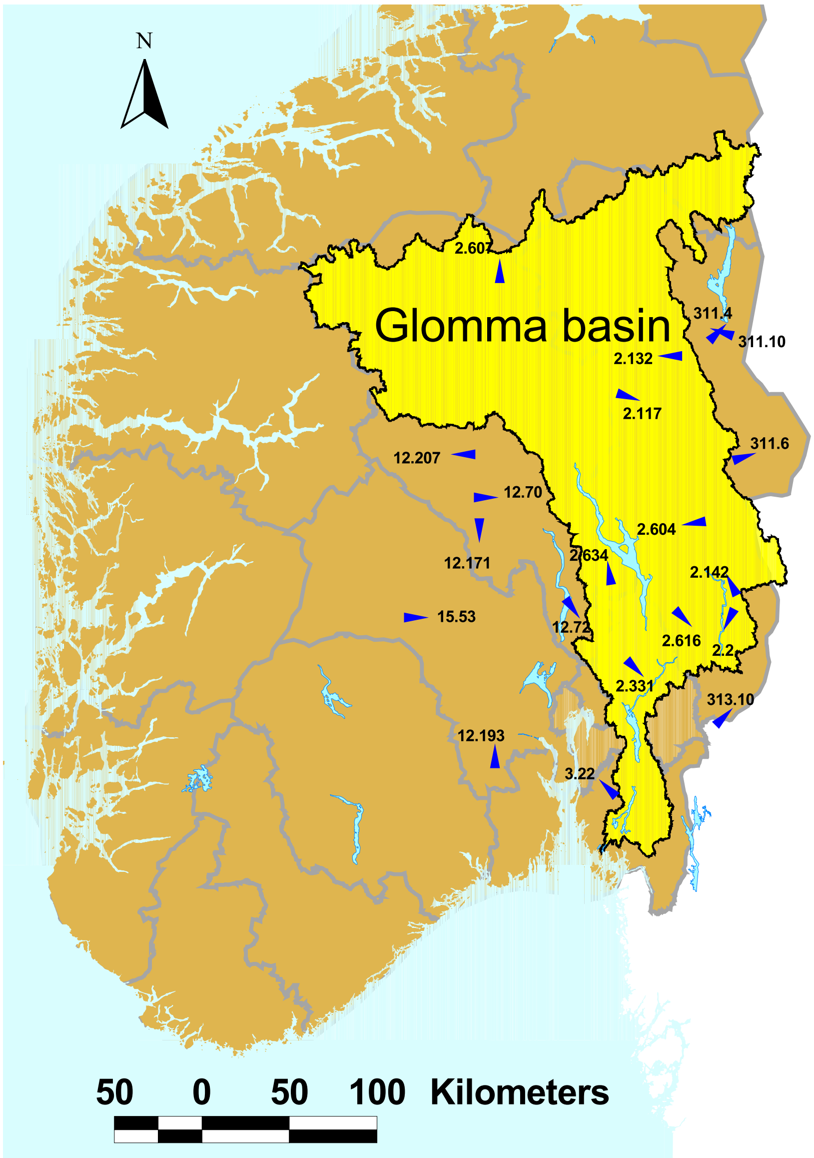

Figure 4. The Glomma region with stations used in the scaling analysis.

Calibrating the scaling procedure

We defined stations with a maximum deviation of the MAD from the global mean of 15% to be from a homogeneous region. Also, when selecting candidate stations to estimate the parameters of the scaling procedure we focused on as small as possible (effective) lake percentage and degree of regulation. Effective lake percentage takes into account the location of the lake in the catchment. Five stations were found to meet these requirements and flood quantiles estimated from observed data for the mean and the return periods of 5, 10, 20 and 50 years were readily estimated through a national project for producing flood inundation maps for certain rivers. The quantiles were estimated by traditional tools of fitting distributions to annual maximum values of observed discharge (Berg et al. 2000 and Pettersson, 2000). We did not try to fit a distribution of the exponential type to the extreme value data, as this is not a prerequisite for the methodology to be used for regional flood frequency purposes.

We estimated Λ_T from pairs of the five stations with equation (12). Then, with a fixed chosen value of A_0, b_T could be estimated by equation (4). In principle, if the five stations were to be considered from a homogeneous region, then pairs from the five stations should, by (12), produce the same value of Λ_T for the different return periods. As Table 1 shows, this was not the case for all pairs.

