Updating snow reservoir in hydrological models from satellite-observed snow covered areas

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Abstract: Operational use of satellite-observed snow covered area (SCA) in the HBV-model has been carried out using two different snow distribution functions. SCA could be included in the model calibration with only small reduction in runoff performance. Updating the runoff models based on the satellite derived SCA showed ambiguous results. The success rate of the updates was only 28% for the traditional snow model. Studies using a dynamic snow distribution function for the snow reservoir showed comparable results to the traditional lognormal snow distribution for prediction of discharge and better for the prediction of SCA. Updating the new model from satellite derived SCA consistently gave improvements on the prediction of discharge.

Keyword: Snow; Runoff simulations; Satellite derived snow covered area
Preface

The research project EnviSnow (Development of Generic Earth Observation Based Snow Parameter Retrieval Algorithms) is supported by the EU commission under the Energy, Environment and Sustainable Development programme (EU Contract number EVG1-CT-2001-00052) for the period 2002-2005. The goal of EnviSnow is to develop and validate new and improved multisensor algorithms for retrieval of snow and soil parameters from Earth Observation (EO) data for use in global climate studies and hydrology, in particular runoff and flood prediction.

EnviSnow involves 11 partners from four European countries. The project is divided into 9 workpackages. Norwegian Water Resources and Energy Directorate (NVE) leads workpackage 5 (WP5): Assimilation and integration of snow parameters in hydrologic models. This report is a deliverable in this workpackage (D2-WP5) and sums up the scientific work carried out in task 1: Snow parameters in a lumped hydrologic model.

Oslo, January 2005

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Project leader
Summary

A study testing operational use of satellite-observed snow covered area (SCA) in the HBV-model was carried out in order to improve the spring flood prediction. Two different snow distribution functions was studied; the traditional lognormal distribution and a dynamic gamma distribution.

The study included a) calibration of HBV-models against both discharge and SCA, and against discharge only, and b) updating of the HBV-models based on satellite observed SCA. Ten test catchments were selected for the study. The results show that the HBV-models calibrated against SCA in addition to discharge simulate discharge nearly as well as models calibrated against discharge only. The simulated SCA was markedly improved when SCA was included in the calibration.

A success rate of 28 percent was found for the updates of models with the traditional snow distribution function. The success and failure of the updates seems to be quite randomly. However, a weak tendency of higher success rates for large SCA values was seen.

A new snow distribution model has been implemented in the HBV model. The new snow distribution model applies a gamma distribution in which the parameters are dynamic, such that they are functions of the number of accumulation and melting events. In this way the modelled spatial distribution of SWE more closely follows the observed spatial distributions of SWE. The new snow distribution model also allows for an automatic updating of the snow reservoir from satellite derived SCA. HBV model with the new snow distribution model predicts discharge (Q) as good as the traditional snow distribution model, predicts SCA better and gives a consistently improved prediction of the discharge when updating the model from satellite derived SCA.
1 Introduction

The amount and timing of snowmelt runoff from snow and glaciers are important information for flood prediction and hydropower operations in Norway. Two examples are the large flood in south-eastern Norway in 1995 and the electricity shortage in Norway during winter and spring 2003. In these situations updated information on snow conditions were of crucial importance for the Norwegian Water Resources and Energy Directorate (NVE). At present, the HBV model is used by the national flood forecasting service at the NVE to simulate runoff in the river systems. Satellite imagery from NOAA AVHRR are used to observe the snow covered area (SCA). However, these observations have not been used for operational model updating so far.

Previous works in the projects Snowtools (Gunerius sen et al. 2000) and Hydalp (Rott et al. 2000) showed that updating of the HBV model with remotely sensed SCA data tended to reduce the model performance. The main reason for this could be that SCA data was not used in the model calibrations. In a pilot study (Engeset and Undæs 2002, Engeset et al. 2003), three catchments were used to test the use of satellite derived SCA in hydrological models. The study showed that when the HBV models were calibrated against satellite-derived SCA, in addition to runoff, the models simulated SCA more consistently with these data, without major reduction in the precision of the simulated runoff. Updating of the model input, in cases where obvious errors in the simulated SCA were detected, gave promising results.

In this study ten test catchments with operational HBV models were selected. The catchments represent different scales and regions in Norway. Time series of satellite derived SCA were used both in the model calibrations, and to detect and update the models when the simulated SCA deviated significantly from the observed SCA. Four years were used to calibrate the models and six years were used for validation.

The objective of this study was to examine if the national flood forecasting could be improved by using satellite-derived snow covered area in the operational hydrological models.
2 Test sites

Ten catchments were selected for this study. The catchments represent different altitude ranges, area sizes and geographical location (Fig. 1 and Tab. 1). They were chosen in order to run models operationally and to cover essential rivers in Norway. For all catchments the snow melt flood in spring and summer is usually the dominating flood each year. To be able to observe SCA from satellites, only catchments with non forested or sparse forested areas were chosen.

Figure 1. Location map of the ten catchments in Norway used in the study.
Table 1 Description of the ten test catchments used in the HBV simulations.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>No. in map (Fig. 1)</th>
<th>Annual runoff (mm)</th>
<th>Area (km²)</th>
<th>Altitude median-max-min (m a.s.l.)</th>
<th>Alpine (%)</th>
<th>Forest (%)</th>
</tr>
</thead>
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<tr>
<td>Akslen</td>
<td>3</td>
<td>966</td>
<td>791</td>
<td>1476 2472 480 84 16</td>
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<td>16</td>
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<td>22</td>
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<tr>
<td>Aursunden*</td>
<td>5</td>
<td>764</td>
<td>835</td>
<td>840 1553 690 59 41</td>
<td>59</td>
<td>41</td>
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<td>Malangsfoss</td>
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<td>847</td>
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<td>719 1677 20 70 30</td>
<td>70</td>
<td>30</td>
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<td>96</td>
<td>4</td>
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<tr>
<td>Heimdalsvatn*</td>
<td>10</td>
<td>840</td>
<td>1192</td>
<td>1231 1531 951 98 2</td>
<td>98</td>
<td>2</td>
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<tr>
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<td>1231 1531 951 98 2</td>
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<td>355 1067 20 51 49</td>
<td>51</td>
<td>49</td>
</tr>
<tr>
<td>Sjodalsvatn</td>
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<td>474</td>
<td>1465 2400 940 100 0</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Vinde-elv</td>
<td>1</td>
<td>487</td>
<td>268</td>
<td>985 1686 560 59 41</td>
<td>59</td>
<td>41</td>
</tr>
</tbody>
</table>

* Catchment where runoff is represented by calculated reservoir inflow.
3 General methods

3.1 Satellite observed SCA

Snow covered area was calculated from NOAA AVHRR satellite images. These images were processed either by NVE according to the method described by Schjødt-Osmo and Engeset (1997) or by the Norwegian Computing Center using the Norwegian-Linear-Reflectance (NLR) method (Solberg and Andersen, 1994). Both methods convert reflectance values from band 2 into SCA. It is assumed that the bare-ground reflectance, and the reflectance of snow covered areas, is constant in space at every AVHRR-scene. Reflectance values for 100 % and 0 % snow cover are found from glaciers and snow-free areas. The snow cover percentage for each 1x1 km² pixel is then calculated as a linear function of the reflectance in the pixel compared to the 100 % and the 0 % reflectance.

3.2 Hydrological modelling

The Nordic HBV model (Sælthun, 1996) used in this study is a modified version of the HBV model (Bergström, 1992). The model structure is a sequence of four submodels for snow, soil moisture, dynamics and routing. The model is divided into ten elevation intervals. The model inputs are observed precipitation and temperature. The main output from the simulations is runoff, but SCA for each elevation interval is also simulated. After snow accumulation the model always simulates 100 % SCA, and simulated SCA is not reduced until the first occurrence of snow melt.

The model was automatically calibrated for the ten test catchments using the parameter estimating routine PEST (Doherty et al., 1994). Data from the four-year period from 1st September 1995 to 31st August 1999 was used for calibration. The model was calibrated in two modes for each catchment: (1) against runoff only (called the Q-models), and (2) against both runoff and SCA (called the QS-models). For two of the catchments the runoff was represented by the calculated reservoir inflow. These runoff data may have large errors in the day to day variations, but the accumulated runoff is supposed to be correct.

As the HBV-model is highly over-parameterised, standard values were assigned to some of the calibration parameters. Internal model parameters, like maximum content of liquid water and the refreezing coefficient, were not calibrated. The snow parameters allowed to be calibrated were the correction factors of the input values (temperature and precipitation), and the degree-day melting factor. This was based on experience from studies of similar models and snow pillow data in Norway (Engeset et al., 2000), where these parameters were found to be of large importance for the dynamics of the snow reservoir. As satellite-based SCA rarely reaches more than 75 % on a catchment scale (Engeset et al., 2003), the satellite-based SCA was transformed linearly to cover the interval 0-100 % before used in the model calibration.

The weighting factor of the observations is of great importance in the automatic calibration process. In this study the simulated results were compared to and evaluated against observed discharge, deviation from accumulated discharge and, when calibrated against SCA, satellite observed SCA. In order to avoid that one of the observation types dominated the calibration, the weighting factors were chosen such that each of the observation types contributed approximately equally to the model performance.
coefficient (Φ). Thus the number of observations and the typical magnitude of each observation type were taken into account in the weighting factor.

