

# **Temporal properties of the spatial distribution of snow**

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### **Temporal properties of the spatial distribution of snow**

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**Abstract:** Investigations of simulated spatial fields of snow water equivalent (SWE) show that the shape of the distribution is a function of accumulation- and ablation events. The coefficient of skew decreases during the accumulation season and increases during the ablation season.

**Key words:** Snow, spatial distribution, gamma distribution

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# **Contents**

<b>Preface .....</b>	<b>4</b>
<b>Summary .....</b>	<b>5</b>
<b>Introduction.....</b>	<b>6</b>
<b>Methodology .....</b>	<b>7</b>
1. The gridded water balance model .....	8
2. Disaggregation of precipitation.....	8
3. Interpolation of temperature .....	11
<b>Results and discussion.....</b>	<b>11</b>
<b>Conclusions .....</b>	<b>20</b>
<b>References .....</b>	<b>21</b>

# Preface

The research project EnviSnow (Development of Generic Earth Observation Based Snow Parameter Retrieval Algorithms) is supported by the EU commission under the Energy, Environment and Sustainable Development programme (EU Contract number EVG1-CT-2001-00052) for the period 2002-2005. The goal of EnviSnow is to develop and validate new and improved multisensor algorithms for retrieval of snow and soil parameters from Earth Observation (EO) data for use in global climate studies and hydrology, in particular runoff and flood prediction.

EnviSnow involves 11 partners from four European countries. The project is divided into 9 workpackages. Norwegian Water Resources and Energy Directorate (NVE) leads workpackage 5 (WP5): Assimilation and integration of snow parameters in hydrologic models. This report is the first deliverable within this workpackage (D1-WP5). The report sums up the scientific work carried out in task 2: Proposed methodology for the study of the spatial distribution of the accumulation and melting of snow.

Oslo, August 2003

  
for   
Kjell Repp  
Director of the Hydrology Department

  
Liss M. Andreassen  
Project leader

# Summary

A simulation exercise has been performed in order to study the temporal development of snow covered area and the spatial distribution of snow water equivalent (SWE). Special consideration has been paid to how the properties of the spatial statistical distribution change as a response to accumulation and ablation events. A distributed rainfall-runoff model at resolution  $1 \times 1 \text{ km}^2$  has been run with time series of precipitation and temperature fields of the same spatial resolution derived from the atmospheric model HIRLAM. The precipitation fields are disaggregated and the temperature fields are interpolated. Time series of the spatial distribution of snow water equivalent and snow covered area for three seasons for a catchment in Norway is generated. The catchment is of size  $3085 \text{ km}^2$  and two rectangular sub-areas of  $484 \text{ km}^2$  are located within the larger catchment. The results show that the shape of the spatial distribution of SWE for all three areas changes during the winter. The distribution is very skewed at the start of the accumulation season, then the skew decreases during the accumulation season, and as the ablation season sets in, the spatial distribution again becomes more skewed with a maximum near the end of the ablation season. For one of the sub-areas, we find a consistently more skewed distribution of SWE, which is addressed to higher variability in precipitation. This indicates that observed differences in the spatial distribution of snow between alpine and forested areas can be a result of the differences in the spatial variability of precipitation. The results obtained from the simulation exercise are consistent with a new approach of modelling the spatial distribution of SWE as summations of a gamma distributed variable.

# Introduction

A major cause of flooding in Norway is the combination of intense snowmelt and precipitation. In order to forecast these events, we need reliable forecast of precipitation and temperature, and a good estimate of the snow reservoir and its coverage in the catchment at the time of the forecast. The Swedish HBV model (Bergstrøm, 1995; Sælthun, 1996) is used operationally for flood forecasting at the Norwegian water Resources and Energy Directorate (NVE) and has been supplemented with a snow routine developed for use in Norway which accounts for the development of the snow reservoir and the snow coverage at different altitude levels (Killingtveit and Sælthun, 1995). The snow routine is developed under the assumptions that snowfall events are lognormally distributed in space with a fixed coefficient of variation and perfectly correlated in space. These assumptions imply that, at all times, the maximum of a new snowfall event will appear in exactly the same location as where the maximum snowfall from previous snowfall events already are found. Also, the distribution of accumulated snow will have a fixed coefficient of skew and not comply with the principles of the central limit theorem (Feller, 1971, p. 258), which would suggest an increasingly less skewed distribution after accumulations. The ablation process is modelled as uniform over the snow covered fraction of the catchment.