Table 1. The parameter Λ estimated for the mean and the return periods of 5, 10 20 and 50 year flood for pairs of the the stations used for estimating the scaling procedure. The bold values of Λ indicate stations (pairs) defining the scaling behaviour of the region.

qm					
Station	2.604	2.117	2.2	2.634	2.142
2.604					
2.117	**0.059**				
2.2	0.017	0.036			
2.634	**0.052**	**0.051**	0.047		
2.142	0.100	0.129	0.071	0.034	

q5					
Station	2.604	2.117	2.2	2.634	2.142
2.604					
2.117	**0.036**				
2.2	0.012	0.024			
2.634	**0.035**	**0.035**	0.027		
2.142	0.066	0.091	0.049	0.023	

q10					
Station	2.604	2.117	2.2	2.634	2.142
2.604					
2.117	**0.027**				
2.2	0.011	0.019			
2.634	**0.028**	**0.029**	0.033		
2.142	0.053	0.076	0.040	0.019	

q20					
Station	2.604	2.117	2.2	2.634	2.142
2.604					
2.117	**0.023**				
2.2	0.009	0.016			
2.634	**0.024**	**0.024**	0.023		
2.142	0.045	0.066	0.072	0.016	

q50					
Station	2.604	2.117	2.2	2.634	2.142
2.604					
2.117	**0.020**				
2.2	0.008	0.014			
2.634	**0.020**	**0.021**	0.019		
2.142	0.039	0.058	0.030	0.014	

Pairs obtained from three of the stations, however, produced very similar values of Λ_T for all return periods and we concluded that these three stations defined the pure scaling behaviour of the homogeneous region. We note that Λ_T decreases as the return periods increase, which implicates higher spatial variability for higher return periods. When Λ_T and b_T are determined, we obtain a general scaling equation for $q(p)_T$ for each return period T as:

$$q(\hat{p})_T = b_T + \frac{1}{\hat{\Lambda}_T}(1 - log\, \hat{p}), \tag{19}$$

where $\hat{A}_0$ in $\hat{p} = \frac{A}{\hat{A}_0}$ is arbitrarily chosen. As already noted, a different value chosen for $\hat{A}_0$, would give a different b_T. Figure 5 a shows estimated quantiles from observed data for the five stations and the theoretical scaling curves calibrated by the three stations considered to be from a homogeneous region and with similar catchment characteristics. As was apparent from Table 1, the figure shows that the two stations, not included in calibrating the scaling procedure, behave incompatible with the other three stations, seen from a scaling context. If we investigate the ratio between the quantiles and the mean annual flood (q_5 / q_m, q_{10} / q_m etc.), we find that these are not constant as assumed by the index flood method. Figure 5 b indicates steeper frequency curves for smaller catchments, which is consistent with empirical observations (Gupta et al. 1994).

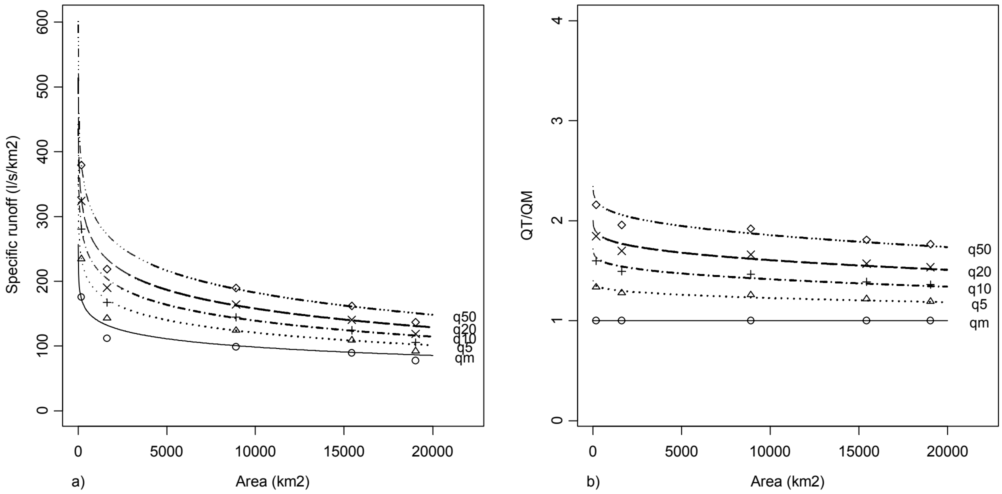

Figure 5. Estimated scaling characteristics (lines) for the Glomma region plotted as area versus specific runoff a), and area versus standardized quantiles with respect to index floods b).