Six independent years (1st September 1994 – 31st August 1995 and 1st September 1999 – 31st August 2004) were used to evaluate the models with respect to runoff and SCA, and to investigate if updating the model input would improve the simulations. Engeset et al. (2003) showed that such updates could be successfully when there was a major difference between observed and simulated SCA.

In this study two approaches in modelling the snow reservoir are presented. The traditional HBV-model using a lognormal snow distribution function and a new snow routine using a dynamic snow distribution function. In the first approach, the lognormal snow distribution, the model input was manually updated with a) a percentage change of the winter precipitation and/or b) temperature modifications immediately ahead of and during the melt season. An updating was triggered by either a) a deviation between observed and simulated SCA greater than 20 percent units at a single satellite scene or b) three succeeding deviations of at least 10 percent units within 10 days. The method and results are presented in section 4. In the second approach, the dynamic snow distribution, the snow reservoir was updated automatically. If the discrepancy between observed and modelled SCA was more than a certain level between 10 and 25 %, the snow reservoir was updated. This was achieved by modelling the snow reservoir as sums of gamma distributed variables. The method and results are presented in section 5. A comparison of the two approaches is presented in section 6 together with a general discussion on prerequisites of remotely sensed data, uncertainty and usefulness. Conclusions are found in section 7.
4 Approach 1: HBV model with the traditional snow distribution

4.1 Methods
The snow distribution in the traditional HBV-models is given as a lognormal distribution function (Bergström, 1992; Sælthun, 1996). Snow accumulation starts when precipitation falls at temperature lower than a given threshold. Up to a specified accumulation level the accumulation is even. Thereafter additional snow falls are distributed according to a lognormal distribution with a model specific coefficient of variation. Snowmelt is modelled using a degree-day method and is uniformly distributed over the area covered by snow. Thus the snow distribution function becomes steeper as snowmelt proceeds.

Satellite observed SCA was used to update the HBV-models when the deviation between the simulated and observed SCA was large. The comparison was done on an aggregated SCA value for the catchment. An updating scenario was triggered by either a) a deviation between observed and simulated SCA greater than 20% at a single occasion or b) three succeeding deviations of at least 10% within 10 days. The model updates was done manually for the hydrological year by a) a percentage change of the winter precipitation and/or b) temperature modifications immediately ahead of and during the melt season.

4.2 Model calibration and simulation
Four years, 1st September 1995 to 31st August 1999, were used for calibration. A total of 96 Q-models and 96 QS-models were automatically calibrated for each catchment using the PEST routine. The five best models of these were chosen for validation. Generally, several of the Q-models and QS-models for each of the catchments were able to simulate runoff well. However, most of the QS-models obtained a small decrease in the coefficient of determination of the discharge ($R^2_Q$) of 0.01 – 0.03 units compared to the Q-models (Tab. 2 and Fig. 2). Looking at the five best models for each of the catchments, the $R^2_Q$ ranged from 0.76 to 0.94 (median = 0.85) for the Q-models, and from 0.73 to 0.94 (median = 0.83) for the QS-models (Tab. 2). The absolute values of $R^2$ for Vinde-elv, Sjodalsvatn and Akslen deviates from those found earlier for those catchments (Engeset et.al., 2003). This is caused by modified weighting factors and different calibration and validation periods used in the two projects and does not influence on the general results of the two studies. The timing of the flood peaks was satisfactory for all catchments in most of the years. However, for some of the catchments the amplitude of the flood peaks matched poorly. As expected, using SCA in the calibration increased the model performance with respect to SCA (Tab. 2 and Fig. 2). While the $R^2_{SCA}$ ranged from 0.45 to 0.98 (median = 0.86) for the ten catchments regarding the five best Q-models for each catchment, the $R^2_{SCA}$ of the QS-models ranged from 0.90 to 0.99 (median = 0.94). The improvement in simulated SCA was remarkable, especially for catchments where the Q-model simulations resulted in low $R^2_{SCA}$. This is in agreement with the results found in the preliminary study using only three of the catchments (Engeset et al., 2003), although it is not as convincing when looking at all ten catchments. Examples from two of the catchments (Aursunden and Orsjoern) are shown in Figure 3 and 4. Aursunden has a generally high $R^2_Q$ whereas Orsjoern has a generally low $R^2_Q$. The improvement in
simulated SCA, when SCA was included in the calibration, was clear for both catchments.

Table 2  Model performance of the five best models from the automatic calibration. The table shows the $R^2$ for the discharge (Q) and the snow covered area (SCA) in the calibration (calib.) and validation (valid.) period and when updating model input (update.) in the validation period. The overall results of the model update are indicated with the +/- signs.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Q models $R^2_Q$</th>
<th>Q + SCA models $R^2_Q$</th>
<th>Q models $R^2_{SCA}$</th>
<th>Q + SCA models $R^2_{SCA}$</th>
<th>Update results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akslen</td>
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<td>0.82 0.83 0.84</td>
<td>0.90 0.52</td>
<td>0.93 0.57 0.87</td>
<td>+</td>
</tr>
<tr>
<td>Atnasjø</td>
<td>0.81 0.80</td>
<td>0.79 0.78 0.74</td>
<td>0.90 0.69</td>
<td>0.91 0.64 0.77</td>
<td>-</td>
</tr>
<tr>
<td>Aursunden</td>
<td>0.92 0.86</td>
<td>0.90 0.85 0.80</td>
<td>0.91 0.64</td>
<td>0.97 0.75 0.86</td>
<td>-/2</td>
</tr>
<tr>
<td>Malangsfoss</td>
<td>0.88 0.81</td>
<td>0.86 0.79 0.73</td>
<td>0.92 0.32</td>
<td>0.94 0.40 0.67</td>
<td>-</td>
</tr>
<tr>
<td>Narsjø</td>
<td>0.88 0.85</td>
<td>0.87 0.85 0.86</td>
<td>0.82 0.50</td>
<td>0.94 0.65 0.68</td>
<td>+</td>
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<tr>
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<td>0.90 0.86 0.88</td>
<td>+/-1</td>
</tr>
<tr>
<td>Orsjoren</td>
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<td>0.75 0.72 0.77</td>
<td>0.75 0.73</td>
<td>0.95 0.87 0.91</td>
<td>+/1</td>
</tr>
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<td>Polmak</td>
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<td>0.98 0.73</td>
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<td>-</td>
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<tr>
<td>Sjodalsvatn</td>
<td>0.79 0.80</td>
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<td>0.95 0.79 0.89</td>
<td>+</td>
</tr>
<tr>
<td>Vinde-elv</td>
<td>0.87 0.85</td>
<td>0.83 0.79 0.83</td>
<td>0.72 0.93</td>
<td>0.92 0.90 0.95</td>
<td>+</td>
</tr>
</tbody>
</table>

1) Improved in two of three years. 2) Worsened in two of three years.

Figure 2. Model performance using the Q- and the QS-models respectively in the calibration and validation period. The QS-models perform slightly poorer than the Q-models with respect to runoff, whereas the performance with respect to SCA are improved in most cases.
Figure 3. Example of calibration results of a) the Q-models and b) the QS-models for Aursunden, which has a generally high $R^2_Q$. $R^2_Q$ decreased from 0.92 (Q-models) to 0.90 (QS-models) and $R^2_{SCA}$ increased from 0.91 (Q-models) to 0.97 (QS-models) when including SCA in the calibration.

As already mentioned, six independent years (the winters 94/95 and 99/00 – 03/04) were used to validate the models. Model runs of the five best models (both Q and QS) with respect to $R^2_Q$ from each catchment were validated against observed runoff and SCA. In general, the QS-models were of the same quality as the Q-models with respect to runoff. Two catchments had higher $R^2_Q$ and eight had lower $R^2_Q$ than in the calibration period. Similar changes in model performance were found both for the Q- and the QS-models. Therefore, the changes in model performance could most likely be attributed to the model input (precipitation and temperature) and the representability of the meteorological observations. The high performance in terms of SCA obtained in the Q+SCA calibration was not maintained in the validation period. The models for all ten catchments experienced a decrease in $R^2_{SCA}$. This indicates a potential for improvement of the model results through the updating procedure.
Figure 4. Example of calibration results of the a) Q-models and b) the QS-models for Orsjoren, which has a generally low $R^2_Q$. $R^2_Q$ decreased from 0.77 (Q-models) to 0.75 (QS-models) and $R^2_{SCA}$ increased from 0.75 (Q-models) to 0.95 (QS-models) when including SCA in the calibration.
4.3 Model updating

Positive trigger response, defined as deviation between simulated and observed SCA, was found in 40 cases (each representing one model year). Of these, 19 cases were subjective rejected from updating because the observed SCA values were assumed unlikely when compared to the stage of melting in the catchment. Updating scenarios were calculated for the remaining 21 cases. The updating scenarios were validated in terms of $R^2$-values of Q and SCA, timing and amplitude of the flood peaks and mass recovery (calculated as simulated discharge divided by observed accumulated discharge of the hydrological year). In the following we describe in detail the updating scenarios for the 10 test catchments.

4.3.1 Akslen

The QS- and the Q-models simulated the SCA with $R^2_{\text{SCA}} = 0.90$ and 0.93, for the Q- and the QS-models respectively, in the calibration period, and 0.52 and 0.57, respectively, in the validation period.