The shape of the distribution is important when the fraction of snow covered area (SCA) starts to play a role in the ablation process. When only a fraction of the catchment produces melt water, the possibility of predicting errors in runoff caused by wrongly estimated SCA increases. The shape of the distribution is central in how the amount of melted snow translates into changes in SCA. Intuitively we see that if the frequency of small values is small, as would be the case for a normal distribution, the response in SCA to a melting event is small. If, on the other hand, the frequency of small values is high, then significant changes in SCA in response to a melting event can be expected.

From studies of the spatial distribution of daily precipitation, a positively skewed distribution has been favoured. The exponential distribution has been a popular choice (Gao and Sorooshian, 1994; Skaugen, 2002), and other studies have indicated that a gamma distribution is suitable (Onof et al., 1998, Mackay et al., 2001). However, studies of the spatial distribution of accumulated SWE in forested areas, often measured at the peak of the accumulation period, show that a normal distribution often is a good model

(Marchand and Killingtveit, 1999; Marchand and Killingtveit, 2002; Alfnes et al., 2004). In alpine areas, however, more skewed distributions are found (Marchand and Killingtveit, 2002; Alfnes et al. 2004). Thus, based on the very limited information at hand, an accumulation-ablation model for snow should take into account that single events are positively skewed, whereas accumulated events tend towards a less skewed and even a normal distribution, indicating a process in accordance with the principles of the central limit theorem.

As data on the temporal development of the spatial distribution of SWE is rare and expensive to obtain, a simulation exercise has been performed in order to produce time series of spatial distribution of snow for a catchment over three winter seasons. A distributed rainfall-runoff model of resolution  $1 \times 1 \text{ km}^2$ , the Gridded Water Balance model (GWB) (Beldring et al. 2002), which is largely based on the Swedish HBV model, computes the water balance elements for Norwegian conditions with precipitation and temperature as input data. In order to produce input for the GWB model of appropriate spatial resolution, time series of precipitation and temperature fields generated by the High Resolution Limited Area Model (HIRLAM) have been disaggregated and interpolated respectively. The intention of this study is to produce realistic, spatial fields of snow as time series. Although none of the models used here to produce the spatial fields of SWE are true representations of the real processes, it is, however, assumed that by using a distributed approach, important features of the dynamical properties of the spatial snow distribution, are captured.

## Methodology

In this section, we present the different models used to produce the time series of the spatial distribution of SWE. Each of the models represents obviously only an approximation to the true process in question. However, as the purpose of this exercise is to study the temporal behaviour of the spatial distribution of SWE, it is assumed that even systematic errors in the models will not disguise the overall features of the dynamical evolution of the snow reservoir. A distributed approach to the estimation of the snow reservoir implies that we, in principle, estimate the accumulation and ablation properties of snow at points, and analyse the collection of points. It is assumed that this procedure limits the impact of errors associated with the individual models.

## **1. The gridded water balance model**

Streamflow data were calculated by a spatially distributed version of the HBV-model (Bergström, 1995). The model carries out water balance calculations for  $1 \text{ km}^2$  grid cell elements which are characterized by their altitude and land use. Each grid cell may be divided into two land use zones with different vegetation, a lake area and a glacier area. The model has components for accumulation, subgrid scale distribution and ablation of snow, interception storage, subgrid scale distribution of soil moisture storage, evapotranspiration, groundwater storage and runoff response, lake runoff response and glacier mass balance, and it considers the effects of seasonally varying vegetation characteristics on potential evapotranspiration. The algorithms of the model are described by Sælthun (1996). The model was run with daily time step, using precipitation and air temperature data as input, and daily runoff data for the individual grid cells were subsequently aggregated to monthly streamflow data for the respective catchments. A globally applicable set of model parameters determined by Beldring et al. (2002) was used. The calibration procedure rests on the hypothesis that model elements having common vegetation characteristics, land use and pedological, topological and geological conditions controlling their hydrological process dynamics should be assigned the same parameter values. The model was calibrated using available information about climate and hydrological processes from gauged catchments in different parts of Norway, and parameter values were transferred to other catchments based on a classification of landscape characteristics. A multi-criteria calibration strategy was applied, where the residuals between model simulated and observed monthly runoff from 141 catchments located in areas with different runoff regimes and landscape characteristics were considered simultaneously.