Calibrating multiple regression model of catchment characteristics

Another data set of 12 discharge stations was introduced to calibrate the regression models for deviations due to catchment characteristics. The stations met the minimum requirement of MAD not deviating more than 15% from the global mean, but varied in catchment characteristics. Figure 6 shows significant deviations between the quantiles estimated from observed data of the new stations plotted with the theoretical scaling curves of Figure 5.

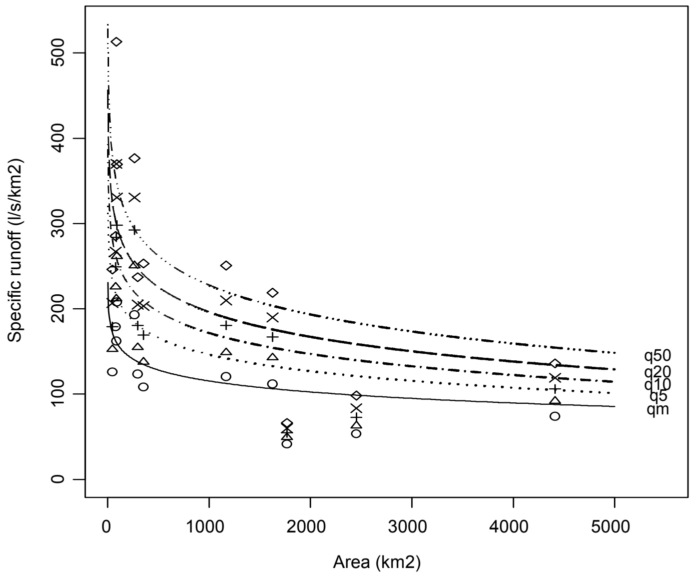

Figure 6. Stations within the range of +/- 15% of MAD and with available catchment characteristics plotted for area versus specific quantiles.

The multiple regression approach described in the previous section was applied, investigating the effects of the following physiographical features: area (km^2), MAD ($l/s/km^2$), effective lake (*%*), bare rock (*%*), catchment length, catchment gradient and river gradient. Multiple regression with the chosen selection of these characteristics was used to estimate e_T of equation (17). To decide the set of catchment characteristics which describes most of the variance, exhaustive stepwise multiple regression method was applied. This approach indicated effective lake percentage (ELP*)* and river gradient (RGD*)* as the set of descriptors. Table 2 shows the stations used with area, ELP and RGD.

Table 2. Catchment characteristics for the station set used for calibrating the regression model.

Estimation set			
Station	**Area**	**ELP**	**RGD**
2.132	1168	0.45	9
2.142	1625	0.07	6
2.331	86.6	0	10
2.616	47	1.04	17
3.22	297	0.65	5
12.171	79	2.39	18
12.207	268	1.25	28
15.53	92.9	0.31	85
311.1	2452	6.7	7
313.1	354	0.45	9
311.6	4410	2.10	2
311.4	1769	11.8	1

The correction terms for flood quantiles of the mean and of the return periods 5, 10, 20 and 50 years for the Glomma region are thus given by:

$$e_m = 0.2083 \cdot log(RGD) - 0.0733 \cdot log(ELP + 0.01) - 0.7961 \qquad (20)$$

$$e_5 = 0.1990 \cdot log(RGD) - 0.0822 \cdot log(ELP + 0.01) - 0.8425 \qquad (21)$$

$$e_{10} = 0.1826 \cdot log(RGD) - 0.1040 \cdot log(ELP + 0.01) - 0.8306 \qquad (22)$$

$$e_{20} = 0.1699 \cdot log(RGD) - 0.1249 \cdot log(ELP + 0.01) - 0.8150 \qquad (23)$$

$$e_{50} = 0.1544 \cdot log(RGD) - 0.1518 \cdot log(ELP + 0.01) - 0.7864 \qquad (24)$$

As we can see from equations 20-24, the river gradient gives a decreasingly positive contribution to flood quantiles for higher return periods, whereas the effective lake percentage has an increasingly negative impact. Table 3 quantifies the differences in quantiles estimated from observed data, estimated by the scaling procedure and estimated by the scaling procedure with corrections for physiographic features. We see that the mean of the corrected quantiles is very close to the observed. The scaling procedure typically overestimates when a catchment has a high lake percentage and a low river gradient. Furthermore, the standard deviation of the ratio between observed and corrected quantiles is reduced.