**Akslen case 1 – increasing the winter precipitation 2001**

In 2001, the main spring flood was highly underestimated and the main decrease in SCA started several weeks too early in the model (Fig. 5a). The total volume of the spring flow was also underestimated with 30%. On 19th June the simulated SCA was 47 percent units lower than the observed SCA. The difference between simulated and observed SCA was decreased to a satisfactory level (within 10 percent units deviation) by increasing the winter precipitation with 200% (results not shown). However, this led to an overestimation of the flow ahead of and during the main flood peak. Decreasing the temperature with 1°C from 1st May to 15th July and increasing the winter precipitation with 150% simultaneously led to a better estimate of both SCA and the early spring flow (Fig. 5). This improved the $R^2_{\text{SCA}}$ from 0.57 to 0.76 and the $R^2_{\text{Q}}$ from 0.83 to 0.84. The main flood was slightly overestimated (although less than in the case when only precipitation was updated), but the mass recovery increased from 0.75 in the original model to 1.08 in the updated model.

**Akslen case 2 – 2002**

In early May 2002, the observed SCA was close to 100% whereas the simulated SCA was between 85 and 90% on three occasions during a five days period. However, since the snowmelt had started three weeks earlier, a SCA of 100% was hardly correct and the model was not updated. The decision was supported by the following SCA observations which showed a rather fast decline of the snow magazine.

**Akslen case 3 – underestimated SCA in 2003**

In end of May 2003, the simulated SCA was 20 – 26 percent units below the observed values. The simulated runoff was too high during a 14 days period at the beginning of the spring flood. Reducing the temperature with 1°C from 17th May led to a good fit of the runoff during flood initiation and improved the simulated SCA slightly. By increasing the winter precipitation with 40% in addition, the simulated SCA got within an acceptable deviation from the observed one (Fig. 5b). The main flood peak was then prolonged slightly too much compared to the observed runoff, but the simulation of the subsequent floods match well to the observed ones.
4.3.2 Atnasjø

The dynamics of the catchment was simulated well in the HBV-models, with respect to both discharge and SCA. The QS-model simulated a slightly larger snow reservoir and estimated SCA a little better than the Q-model in the calibration period. In the validation period the Q-model was better than the QS-model. The performance with respect to discharge was very similar for the two models.

**Atnasjø case 1 – winter precipitation decreased in 2000**

On 29th April 2000, the simulated SCA was 40 percent units higher than the observed one (Fig. 6a). This was followed by succeeding overestimations of SCA. Reducing the winter precipitation with 70 % led to a much better fit of SCA, $R_{SCA}^2$ increased from 0.66 to 0.77, but the flood was dramatically underestimated (Fig. 6a).

**Atnasjø case 2 – overestimated SCA in 2001**

In the middle of the melt season 2001, a triggering observation where simulated SCA was 30 percent units above the observed one was recorded. The SCA observations indicated a very rapid reduction in SCA (Fig. 6b) which could only be caused by very high temperatures or very little snow in the catchment. Since the simulation so far reproduced the observed flood event almost perfect, no update of the model input was performed.

**Atnasjø case 3 – over- and underestimation of SCA in 2003**

On 23rd April 2003, the satellite observation showed a SCA of 60 %, 22 percent units lower than the simulated one (Fig. 6c). Based on that the snow melt hardly had started this observation was considered unlikely and thereby rejected. On 11th May the observed SCA was 100 %, 25 percent units higher than the simulated one (Fig. 6c). This could be cased by a small snowfall, covering the catchment with a thin layer of snow. As both the observation and the model show increasing runoff, it is likely to believe that the snow melted away during the day. No action to update the model was therefore performed. A large deviation between observed and simulated SCA was detected on 16th June.
However, because the main melt flood was already reaching its end, the observation was treated as misclassification.

Atnasjø case 4 – overestimated SCA 2004

A 28% overestimation of SCA was detected on 23rd May 2004. This was during a very rapid rise and decrease of the SCA, cased by precipitation during a temporary temperature depression (Fig. 6d). Because the main spring flood had ceased, no update of the model was made.

4.3.3 Aursunden

The catchment Aursunden is a reservoir, with regulated outlet. The discharge used in the model is therefore calculated from reservoir inflow. An increase in $R^2_{SCA}$, from 0.91 to 0.97, was obtained by calibrating against SCA in addition to the discharge. The improvement was largest in the validation period, which also maintained the $R^2_Q$ of the Q-models. Large discrepancies between simulated and observed SCA were found for several of the validation years. However, the simulated discharge was close to the observed also when the simulation of SCA failed.

Aursunden case 1 – updating the winter precipitation 2000
In 2000, the SCA was overestimated although the simulated discharge fitted well to the observations. Reducing the winter precipitation in order to obtain a better fit to the observed SCA (29th May and later) led to an underestimation of the flow peaks (Fig. 7a).

**Aursunden case 2 – no update of model input winter 2001**

On 15th May 2001, the observed SCA was 25 percent units lower than the simulated (Fig. 7b). The observed value was interpreted unlikely because the flood peak had already cumulated and the simulated discharge and SCA fitted well before that date.

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![Graphs showing simulated SCA and discharge compared to the observed values with and without updating the input data of the catchment Aursunden.](image)
Aursunden case 3 – updating the winter precipitation 2002

In 2002, the simulated SCA was 36 percent higher than the satellite observed SCA on 7th May. A reduction to 60 % of the observed winter precipitation resulted in a good fit to observed SCA just before the main flow peak (Fig. 7c). The mass recovery was reduced from 1.13 to 0.78 of that observed. This clearly points on the risk of reducing the precipitation, as it led to a highly underestimated spring flood. Reducing the winter precipitation somewhat less, to 80 % of that observed, gave a better fit to the discharge curve (Fig. 7d) and a mass recovery of 1.02. The $R^2_{SCA}$ was improved from 0.71 to 0.78, whereas the $R^2_Q$ remained almost unchanged. However, the deviation between simulated and observed SCA early in the melt season was still larger than 20 %.

Aursunden case 4 – updating the spring temperature 2003

A SCA observation 11th May 2003 indicated a 100 % SCA-cover whereas the modelled simulated 68 % SCA. Three weeks earlier the modelled had simulated a non-existent flood event. Decreasing the temperature with 3°C from 15th to 24th April corrected this error and simultaneously improved the simulation of both the runoff and SCA during the main flood event (Fig. 7e). The next SCA observation, on the 28th May, still indicated an underestimation of SCA in the model. However, since the main flood event was ceasing no further update of the model input was done.

Aursunden case 5 – updating the spring temperature 2004

On 1st May 2004, a SCA observation indicated that there where too much snow in the model. An inspecting the runoff curves revealed a slightly too late initiation of the snow melt in the model. Increasing the spring temperature with 2°C from 1st to 15th April improved the simulated runoff to an almost perfect fit and reduced the deviation between simulated and observed SCA to an acceptable level (Fig. 7f). However, the subsequent floodpeak was more underestimated than without updating the model input.

4.3.4 Malangsfoss

The HBV-models of the catchment Malangsfoss had high $R^2$ values for discharge and SCA both in the Q- and the QS-models. The $R^2_Q$ was 0.88 and 0.86, respectively, and the $R^2_{SCA}$ 0.92 and 0.94, respectively, in the calibration period. In the validation period the $R^2_{SCA}$ was markedly lower, indicating that updating the model input could improve the simulations.

Malangsfoss case 1 – 1995

An observation in late spring 1995 indicated a SCA 21 percent units below the simulated one. No update was performed because the main flood event was ceasing.

Malangsfoss case 2 – increasing the winter precipitation 2001

The first observation of SCA in 2001 fitted well with the simulated one (Fig. 8a). Three weeks later, when the snowmelt had proceeded, the observed SCA was 32 percent units higher than the simulated. Simultaneously the simulated discharge agreed well with the observed one. Increasing the winter precipitation by 60 % reduced the deviation between simulated and observed SCA to a satisfactory level. However, the flood was then overestimated through most of the melt period, except for the main flood peak which was better estimated by the updated model.
Figure 8. Simulated SCA and discharge compared to the observed ones with and without updating the input date in Malangsfoss.

Malangsfoss case 3 – increasing the winter precipitation 2002

The simulated SCA was lower than the observed ones through the main part of the melt season in 2002. On 25th May the simulated SCA was 25 percent units below the observation. Increasing the winter precipitation with 40 % gave a much better correspondence to the observed SCA (Fig. 8b). However, the main flood peak was highly overestimated and the mass recovery much too high (1.28 compared to 1.00 in the original simulation).

Malangsfoss case 4 – underestimated SCA in late spring 2003

In late spring 2003, the several observation of SCA was 15 to 28 percent units higher than the simulated ones. Because of the late time of the spring and the fact that the spring flood had ceased, the observations were assumed to be unreliable and no update was performed.

4.3.5 Narsjø

A marked improvement of the model performance with respect to SCA was achieved when including satellite observations in the calibration. The R^2_{SCA} increased from 0.82 to 0.94 in the calibration period and from 0.50 to 0.65 in the validation period.

In total, seven triggering observations was detected during the validation period. On 22nd May 1995, 29th April 2000 and 24th April 2002 the observed SCA was 30 percent units or more below the simulated. The simulated discharge, both with respect to snow melt start and volume, fitted well with the observed ones up to the time of the triggering observations (see Fig. 9a and b for 1995 and 2002, respectively). In Narsjø, snow melt starts early compared to the reference points for the snow signature, which may increase the uncertainty of the satellite observed SCA considerable. The observed SCA was therefore considered unlikely, and no updating of the model was performed.

On 11th May 2003, the observed SCA was 25 % higher than the simulated one. The simulation also overestimated the snowmelt in late April. Reducing the temperature with
1°C from 15th to 30th April the simulation of both the runoff ahead of the triggering observation and the SCA was improved (Fig. 9c). The following melt flood was slightly better simulated with the updated model.