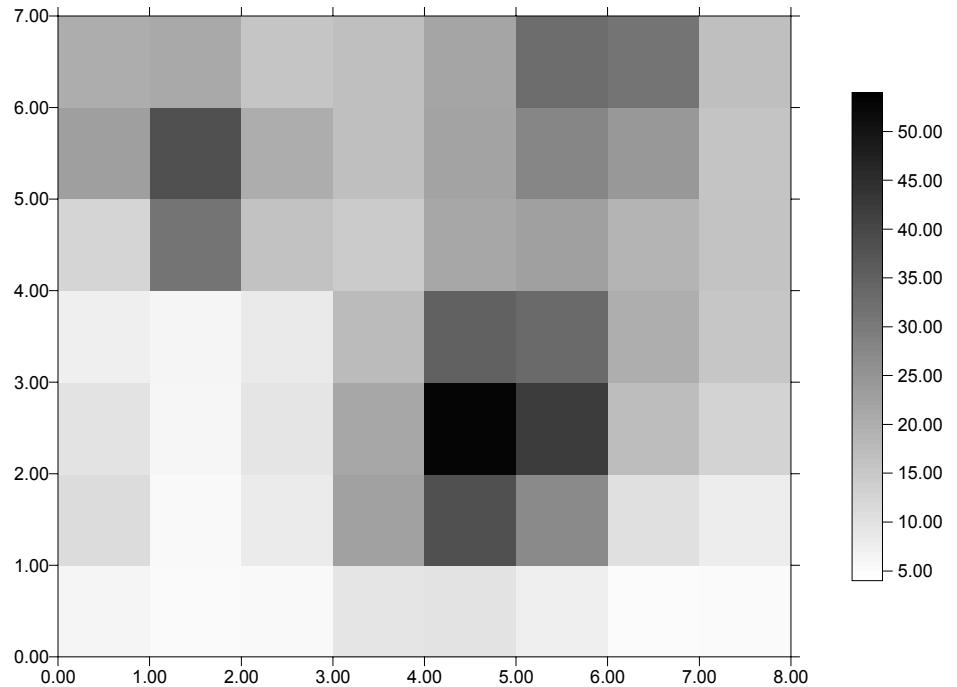
## **2. Disaggregation of precipitation**

Meteorological and hydrological processes are currently described on different spatial scales. Meteorological operational atmospheric models such as the HIRLAM use grid sizes of  $11 \times 11 \text{ km}^2$  and  $50 \times 50 \text{ km}^2$  whereas hydrological distributed models, such as the GWB, use  $1 \times 1 \text{ km}^2$ . The problem represented by this discrepancy in spatial scale was addressed in Skaugen (2002), where the spatial rainfall fields are simulated according to a mixture of exponential distributions. This methodology was applied in this study to provide continuous precipitation fields of spatial resolution  $1 \times 1 \text{ km}^2$ . From a rainfall field consisting of grid cells of  $11 \times 11 \text{ km}^2$ , precipitation values for pixels of resolution