Table 3. Comparison of the ratio between quantiles estimated from observed data and quantiles estimated by the scaling procedure and estimated by the scaling procedure with corrections for physiographic features for the station set used for calibrating the regression model.

	qm		q5		q10		q20		q50	
Station	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$
2.132	0.87	1.15	0.82	1.15	0.84	1.19	0.85	1.19	0.87	1.21
2.142	0.85	1.07	0.83	1.09	0.81	1.04	0.81	0.98	0.80	0.91
2.331	0.87	0.85	0.83	0.84	0.92	0.86	1.04	0.89	1.23	0.94
2.616	0.64	0.78	0.56	0.74	0.54	0.75	0.54	0.76	0.55	0.79
3.22	0.75	1.15	0.70	1.14	0.68	1.11	0.67	1.09	0.66	1.07
12.171	0.95	1.23	0.88	1.23	0.80	0.19	0.74	1.14	0.67	1.09
12.207	1.16	1.31	1.13	1.37	1.09	1.40	1.07	1.41	1.04	1.42
15.53	1.21	0.91	1.04	0.90	0.97	0.88	0.93	0.86	0.89	0.83
311.1	0.43	0.73	0.39	0.72	0.38	0.75	0.38	0.79	0.39	0.85
313.1	0.67	0.89	0.64	0.90	0.66	0.93	0.68	0.96	0.73	1.01
311.6	0.65	1.31	0.64	1.37	0.63	1.37	0.62	1.36	0.61	1.34
311.4	0.32	0.84	0.29	0.82	0.27	0.81	0.26	0.80	0.25	0.79
Mean	0.77	1.02	0.73	1.02	0.72	1.02	0.72	1.02	0.73	1.02
Std.dev	0.25	0.21	0.24	0.23	0.24	0.23	0.25	0.22	0.27	0.21

Validation of methodology

For validating the procedure for estimating quantiles in ungauged catchments, another five stations with similar MAD to that of the station set used for estimation were selected. Table 4 shows the stations used for validation with area, ELP and RGD.

Table 4. Catchment characteristics for the station set used for validating the model.

Validation set			
Station	**Area**	**ELP**	**RGD**
2.607	126.9	1.02	31
12.7	557	0.13	16
12.193	50	0.28	10
12.72	108	1.07	19
246.4	49	14.8	10

Equations (19) and (20-24) were applied and a comparison between quantiles estimated from observed data and quantiles estimated by the scaling procedure and estimated by the scaling procedure with corrections for physiographic features is shown in Table 5. The performance of the scaling procedure alone is comparable to that of the estimation set. The correction procedure for physiographic features improves on average the estimation, but extremely high ELP for one of the validation catchments is not sufficiently corrected for and the validation set is, on the average, slightly overestimated. Also here, the standard deviation for the ratio between observed and corrected quantiles is reduced.

Table 5. Comparison of the ratio between quantiles estimated from observed data and quantiles estimated by the scaling procedure and estimated by the scaling procedure with corrections for physiographic features for the station set used for validating the model.

	qm		**q5**		**q10**		**q20**		**q50**	
Station	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$	$\frac{q_{obs}}{q_{scl}}$	$\frac{q_{obs}}{q_{corr}}$
2.607	0.80	0.87	0.77	0.91	0.77	0.95	0.79	1.00	0.81	1.06
12.7	1.20	1.29	1.13	1.29	1.03	1.17	0.95	1.05	0.86	0.91
12.193	0.98	1.22	0.90	1.18	0.85	1.13	0.83	1.09	0.81	1.04
12.72	0.55	0.66	0.53	0.69	0.54	0.74	0.57	0.79	0.61	0.86
246.4	0.34	0.57	0.31	0.57	0.28	0.57	0.26	0.56	0.24	0.56
Mean	0.77	0.92	0.72	0.93	0.70	0.91	0.72	0.90	0.67	0.88
Std.dev	0.34	0.33	0.32	0.31	0.29	0.26	0.25	0.22	0.26	0.20