The triggering observations, SCA 15 – 25 percent units higher than simulated, in late May 2003 was considered unlikely because the melt flood was declining.

Two triggering observations occurred during the melt season 2004 (Fig. 9d). The first one, on 1st May, indicated an overestimation of SCA of 25%. This followed only three days after a perfect fit between observed and simulated SCA. Because the flood was declining and the runoff so far was simulated very well by the model, no update was made. The next triggering observation was on 10th and 12th May when the simulated SCA was 16 to 18 % below the observed one. Also in this case, the flood was declining and the simulated runoff agreed very well with the observed one up to the triggering observation. Therefore, no update was done.

**Figure 9.** Simulated SCA and discharge compared to the observed values in the catchment Narsjø 1995 (a) and 2002 (b).
4.3.6 Nedre Heimdalsvatn

The catchment Nedre Heimdalsvatn is a reservoir, with regulated outlet. As for Aursunden the discharge is calculated from reservoir inflow. The model performance with respect to SCA, was markedly better with the Q+SCA than the Q-model both in the calibration and the validation period, except for 2001 where the Q-model simulated SCA better than the QS-model.

Nedre Heimdalsvatn case 1 – updating the winter precipitation in 2000

A SCA observation early in the melt season in 2000 indicated that the snow reservoir was overestimated in the model. Reducing the winter precipitation with 50 % decreased the deviation from 24 to 13 percent units and led to a large underestimation of the flood (Fig. 10a). However, the next SCA observation, 9 days later, corresponded well with the original model, which also simulated the discharged much better than the updated model.

Nedre Heimdalsvatn case 2 – updating the winter precipitation in 2001

The simulated SCA curves for the melt season 2001 declined too early and the largest flood peaks were underestimated (Fig. 10b). On 19th June, the simulated SCA was 23 percent units lower than the satellite observed SCA. Increasing the winter precipitation with 40 % eliminated most of the discrepancies between simulated and observed SCA. The mass recovery of the flow was improved from 0.67 to 0.89. Although the total difference between simulated and observed discharge was reduced in this case, the model still failed to predict the two largest flood peaks. However, these flood peaks may not be correct on a daily basis since they are calculated as reservoir inflow based on measured water level in the reservoir and discharge out of the reservoir.

Figure 10. Simulated SCA and discharge compared to the observed values with and without updating the input data of the catchment Nedre Heimdalsvatn.
Nedre Heimdalsvatn case 3 – updating the spring temperature 2003

The SCA observation on 31st May 2003 indicated an underestimation in the modelled SCA of 25%. A comparison between the simulated and observed runoff showed that the modelled runoff was higher than the observed one in early spring. Reducing the temperature with 1°C from 15th April to 31st May improved the modelled runoff and reduced the deviation between simulated and observed SCA to an acceptable level (Fig. 10c). The model update result in a slight improvement of the following flood peak, although still underestimated.

Nedre Heimdalsvatn case 4 – overestimated SCA in 2004

On 23rd May 2004, a SCA observation of 45% was recorded whereas the model simulated 86% SCA. A snowfall on the 22nd led to a rapid increase in the simulated SCA and a corresponding decrease to the observed level 2 days later. The deviation between the observed and simulated SCA on the 23rd was probably due to a small lag in the model response. Therefore, no update of the model was done.

4.3.7 Orsjoren

Including SCA in the calibration gave a small shift in the SCA curve and advanced the decrease in SCA with a few days. This led to an improved simulation of SCA by the QS-model compared to the Q-model. $R^2_{SCA}$ increased from 0.75 to 0.95 in the calibration period and from 0.73 to 0.87 in the validation period.

Orsjoren case 1 – rejected triggering observations 1995.

A SCA observation shortly after the initial rise of the 1995 flood indicated that the snow reservoir in the model was underestimated (data not shown). During the flood rise, the simulated discharge had reproduced the observed flow well, although it was slightly too high at the time of the SCA observation. An increase of the snow reservoir, in order to fit the observed SCA, would cause an even higher discharge at this time. Therefore no updating of the model was carried out.


In 2000, the simulated SCA fitted well to the observed during the first flood rise although the increase in discharge starts a few days to early (Fig. 11a). Later in the melt season the simulated SCA was much lower than the observed and a second flood rise was not captured by the model. Decreasing the temperature with 1°C during the melt period, 20th April to 30th June, improved the timing of the first flood rise (Fig. 11a). As a consequence the SCA was slightly overestimated during the first flood. However, the modelled SCA fitted better to the observed SCA later in the melt season. A small improvement was achieved for the second flood, although it was still underestimated. The $R^2_Q$ improved from 0.72 to 0.73 and the $R^2_{SCA}$ from 0.87 to 0.88.
a) Case 2 - Temperature reduced with 1°C from 20th April to 30th of June 2000.

b) Case 3 - Temperature reduced with 2°C from 1st May to 15th May 2001 and winter precipitation 00/01 reduced with 40%.

c) Case 4 - Temperature reduced with 2°C from 15th April to 15th May 2003.

Figure 11. Simulated SCA and discharge compared to the observed values with and without updating the input data of the catchment Orsjoren.


In 2001, both the SCA and the melt flood was overestimated by the model (Fig. 11b). The rise in discharge started too early and a small flow peak, which was not seen in the observations, was simulated by the model. The main flood peak was simulated well by the model, but an later flood event was highly overestimated. On 12th and 19th June the SCA was overestimated with 46 and 13 percent units, respectively, by the model. Decreasing the winter precipitation with 40 % led to a better correspondence between observed and simulated SCA. The accumulated runoff was also improved, although the overestimated initial flow peak was still present. Reducing the temperature in the beginning of May, in addition to the reduction in winter precipitation, improved the fitting of the first flood rise (Fig. 11b). The corresponding delay in the decrease of SCA results in an overestimated SCA on 12th June. The main flood peak was still underestimated, whereas the tail of the flood was much closer to the observed one although slightly to high.


From 28th May to 2nd June, four satellite observations showed 10 to 15 % higher absolute value of SCA than the model. Simultaneously the modelled runoff in May was higher than the observed one, indicating a too early start of the snow melt. A reduction of the temperature with 2°C from 15th April to 15th May improved the simulated SCA and the initiation of the spring flood runoff. However the runoff volume during the flood was overestimated (44 % higher than observed) (Fig. 11c). Obviously the snow magazine was
too large, which could not be detected neither by the observed SCA or runoff before the spring flood ceased.

4.3.8 Polmak

The melt season in the Polmak catchment is short. The catchment has a large area, but still a rather uniform response in terms of snow melt. The HBV-models reflected the dynamics very well with $R^2_Q = 0.94$ and $0.89$ respectively, in the calibration and validation period. Only a few satellite observations of SCA were available. The observations, fitted relatively well with the simulations, $R^2_{SCA} = 0.98$ (Q-model) and $0.99$ (QS-model) in the calibration period, and $0.73$ (Q-model) and $0.72$ (QS-model) in the validation period.

Polmak case 1 – overestimated SCA 2000

One triggering observation was found in the validation period, on 20th May 2000 (Fig. 12a). It occurred at the very beginning of the snow melt and the observed SCA value was considered as unlikely low. Therefore, and because the simulated and observed discharge agreed well up to that date, no updating of the model input was performed.

Polmak case 2 – overestimated SCA 2002

On 28th May 2002, the simulated SCA was 20 percent units lower than the observed one. Simultaneously the simulated and observed runoff showed a perfect match (Fig 12b). In order to simulated a SCA close to the observed one, the winter precipitation was increased by 50 %. Two days later a SCA observation contradicted the previous one, indicating that the original model was correct. This was also the best model for the proceeding flood. Without the later SCA observation the updated model would have largely overestimated the spring flood.

Polmak case 3 – overestimated SCA 2004

On 6th May 2004, the simulated SCA was 53 percent units higher than the observed one. Reducing the winter precipitation by 40 % led to a good fit of the SCA. However, the following flood peak was then highly underestimated (Fig 12c). Reducing the winter precipitation with 20 % resulted in a better overall fit of the runoff during the flood, but the simulated SCA was then 30 percent units above the observed one.
a) Case 1 - Simulated SCA 30 percent units higher than the observed one. Model input not updated.

b) Case 2 - Simulated and observed Q and SCA spring 2002.

c) Case 3 - Winter precipitation 03/04 reduced by 40 %.

Figure 12. Simulated and observed SCA and discharge in the catchment Polmak 2000.

4.3.9 Sjodalsvatn

The QS-model simulated SCA much better in the calibration period than the Q-model, $R^2_{SCA} = 0.95$ compared to 0.76. In the validation period the SCA was better simulated with the QS-model than the Q-model, $R^2_{SCA} = 0.79$ compared to 0.58.

Sjodalsvatn case 1 – updating the winter precipitation 2001.

In 2001, the SCA was underestimated with approximately 20 % by the QS-model. The main spring flood was also underestimated. By increasing the winter precipitation with 40 and 60 % the underestimation of SCA was decreased to 18 and 14 percent units respectively (Fig. 13a and b). A better mass recovery was also obtained, increasing from 0.77 in the original simulation to 0.94 and 1.0, respectively, in the updated simulations. The flood peak were still underestimated and the tail of the main flood event became too large.

Sjodalsvatn case 2 – updating the spring temperature 2003.

In end of May 2003, three SCA observations within five days showed 13 to 16 % higher SCA than the model. Simultaneously the model simulated a too early rise of the spring flood. By reducing the spring temperature with 2°C, from 15th April to 25th May, simulated SCA showed a perfect fit to the observed ones and the initiation of the spring flood agreed better with the observed runoff (Fig. 13c). The first flood peak was simulated very well, however the later flood peaks were overestimated, as without updating, and in particular the latest of them. Obviously the snow magazine was
overestimated in the model and thereby leaving too much snow to the end of the melt season.

![Graph](image)

a) Case 1 - Increasing the winter precipitation 00/01 with 40%.

![Graph](image)

b) Case 1 - Increasing the winter precipitation 00/01 with 60%.

c) Case 2 - Temperature reduced with 2°C from 15th April to 25th May 2003.

Figure 13. Simulated SCA and discharge compared to the observed values with and without updating the input data of the catchment Sjodalsvatn.

4.3.10 Vinde-elv

The QS-model simulated SCA better in the calibration period than the Q-model, $R^2_{SCA} = 0.92$ compared to 0.72 for the Q-models. In 2001 the melt started too early in the QS-model, advancing the flood peak and the decreasing the SCA (Fig. 14a). Actually, the Q-model timed the flood better this year. Both the Q and the QS-model overestimated the flood peak in 2001. In 2002 the simulated decrease of the SCA started too early. This could be due to error in the amount of accumulated snow. An observed flood peak in the middle of May 2002 was not captured in the model (Fig. 14b). Most likely this flood peak was caused by a precipitation event not measured by the meteorological station.

Vinde-elv case 1 – updating the temperature in the melt season 2001.

On 20th May 2001, the simulated SCA was 35 percent units lower than the satellite observed SCA. Decreasing the temperature with 2°C from 21th April to 16th May led to a simulated SCA very similar to the observed one, $R^2_{SCA} = 0.95$ compared to 0.90 without updating, and the simulated discharge became more similar to the observed one during most of the melt period (Fig. 14a). At the end of the melt flood the discharge was slightly overestimated in the updated model, probably because some rain events in late fall 2000 was simulated as snow in the model. The mass recovery for the hydrological year 2000/2001 was 1.0 in both cases.
4.3.11 Summary of the model updating

The model updating revealed quite diverging results. The overall change in model performance for each of the catchments are given in Table 2 and Figure 15, whereas the numbers of successful and unsuccessful updates are listed in Table 3. Eleven of the model updates, mainly in 2001 and 2003, gave better amplitudes of the flood peak(s) and the accumulated discharge volume. During the winter 2001 the main wind direction deviated from normal (prominent snow-producing weather circulation from south-east as opposed to from west which is normal). As a result the HBV-model simulated the snow magazine incorrectly in several catchments. For the catchments with high mean altitude (Akslen, Sjodalsvatn, Orsjoren and Nedre Heimdalsvatn) the snow magazine was successfully adjusted by using the satellite observed SCA. However, the catchments with lower mean altitude, although still alpine, did not show a positive response when using the observed SCA values. In 2003, positive response on the model update was seen also in the lower located catchments, whereas one of the high alpine catchments showed negative response. The snow melt started one to two week earlier than normal both in 2003 and 2004. Since no successful updates was achieved in 2004, the early snowmelt alone cannot explain the positive response on the updates in 2003.
In the remaining cases, the change of model input led to larger deviation between simulated and observed discharge than the non-updated models or, in one case, non-significant change of model performance. In addition about half of the triggering SCA observations were rejected from updating the model because they were assumed unlikely when compared to the stage of snowmelt. The overall success rate of the updates was 28\%. Taken into account that only satellite images of high quality were included in the time series, the results are not convincing.

Table 3 Results of the update scenarios.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>No. of triggering dates</th>
<th>Number of cases within each result class.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Successful</td>
<td>Insignificant</td>
</tr>
<tr>
<td>Akslen</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Atnasjø</td>
<td>6</td>
<td>-</td>
</tr>
<tr>
<td>Aursunden</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Malangsfoss</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Narsjø</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>N. Heimdalsvatn</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Polmak</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>Orsjoren</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Sjodalsvatn</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Vinde-elv</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Sum 40 11 1 9 19

$^1$Cases where the update was rejected either because the main flood event had ceased or because the observed SCA was considered unlikely or very uncertain.

The pattern of successful and unsuccessful updates versus observed SCA does not show a clear trend, Fig. 16. Only a weak tendency of more successful updates at high observed SCA values and more rejection of updates at low observed SCA values are seen. There were also no direct connection between the altitude of the catchments and the rate of success. Unsuccessful updates and rejection of satellite SCA observations occurred for all of the test catchments, and successful updates were achieved for eight of the catchments.

![Figure 16. Success of model updates versus simulated and observed snow covered area (SCA).](image-url)
5 Approach 2: New snow distribution model

This chapter describes the latest developments of the new snow distribution model using gamma sums. Two major features are described; 1) estimating parameters from precipitation observations and 2) automatically updating the snow reservoir from remotely sensed data.

Differences in spatial distributions of SWE observed at the peak of accumulations are, in this study, associated with the differences in the spatial variability of precipitation rather than on landscape specific features (alpine, forest, topography etc.) as suggested by numerous authors (Alfnes et al. 2004; Marchand and Killingtveit, 2004 and Shook and Gray, 1997). In this study we want to demonstrate that the parameters of the snow distribution model presented in Skaugen et al. (2003, 2004) can be estimated from an assessment of the spatial and temporal variability of precipitation for the catchments of interest. The proposed approach is attractive firstly in that the variability of snow is linked to the variability of precipitation and that this link is analytical, and secondly that it facilitates the parameterisation of the snow distribution model from readily available precipitation information, and consequently reduces the number of tuning parameters in the rainfall-runoff model.

Linear relationships between spatial extent of the snow cover and the mean areal SWE have been used as a rule of the thumb by hydropower companies (Dan Lundquist, GLB, pers. comm.), and such relationships have been studied by several authors (Buttle and McDonald, 1987; Dey, et al. 1992, Häggström, 1994). However, experiences tells us that observations of only the snow cover extent are not sufficient because it is well known that a large snow cover extent may sometimes be associated with a small snow reservoir and vice versa (Rango, 1996). In the proposed methodology we do not reject the idea of a relationship between spatial extent of the snow cover and the mean areal SWE, but this relationship is conditioned on the spatial distribution of SWE and is indeed the key to how we can develop an explicit link between SCA and mean areal SWE. This feature also provides a theoretical framework for developing updating algorithms of the snow reservoir from remotely sensed data (satellite images).

5.1 General methodology for modelling the snow reservoir with sums of gamma distributed variables.

5.1.1 Temporal and spatial distribution of SWE

Modelling the spatial distribution of SWE, taking into account the history of accumulation and ablation events, as sums of gamma distributed variables was initially described in Skaugen (1999) and in Skaugen et al. (2004). The major points of the derivation of the model will be revisited here in the following.

Let us consider \( y(x = x_j, t = t_i) \) to be the SWE for a snowfall event measured at time \( t_i \) at position \( x_j \) in some catchment. The variable \( y \) constitutes a stochastic process in time and space, and initially we assume the stochastic process \( y \) to be stationary and independent in time and space. Then the temporal distribution of \( y \) at any point \( x \), must
coincide with the spatial distribution of $y$ at any time $t$. Under these assumptions we have the rather unrealistic implication that the mean areal SWE is equal for every snowfall event and that the individual snowfall events are uncorrelated in space. In the further development of the model we take into account temporal and spatial deviations from the assumptions of stationarity and independence of the process.

**Temporal distribution of $y$**

Let us fix the point $(x = x_j)$ and assume that the temporal distribution of $y$ is a two-parameter Gamma distribution, $y(x = x_j, t) = G(v, \alpha)$, with probability density function (PDF):

$$f_{\alpha, v}(y) = \frac{1}{\Gamma(v)} \alpha^v y^{v-1} e^{-\alpha y} \quad \alpha, v, y > 0$$  \hspace{1cm} (1)

where $\alpha$ and $v$ are parameters. The mean equals $E(y(x = x_j, t)) = v/\alpha$ and the variance equals $Var(y(x = x_j, t)) = v/\alpha^2$. In order to take into account the temporal fluctuations of $y$ around its mean, $E(y)$, a Gamma variable $u$, is introduced by scaling the distribution of $y$ with its mean, $E(y(x = x_j, t)) = v/\alpha$. The mean of $u$ is $E(u) = 1$, and variance is $Var(u) = 1/v$. We can thus rewrite the process $y$ as:

$$y(x = x_j, t) = u v / \alpha = (v/\alpha)G(v, v)$$  \hspace{1cm} (2)

**Spatial distribution of $y$**

In order to take into account the spatial fluctuations of $y$, let us assume that the spatial distribution of $y$, at a fixed time $t_i$, that is $u = u_i$, also is Gamma distributed, $y(x, t = t_i) = u_i G(v, \alpha)$ with mean equal to

$$E(y(x, t = t_i)) = u_i v / \alpha$$  \hspace{1cm} (3)

and variance equal to

$$Var(y(x, t = t_i)) = u_i^2 v / \alpha^2$$  \hspace{1cm} (4)

By introducing a Gamma variable $w$ with mean $E(w) = 1$, and variance $Var(w) = 1/v$, the spatial process of $y$ for a fixed time $t_i$ can be written as:

$$y(x, t = t_i) = w u v / \alpha = (u_i v / \alpha)G(v, v)$$  \hspace{1cm} (5)