Comparison to previous regional flood frequency analysis studies in Norway

In order to investigate how the proposed method for regional flood frequency analysis compares to previous studies, the scaling procedure, not including the corrections due to catchments characteristics, was applied to several stations around in Norway. The same stations are used in an earlier report, Sælthun et al. (1997). It is therefore possible to directly compare the results of the scaling procedure with a more traditional method of quantile estimates in ungauged catchments. In the report of Sælthun et al. (1997), they estimated index floods for two seasons, the spring season and the autumn season. The index flood for ungauged catchments was estimated based on regression relations with physiographical catchments features. It is noticeable that catchment size was not found to be a significant descriptor. The reason why they chose to use seasonal values was because floods originated from rainfall events versus snowmelt are considered two different processes giving two different populations. In our approach we have also estimated index floods for annual maximum series (AMS).

A comparison between observed and estimated index floods for different seasons are given in Table 6. The results indicate that spring index floods are best estimated, autumn index floods are worst estimated, while annual index floods are somewhere in between. If we compare the results with the overall performance of the method used by Sælthun et al. (1997) and Wingård et al (1978), (see Table 7), we see that the proposed scaling procedure outperforms the previous studies. The improved results may be explained by that the scaling procedure is calibrated on local data, whereas in the report of Sælthun et al. (1997), the equations are based on far more general regions. The scaling method only uses catchment area as descriptor, but it is to be expected that other catchment features influence the floods. Previous sections show that the estimation results improve by including these catchment characteristics in the estimation routine.

For one of the stations in the initial analysis, we got a large difference between observed and estimated index flood. A closer inspection of the rating curve, in collaboration with a field hydrologist, showed indications that large errors could be expected for high water levels. The station was removed from further analysis. A method for detecting errors and inconsistencies in discharge data is thus suggested.

Table 6. Difference between observed and estimated specific index floods (l/s/km^2) for three different seasons. Estimates are based on the proposed scaling method.

Station		Observed (spring)	Sim. (spring)	Dev. (%)	Observed (autumn)	Sim. (autumn)	Dev. (%)	Observed (annual)	Sim. (annual)	Dev. (%)
2.323	Fura	247.9	236.8	-4.48	217.6	158.3	-27.25	280.4	250.3	-10.73
2.331	Kauserud	139.2	121.1	-13.00	101.8	69.5	-31.73	162.0	124.3	-23.27
8.6	Sætern bekken	156.2	180.8	15.75	227.2	312.1	37.37	252.7	331.2	31.06
15.53	Borgåi	202.0	208.3	3.12	87.3	109.2	25.09	207.8	212.9	2.45
26.20	Årdal	439.3	433	-1.43	549.8	514.7	-6.38	596.1	562.9	-5.57
35.5	Moavatn	442.3	477.7	8.00	383.8	558.5	45.52	484.6	609.3	25.73
36.9	Middal	527.9	467.3	-11.48	383.3	458.2	19.54	543.0	543.4	0.07
62.10	Myrkdals vatn	446.6	494.7	10.77	471.4	547.3	16.10	529.4	615.8	16.32
109.12	Bruøy	185.5	238.6	28.63	130.8	150.2	14.83	198.1	243.4	22.87
111.8	Nerdal	354.7	425.8	20.05	270.5	416.8	54.09	375.8	482.5	28.39
156.15	Forsbakk	773.3	765.9	-0.96	988.3	772.5	-21.84	1028.0	928.6	-9.67
200.3	Skogsfjord vatn	266.9	342.4	28.29	230.5	262.6	13.93	304.5	369.2	21.25
212.10	Masi	115.0	110.4	-4.00	21.3	28.5	33.80	115.0	110.3	-4.09
246.4	Lille Ropelv vatn	66.1	48.4	-26.78	25.7	30.8	19.84	66.9	48.0	-28.25
		Mean error		3.75	Mean error		13.78	Mean error		4.76
		Standard dev.		15.98	Standard dev,		26.54	Standard dev.		19.53

Table 7. Performance of three different estimation routines of index floods. Values are deviation (in percent) between observed and estimated index floods. Sim-92 and Sim-78 are based on a multiple regression analysis methods (Sælthun et al. 1997, Wingård et al. 1978), whereas Scaled-03 have used the proposed scaling method.