If we let both $x$ and $t$ vary, we see that the process $y$ is not gamma distributed, but distributed as the product of two gamma distributions scaled with $v/\alpha$:

$$y(x, t) = w u v / \alpha = (u_i v / \alpha)G(v, v)G(v, v)$$  \hspace{1cm} (6)

To approximate the spatial- or the temporal distribution as a Gamma distribution, we thus have to keep one of the variables $u$, or $w$ constant. To approximate the temporal distribution, $w$ is constant. This implies that the measured snowfall at each point is the
mean areal SWE, which is exactly the applied procedure when using rainfall-runoff models driven with precipitation measurements. To approximate the spatial distribution, \( u \) is constant, which implies that every snowfall event is equal to the mean areal SWE, which is clearly not very realistic. However, as our primary concern is the change in the spatial distribution due to accumulations in time, we approximate the spatial distribution of accumulations by assuming that when the number of accumulations increase, the resulting distribution of accumulations can be approximated as if each event was equal to the mean areal SWE, i.e. a constant \( u \) and equal to its mean \( E(u) = 1 \). We thus want to consider the spatial distribution of the accumulations of \( y(x, t = t_i) \), for \( i = 1, \ldots, n \) events, which we denote \( z'(x, t_n) \):

\[
z'(x, t = t_n) = y(x, t_1) + y(x, t_2) + \ldots + y(x, t_n), \quad y(x, t_i) > 0
\]  

(7)

According to Feller (1971, p.47), the variable \( z'(x, t_n) \) is gamma distributed with parameters \( \alpha \) and \( n \nu \) if \( z' \) is the sum of identically and independent gamma distributed variables,

\[
y(x, t) = \frac{w \pi \nu}{\nu} = \frac{\pi \nu}{\alpha} G(\nu, \nu)
\]  

(8)

where \( \bar{u} \) is the average value of \( u \), for the times \( t_1, \ldots, t_n \). We see that when \( n \) grows large, \( \bar{u} \) converges to the expectation of \( u \) which is equal to one. The spatial distribution of \( z'(x, t = t_n) \) is thus approximated as a gamma distribution. \( z'(x, t = t_n) = G(n \nu, \alpha) \), with mean:

\[
E(z'_i, (x, t = t_n)) = n \nu / \alpha
\]  

(9)

and variance:

\[
Var(z'_i, (x, t = t_n)) = n \nu / \alpha^2
\]  

(10)

The above derivation is basically thought appropriate for accumulation of a stationary variable for a certain amount of time i.e. precipitation as snow. The melting process is more complicated as the melting is more intense as the temperature increases during the spring, introducing a temporal non-stationarity of the process. However, we approximate ablation also with the presented approach and keep account of the variable \( n \) by letting accumulated or melted amounts of snow, be, at any time, gamma distributed with parameters \( u \nu \) and \( \alpha \). The accounting is done by keeping track of \( u \) and \( n \) and update \( n \) as \( n_{i+1} = n_i + u_i \), for accumulation, and \( n_{i+1} = n_i - u_i \), when a melting event has occurred so the resulting distributions of accumulations stays distributed with parameters \( n \nu \) and \( \alpha \).

### 5.1.2 Modelling snow free areas

We need to incorporate the presence of snow-free areas in the catchment into the methodology presented above. In Skaugen et al. (2004) this was achieved by postulating how the snow coverage (SCA), here termed \( p \), relates to a melting event and deciding on
the functional relationship between melted amount, \( u, \nu / \alpha \), the updated mean of SWE, \( (n, u, \nu / \alpha) \) and SCA. For a given spatial distribution of SWE, where we initially have full coverage (\( p=1 \)), there exists a set of \( p \)-values corresponding to different melting events. Typically, for a certain amount to be melted, we would expect significant reduction in \( p \) if the distribution is very skewed, and not so if the distribution is more normal (see Skaugen et al. 2004 for more details). This implies that, conditioned on the different spatial distributions of SWE, we have different sets of \( p \)-values and corresponding melting amounts. When we map such sets of \( p \)-values and corresponding melting amounts they take on the shape of the spatial distribution of SWE itself, and can thus be seen as scaled versions of the gamma distribution of SWE. The scaling parameter is estimated so that the probability of melting less or equal to the entire present mean areal SWE is equal to one, i.e. if the entire present snow reservoir was to melt, the corresponding SCA is zero. We thus estimate the new scale parameter \( \alpha' \) so that:

\[
\int_0^{n, \nu / \alpha} f(z; n, \nu, \alpha') \, dz - 1 \leq \Delta \pm 0.1 \Delta
\]  

(11)

where \( f() \) is the PDF of the gamma distribution, \( n, \nu / \alpha \) is the mean areal SWE at the time \( t \) and \( \Delta \) is some small chosen measure (e.g. \( \Delta = 0.001 \)). The choice of \( \Delta \) represents the level of truncation of the distribution and should not be arbitrary in that it will define the minimum spatial resolution of our estimates of SCA. The skew of the distribution is not affected by the new scale parameter. The new coverage is thus estimated as the complementary probability of melting \( u, \nu / \alpha \) from the scaled distribution of SWE,

\[
1 - \int_0^{n, \nu / \alpha} f[z'; n, \nu, \alpha'] \, dz : \text{With this approach the evolution of SCA in the melting season is directly linked to the dynamic shape parameter, } n \nu \text{ of the spatial distribution of SWE.}
\]

**Updating SCA after an ablation event:**

The updated SCA at time \( t \) after melting \( u \), equivalents is:

\[
p_t = p_{t-1}(1 - \int_0^{u, \nu / \alpha} f[z'; n, \nu, \alpha'] \, dz), \quad p_t < p_{t-1}
\]

(12)

where \( \alpha' \) is the new scale parameter and estimated with (11).

**Updating SCA after an accumulation event:**

For updating the SCA after accumulation we apply the same reasoning as for ablation. The snowfall at time \( t \) of \( u \), equivalents, gives us a new scaled version of the gamma distribution \( f(z'; (n, u, \nu) + \alpha') \), where \( \alpha' \) is estimated as shown below. The previous \( p_{t-1} \), (before the new snowfall, \( u \)) which is known, is seen as if a similar amount, \( u \), was melted from the new \( p \). The updated SCA at time \( t \), after accumulating \( u \), equivalents, will be:
\[
p_t = p_{t-1} l(1 - \int_0^{\frac{u, \nu}{\alpha}} f(z; (n_{t-1} + u_t)\nu, \alpha_{\text{acc}}) \, dz), \quad p_t > p_{t-1}
\]

where \(\alpha_{\text{acc}}\) is estimated from

\[
\int_0^{\frac{n_t + u_t}{\nu}} f(z; (n_t + u_t)\nu, \alpha_{\text{acc}}) \, dz - 1 \leq \Delta \pm 0.1\Delta
\]

### 5.2 Methodology for estimating parameters from precipitation observations

If information on the spatial variability of snowfall/precipitation exists, we would like to use this information when estimating the parameters \(\nu\) and \(\alpha\). In order not to involve the product of the distributions of \(u\) and \(w\), we simplify the derivation by assuming that \(u\) is constant and equal to 1. This is equivalent of stating that the spatial mean of each snow fall event is equal to \(\nu / \alpha\), but that snow fall events are allowed to vary in space. The variable \(y\) is now described by (see equation (5)):

\[y(x, t) = w \nu / \alpha = \nu / \alpha \mathcal{G}(\nu, \nu)\]

with mean \(E(y) = \nu / \alpha\) and variance \(\text{Var}(y) = \nu / \alpha^2\), from which expressions for \(\alpha\) and \(\nu\) are straightforwardly obtained as:

\[
\nu = \frac{E(y)^2}{\text{Var}(y)}
\]

and

\[
\alpha = \frac{E(y)}{\text{Var}(y)}
\]

The values of \(E(y)\) and \(\text{Var}(y)\) can be estimated from time series of precipitation, excluding zero events.

**The gamma model and independence in time and space**

We find, when measuring the spatial variability of SWE from snow courses, that the variance is much higher than what can be obtained by using the theoretical expression for \(\text{Var}(z')\) in equation (10), with parameters \(\alpha\) and \(\nu\) estimated from time series of precipitation. Also, when inspecting the spatial variability of SWE for two sub catchments, where the time series of SWE were simulated for grid cells of 1X1 km\(^2\) (Skaugen et al., 2003), we find values of spatial variability much higher than consistent with the theoretical expression for \(\text{Var}(z')\). These findings confirm a notion that spatial and temporal independence cannot be neglected, as is the case for the model for spatial distribution of SWE put forward in Skaugen et al. (2004a), and which is reviewed above. The variance of a sum of correlated variables receives a contribution from correlations according to (Haan, 1977, p.56):
\[
\text{Var}(z') = \sum_{i=1}^{n} \text{Var}(y_i) + 2 \sum_{i<j} \text{Cov}(y_i, y_j) \tag{16}
\]

where \(n\) is the number of accumulations. In the following, we assume for the sake of simplicity, that there is a constant covariance contribution (a constant fraction \(c\) of the variance \(\text{Var}(y) = \nu / \alpha^2\) for the individual \(y\)'s,) associated with each summation in expression (13) and we get:

\[
\text{Var}(z') = n \frac{\nu}{\alpha^2} + n(n-1)c \frac{\nu}{\alpha^2} = n \frac{\nu}{\alpha^2} (1 + (n-1)c) \tag{17}
\]

Let us maintain that at all times we want the spatial distribution of snow to be represented by a sum of gamma distributed variables, that is, the distribution of the accumulations is a two parameter gamma distribution where the number of accumulations determines the statistical moments of the distribution, as developed in the previous section. The mean after \(n\) accumulations equals \(\bar{E}(z') = n\nu' / \alpha'\) and the variance equal to \(\text{Var}(z') = n\nu' / \alpha^2\), where \(\alpha'\) and \(\nu'\) are parameters of the two parameter gamma distribution of SWE. We further have that:

\[
\bar{E}(z') = n\nu' / \alpha' = n\nu / \alpha \tag{18}
\]

because there is no covariance contribution for the mean (Haan, 1977, 56). For the variance, we have that:

\[
\text{Var}(z') = n\nu' / \alpha^2 = n \frac{\nu}{\alpha^2} (1 + (n-1)c) \tag{19}
\]

where \(\alpha\) and \(\nu\) are parameters derived from studying the spatial variability of precipitation according to (11) and (12). If we solve for \(\alpha'\) and \(\nu'\) from (18) and (18) we get the following expressions:

\[
\alpha' = \frac{\alpha}{1 + (n-1)c} \tag{20}
\]

and

\[
\nu' = \frac{\nu}{1 + (n-1)c} \tag{21}
\]

We see that the parameters decrease as \(n\) grows with a rate \(c\) determined by how large the fraction of the covariance contribution is compared to the variance. An interesting feature appears when we study the expression for the skew of the variance for the gamma distribution. The skew parameter for a two parameter gamma distribution can be written as \(\gamma = 2 / \sqrt{\nu}\), where \(\nu\) becomes the shape parameter. If we exchange \(\nu\) for \(\nu'\) (21), in the expression for the skew we get:

\[
\gamma = \frac{2}{\sqrt{\nu' \frac{n\nu}{1 + (n-1)c}}} \tag{22}
\]
which varies far less dramatically than when using a fixed $\nu$.

**Comparisons to observed data**

Comparisons of modelled statistical moments of the spatial distribution of SWE to observed data were performed using snow course data measured at catchment, Atnasjø, located in Central Southern Norway (see section 2). The parameters $\alpha$ and $\nu$ were estimated using equations (14) and (15) with mean and variance estimated from time series of precipitation (excluding zero events), from a representative precipitation station. The parameter $n$ could be estimated from using equation (18), and the covariance contribution (a constant fraction of $\text{Var}(y)$), was set equal to $c = 0.1$. Table 4 and Figure 17 show the comparison between observed values of standard deviation and skew from snow courses (two sets of 5 snow courses were carried out within two weeks) inside the catchment and values modelled with and without covariance contribution. When using a covariance contribution, the modelled statistical parameters are much closer to the observed.

**Table 4 Comparison of mean, standard deviation and skew between observed snow course data and modelled with and without covariance contribution.**

<table>
<thead>
<tr>
<th>Observed data</th>
<th>Without covariance</th>
<th>With covariance, $c = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MeanObs</td>
<td>StdObs</td>
<td>SkewObs</td>
</tr>
<tr>
<td>108</td>
<td>49</td>
<td>0.07</td>
</tr>
<tr>
<td>216</td>
<td>86</td>
<td>1.26</td>
</tr>
<tr>
<td>347</td>
<td>217</td>
<td>0.94</td>
</tr>
<tr>
<td>232</td>
<td>154</td>
<td>1.04</td>
</tr>
<tr>
<td>365</td>
<td>323</td>
<td>1.97</td>
</tr>
<tr>
<td>19</td>
<td>34</td>
<td>1.65</td>
</tr>
<tr>
<td>155</td>
<td>116</td>
<td>0.91</td>
</tr>
<tr>
<td>182</td>
<td>200</td>
<td>1.86</td>
</tr>
<tr>
<td>153</td>
<td>189</td>
<td>2.32</td>
</tr>
<tr>
<td>233</td>
<td>314</td>
<td>2.26</td>
</tr>
</tbody>
</table>
Figure 17 Comparison between observed and modelled values of standard deviation and skew. The squares are modelled with an autocovariance contribution and the asterisks are modelled without a covariance contributions. The straight line signifies perfect correspondence.

5.3 Methodology for updating the snow reservoir from remotely sensed data

Let us say that we obtain an estimate of SCA from remotely sensed data that differs from that estimated from our rainfall-runoff model. We are then faced with two choices. One is that the distribution of SWE we use in our model is incorrect and that the meteorological input (precipitation and temperature), and thus the water balance, is assumed correct. Then we simply update the mean areal SWE with the observed SCA ($p$) using $E(z) = pE(z')$, where $z$ and $z'$ denote accumulated SWE including and not including zeros. $E(z)$ is thus the mean areal SWE in the catchment and $E(z')$ is the mean areal SWE when snow free areas are excluded. The other option is that the water balance is incorrect due to wrong input (precipitation and temperature) or that the melting procedure is wrongly calibrated so that more or less water has left the catchment. Experience with
use of the HBV model tells us that when large discrepancies are found between modelled and observed SCA, wrong input (precipitation and temperature) is usually to blame. So the problem at hand is to develop tools to use information on SCA directly on the snow reservoir, i.e. SWE. This case is more complicated than the former in that we have to update the water balance conditioned on an observed SCA and we have to “translate” the information on SCA into changed mean and variability and thus a changed distribution of SWE. Because of the link proposed in Skaugen et al. (2004), reviewed above, between the evolution of snow free areas and the shape parameter of the spatial distribution of SWE, we apply a similar reasoning to update the spatial distribution of SWE from information on SCA. By assuming that the general statistical model is correct, we can increase or decrease the parameter $n$, according to the equations below until we have an SCA that corresponds with the observed. The procedure for updating the snow reservoir from known SCA differs from the procedure above, estimating SCA from known snowfall or melting amount, in that we have to model a change in the statistical parameters of the distribution of SWE based on known SCA.

Case of observed SCA less than modelled ($p_{sat} < p_{mod}$)

The following equation have to solved for $u_{abl}/\alpha$, which represents the amount SWE of overestimation by the model.

$$p_{sat} = p_{mod} \left(1 - \int_0^{u_{abl}/\alpha} f(z'; n, \nu, \alpha') \, dz\right), \quad p_{sat} < p_{mod} \quad (23)$$

where $\alpha'$ is the scale parameter and estimated so that:

$$\int_0^{n\nu/\alpha} f(z'; n, \nu, \alpha') \, dz - 1 \leq \Delta \pm 0.1\Delta \quad (24)$$

where $f()$ is the PDF of the gamma distribution, $n\nu/\alpha$ is the mean area SWE at the time $t$ and $\Delta$ is some small chosen measure (e.g. $\Delta = 0.001$). Note that $\alpha'$ is the scale parameter estimated for the modelled SWE before any corrections and is thus previously known. For different values of $u_{abl}/\alpha$, $\alpha'$, is estimated and expression (23) is evaluated. This procedure is repeated until a satisfactory evaluation of (23) is obtained. The new mean and variance of the SWE are: $E(z'_{t+1}) = (n_t - u_{abl})\nu/\alpha$ and $Var(z'_{t+1}) = (n_t - u_{abl})\nu/\alpha^2$

Case of observed SCA higher than modelled ($p_{sat} > p_{mod}$)

For the updating distribution of SWE from a higher SCA than modelled, we apply the same reasoning as above. The snowfall corresponding to a higher SCA, giving an increased SWE of $u_{acc}$ equivalents, gives us a new scaled version of the gamma distribution $f(z; n_t + u_{acc}\nu, \alpha'_{sat})$, where $\alpha'_{sat}$ is estimated as shown below. The modelled SCA, $p_{mod}$, is known and is seen as if
an amount, $u_{acc}$ was melted from the new $P_{sat}$. The updated SWE after updating by a new SCA is found by solving:

$$P_{sat} = P_{mod} / \left(1 - \int_0^{u_{acc} + \alpha'} f(z'; (n_t + u_{acc}) \alpha', \alpha_{sat}') \, dz \right), \quad P_{sat} > P_{mod} \quad (25)$$

for $u_{acc} \alpha'$. Where $\alpha_{sat}'$ is estimated by

$$\int f(z'; (n_t + u_{acc}) \alpha', \alpha_{sat}') \, dz - 1 \leq \Delta \pm 0.1 \Delta$$

This is carried out by for different values of $u_{acc} \alpha'$. For each value of $u_{acc} \alpha'$, a new $\alpha_{sat}'$ is estimated and (25) is evaluated until a satisfactory result is achieved.

The new mean and variance of the SWE are:

$$E(z_{t+1}^i) = (n_t + u_{sat}) \alpha'$$

$$Var(z_{t+1}^i) = (n_t + u_{sat}) \alpha'^2$$

### 5.4 Results and discussion

Five catchments (Akslen, Atnasjø, Orsjoren, Sjodalsvatn and Vinde-elv) with location and physiography described in section 2 were tested using the new snow distribution model. The parameters of the snow distribution model were calibrated such that the mean obtained from the precipitation stations was conserved, but the variability was allowed to vary. The covariance contribution used was $c = 0.1$ which originated from an analysis of the precipitations time series measured at stations associated with one of the catchments in question. The models were calibrated both on discharge $Q$ and snow covered area, SCA.

Table 5 shows that the Gamma sum model is similar to the original HBV model for predicting $Q$. The prediction of SCA, however, is consistently better with the new model. When updating the Gamma model, the periods for calibration and validation were run with updating whenever a satellite observation was available and a certain level of divergence (between 10 and 25%) between observed and simulated SCA was recognised. When updating from SCA, a corresponding shift in SWE was calculated by the procedure described in section 5.3 until a satisfactory consistency in observed and simulated SCA was obtained. Thus, updating was not carried out for every satellite observation and in every elevation zone, and a complete consistency with observations was not obtained.