	Sim-92 (spring)	Sim-78 (spring)	Scaled-03 (spring)	Sim-92 (autumn)	Sim-78 (autumn)	Scaled-03 (autumn)
Mean error (%)	13	11	4	-6	23	14
Standard deviation	27	32	16	46	47	27

Conclusions

The model put forward in this report captures the scaling behaviour of flood quantiles. Flood quantiles for an ungauged candidate catchment in an homogeneous region can thus be estimated by using the general scaling equation (19) with a correction term e_T, determined from physiographic catchment features that can be obtained from maps.

The proposed scaling model illustrates that the ratio of floods of a given return period to the mean annual flood is dependent of drainage area, which is contrary to the principal assumption of the index flood but consistent with observations.

A natural next step would be to develop regional estimates of the spatial variability (i.e. Λ) for different quantiles nation-wide from existing observations, following the procedure outlined in this study. Together with physiographical features of catchments derived from maps, flood quantiles can, in principle, be estimated catchment everywhere and of any size. The methodology can aid consultant engineers in design work and be a tool to check existing design discharge values for spatial scaling consistence.

References

Bell, F.C. (1976) *The areal reduction factor in rainfall frequency estimation*. Rep. 35, Institute of Hydrology, Wallingford, UK.

Berg. H, Høydal, Ø., Voksø, A. and Øydvin, E. (2000) *Flood inundation maps- Mapping flood prone areas in Norway*. Symposium abstracts, for the conference Extremes of the Extremes, Iceland, OS-2000/036.

Blöschl, G and Sivapalan, M. (1995) Scale issues in hydrological modelling: a review. *Hydrological. Processes*, **9**, 251-290.

Beven K. (1991) Scale Consideration. In D.S. Bowles and P.E. O'Connel (eds.) Recent *Advances in the modeling of Hydrologic Systems*, Kluwer Academic Pub. 357-371.

Feller, W. (1971) *An introduction toprobability theory and its applications*. John Wiley & Sons, Ltd., New York.

Gupta, V.K., Mesa, O. J. and Dawdy, D. R. (1994) Multiscaling theory of flood peaks: Regional quantile analysis. *Water Resour. Res*. 30, 12, 3405-3421.

Gupta, V.K., Castro, S. L. and Over, T. M. (1996) On scaling exponents of spatial peak flows from rainfall and river network geometry. *J. Hydrol*. 187, 81-104.

IAHS , International Association of Hydrological Sciences, (2002) Newsletter no 75.

NERC, (1975) *Flood studies Report*, Natural Environment Council, London.

Pettersson, L.E. (2000) *Flomberegning for Glommavassdraget oppstrøms Vorma*, Norwegian Water Resources and Energy Directorate, NVE-dokument no. 10 (in Norwegian).

Robinson J.S. and Sivapalan, M. (1997) An investigation into the physical causes of scaling and heterogeneityof regional flood frequency. *Water Resour. Res*., 33, 5, 1045-1059.

Skaugen, T. (1997) Classification of rainfall into small- and large-scale events by statistical pattern recognition. *J. Hydrol.,* 200, 40-57.

Sælthun, N.R., Tveito, O.E., Bønsnes, T.E. and Roald, L.A. (1997). Regional flomfrekvensanalyse for norske vassdrag. Rapport nr. 14, NVE, Oslo.

Wingård, B., Hegge, K., Mohn, E., Nordseth, K. and Ruud, E. (1978). Regional flomfrekvensanalyse for norske vassdrag. Rapport nr. 2, NVE, Oslo.

Veitzer, S.A. and Gupta, V.K. (2001) Statistical self-similarity of width function maxima with implications to floods. *Adv. in Water Res*. 24, 955-965.

Wood, E.F., Sivapalan, M. Beven, K. and Band, L. (1988) Effects of spatial variability and scale with implications to hydrologic modelling. *J.Hydrol.* 102, 29-47

Woods, R., Sivapalan, M. and Duncan, M. (1995) Investigating the representative elementary area concept: an approach based on field data. In J.D. Kalma and M Sivapalan (eds) *Scale issues in hydrological modelling*. Wiley & Sons. 49-70.