This is the reason why the efficiency measure $R^2$ is not equal to 1.0 for the forecasting of SCA. When updating the model we found that for some catchments the forecasting of $Q$ deteriorated when the level of discrepancy was lower and approaching zero, i.e. when approximating a prefect simulation of SCA. This feature either signifies inability of the model to make use of observed SCA or that the estimates of SCA are incorrect.
Table 5 Nash-Sutcliffe criterion for model performance for 5 catchments. The Gamma model uses calibrated estimates of $V$ and $\alpha$.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Modell</th>
<th>Predicting Q</th>
<th>Predicting SCA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Kal R2</td>
<td>Val R2</td>
</tr>
<tr>
<td>Akslen</td>
<td>HBV</td>
<td>0.82</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>HBV (updated)</td>
<td>-</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.84</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>Gamma (updated)</td>
<td>0.85</td>
<td>0.81</td>
</tr>
<tr>
<td>Atnasjø</td>
<td>HBV</td>
<td>0.79</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>HBV (updated)</td>
<td>-</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.79</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>Gamma (updated)</td>
<td>0.81</td>
<td>0.67</td>
</tr>
<tr>
<td>Orsjoren</td>
<td>HBV</td>
<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>HBV (updated)</td>
<td>-</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.73</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>Gamma (updated)</td>
<td>0.74</td>
<td>0.77</td>
</tr>
<tr>
<td>Sjodalsvatn</td>
<td>HBV</td>
<td>0.78</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>HBV (updated)</td>
<td>-</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>0.77</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>Gamma (updated)</td>
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<td>0.79</td>
</tr>
<tr>
<td>Vinde-elv</td>
<td>HBV</td>
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<td>0.79</td>
</tr>
<tr>
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<td>HBV (updated)</td>
<td>-</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
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<td>0.79</td>
</tr>
<tr>
<td></td>
<td>Gamma (updated)</td>
<td>0.81</td>
<td>0.80</td>
</tr>
</tbody>
</table>

The improvements on predicted Q from updating are modest, as was the case when updating the traditional model. From Figure 18 below we can see that the updating routine works favourably for certain events, so it is tempting to address the failure in improving the discharge forecast on the inaccuracy of the satellite scenes. Unlike the updating procedure in the previous section, where each of the satellite scenes were carefully evaluated before any updating were performed, here, all satellite scenes where the discrepancy between modelled and “observed” SCA were larger than a specified level (between 10 and 25% for the different catchments, see discussion above) were taken into account and the snow reservoir updated. Therefore, it is entirely possible that the very modest improvements on predicting Q are due to the inclusion of inferior satellite scenes.
Figure 18. Simulated snow covered area and discharge for selected seasons for the five catchments. An improvement in the predicted discharge is seen for the updated models.
6 Comparing updating results from the two model approaches.

6.1 Note on observed SCA

During this study using two different approaches of snow distribution modelling, it became apparent that the methodology used for estimating SCA from satellite images contained flaws that where of significance for the testing of the two methods. As noted in section 3.1 reflectance values for 100 % and 0 % snow cover are found from glaciers and snow-free areas. The snow cover percentage for each 1x1 km² pixel is then calculated as a linear function of the reflectance in the pixel compared to the 100 % and the 0 % reflectance. It has become evident that the estimate of areal SCA can differ significantly whether this procedure is carried out for the whole catchment as a bulk estimate or for each elevation zone separately. This is due to the linear “stretching” that takes place when the observed SCA is reluctant to be 100% even when temperature data signifies that no melting has occurred. The linear method forces the mean areal SCA value to be 100%, by multiplying the mean areal SCA value by some factor. This factor is considered to be a constant. However, events occur when applying this factor provides SCA values higher that 100%. The chosen procedure is then to adjust the factor so that, again, the mean areal SCA value is 100%. When each elevation zone is considered independently, the final correction factor is limited only to the particular elevation zones that with the original correction factor give a higher the mean areal SCA than 100 %. This implies that the stretching, seen from a catchment point of view, is no longer linear, in that a different factor is applied for the different elevation zones. It is assumed that estimating for the different elevation zones gives more correct SCA because it is possible to take into account the fact that reflectivity for 100 % SCA in the lower elevation zones is different than reflectivity for 100 % SCA in the higher elevation zones. This is due to different optical properties and morphological processes of the snow in the different elevation zones.

For the present study the following observed SCA values were used: In the traditional HBV-model (section 4) the linear stretching of catchment SCA-values was applied whereas in the new model (section 5) stretching of the SCA for each elevation zone was applied.

6.2 Lognormal versus gamma distributed snow

The model approaches presented in this study have two fundamental differences, the distribution function used to describe the snow reservoir and the method used for updating the models.

In the traditional model the skew of the snow distribution remains unchanged during accumulation above a specified minimum level of snow, of which the distribution is uniform, and snowmelt is assumed uniformly distributed. The model updates is made manually, adjusting winter precipitation and spring temperature in order to fit the simulated SCA and runoff to the observed ones. Aggregated SCA values for the whole catchment are used. When several triggering SCA observations occur in the melt season, previous updates are overwritten by the new ones.
The distribution of SWE from the Gamma model, on the other hand, changes dynamically with the snowfalls and snowmelt events from very skewed at the start of the accumulation season towards a normal distribution as snow accumulations proceeds. During snowmelt, the distribution is again increasingly skewed. The updates of the model are automatic, changing the SCA and thereby the snow reservoir abruptly when a triggering observation occurs. The SCA values are compared and updated for each elevation zone individually.

It is difficult to compare the updating performance between the traditional and the new model, because the traditional model was only updated for a selected few events, whereas the new model was updated for every SCA observation available. The results in Table 5 show, however, that the estimates of Q from the Gamma model were improved consistently in the calibration period and in three of five catchments in the validation period, when updated against observed SCA. Generally, the Gamma model performed better than the traditional model with respect to SCA. This indicates that the new snow distribution function represents the dynamics of the snow reservoir more correctly.

6.3 Uncertainties in satellite observed SCA

As shown by the simulations for Narsjø in 2002 (Fig. 9b) an almost perfect runoff simulation can be related to a very poor simulation of SCA compared to the satellite data. In such cases it is relevant to mistrust the quality of the satellite derived SCA. Undetected clouds, low precision in the geometrical correction of the satellite image or regional variations in snow reflectance can cause such errors. For the Narsjø catchment the snowmelt usually starts early and the snow reflectance is reduced compared to snow reflectance in the higher elevated training areas used to estimate the 100 % SCA reflectance. This effect leads to an underestimation of SCA, especially early in the melting period before melting starts in the training areas. Both in 2000 and 2002 melting started earlier than normal in the Narsjø catchment compared to the higher elevated Akslen catchment. Particularly in 2000 the observed SCA seemed to be far too low (Fig. 18). Using these data to update the models would have lead to a total underestimation of the flood. The estimated SCA at 29th April was about 40 %. At this point only 20 millimetres accumulated runoff was observed since the start of the melting runoff at 21st April. The runoff data reveals a rather large flood after 29th April and it was not observed any precipitation in the days until the flood peak at May 2nd. This shows that the simulated SCA was rather too low than too high before the flood event.

![Graph](image_url)
7 Conclusions

This study shows that the HBV-models with the traditional lognormal snow distribution can be calibrated against SCA in addition to Q with only small reduction in runoff performance. The improved performance in SCA was considerable higher than the loss of performance in runoff. Generally, both the Q- and the QS-models simulated runoff well in the calibration period. In addition, the QS-models showed a good fit to the observed SCA. Similar performance in predicting Q was found for the lognormal and the gamma distributed snow function when the HBV-models were calibrated against SCA in addition to runoff. The gamma model, however gave consistently better estimates of SCA.

Using satellite observed SCA to update the HBV-models showed diverging results using the traditional snow distribution model. The success was quite random, though a weak tendency of higher success rate at large SCA values was found. This is consistent with the increased uncertainties in satellite SCA products at the end of the snowmelt period. Using SCA from satellite images is not straightforward during snow melt, since the spectral signature may vary considerable in space. In order to improve the SCA product during snowmelt, information of the snow state could be included in the SCA algorithm. The model using the gamma model for snow distribution responded favourably to updating with observed SCA. The improvements, however, are modest and probably suffer from uncertainty in the SCA estimates as do the traditional model. In the new model the updating procedure gives an immediate change in snow reservoir and is easily implemented an applied.

The results of this study illustrates that satellite observed SCA in hydrological models can be useful, especially in years with unusual weather conditions. However, the uncertainties in the satellite SCA products are still too large to develop a system where the snow reservoir is automatically updated from satellite observed SCA. In order to use satellite observations of SCA in operational flood warning models, a careful evaluation of each scene and its calculated SCA compared to the stage of snow melt is needed.

As a final remark, one should also bring attention to the obvious shortcomings of the hydrological model (HBV) itself. It is unsatisfactory that the performance in predicting Q deteriorates when we include additional information like that of SCA. This is a commonly known feature of the HBV model and the same behaviour is observed when including other observations of, say, groundwater. This brings attention to the problem of overparameterization in the HBV model that is quite effective in disguising flawed process descriptions. This study has obtained positive results when including SCA data and a new dynamic snow distribution model, but part of the problem of not obtaining even better results are not entirely related to imperfect SCA observations but also on inadequate description of the hydrological processes in the HBV model.
8 References


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