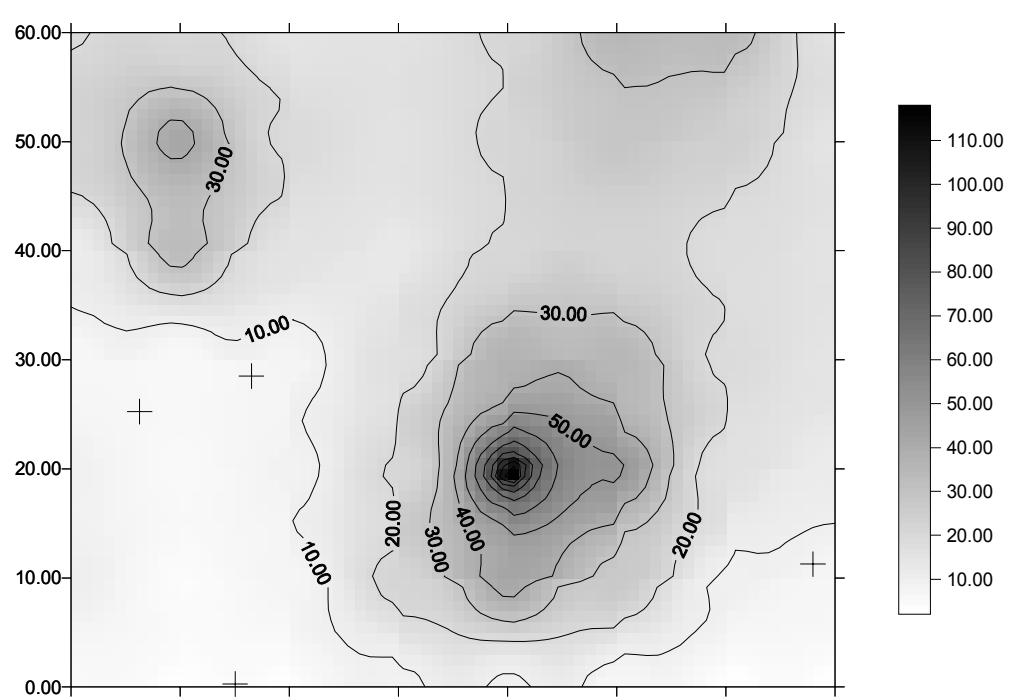
$1 \times 1 \text{ km}^2$  were estimated. The (two) parameters of the shifted exponential distribution were estimated locally from the nodal values of the atmospheric model, and intermittency was estimated based on (local) values of the spatial mean and variance. The disaggregation procedure is carried out according to the following points:

- 1) Let each grid cell be subdivided into 121 pixels. For the 121 pixels in each grid cell, each pixel is assigned a value interpolated from the nodal precipitation values (the four corner values) of the grid cell. This procedure is repeated for the N grid cells. Each pixel in the field is assigned a rank (1 to  $N \times 121$ ), according to its interpolated value.
- 2) For each grid cell, the spatial mean and the spatial variance of the grid cell are estimated from the nodal values of the grid cell.
- 3) For each grid cell, the spatial mean and variance is calculated and intermittency,  $p$ , is estimated by  $p = 2 / ((\text{var}(z) / E(z)^2) + 1)$ , where  $z$  is precipitation. Dependent on whether  $p$  is higher or less or equal to 1, it is decided if the grid cell is i) completely covered with a minimum intensity  $b$ , and exponentially distributed precipitation,  $f(z; \lambda, b)$ , ( $p > 1$ ), where  $\lambda$  is the parameter of the exponential distribution and  $b$  is a location parameter, or ii) intermittent with fractional coverage  $p$  and positive precipitation is exponentially distributed,  $f(z; \lambda)$ , ( $p \leq 1$ ).
- 4) In case of full coverage, 121 values are simulated from  $f(z; \lambda, b)$ , and in the case of intermittency,  $p \times 121$  values are simulated from  $f(z; \lambda)$ .
- 5) Point 2) to 4) is repeated for every grid cell 1,.., N
- 6) The  $N \times 121$  simulated values are then ordered and the ranked pixels from 1) are assigned the simulated value of equal order. In case of intermittency, the  $(1-p) \times 121$  lowest ranked pixels from each grid cell are assigned the value zero.

Figure 1 shows an example of the initial HIRLAM field of grid cells (Fig. 1 a), and the disaggregated field (Fig. 1 b). The disaggregation procedure respects intermittency, the mean and the spatial correlation structure of the original HIRLAM field, whereas the spatial variance is somewhat higher for the disaggregated field (Skaugen, 2002).



a)



b)

**Figure 1.** HIRLAM field a) and disaggregated field b) of the 6th August 2000. + marks locations of precipitation gauges. The unit is mm/day.

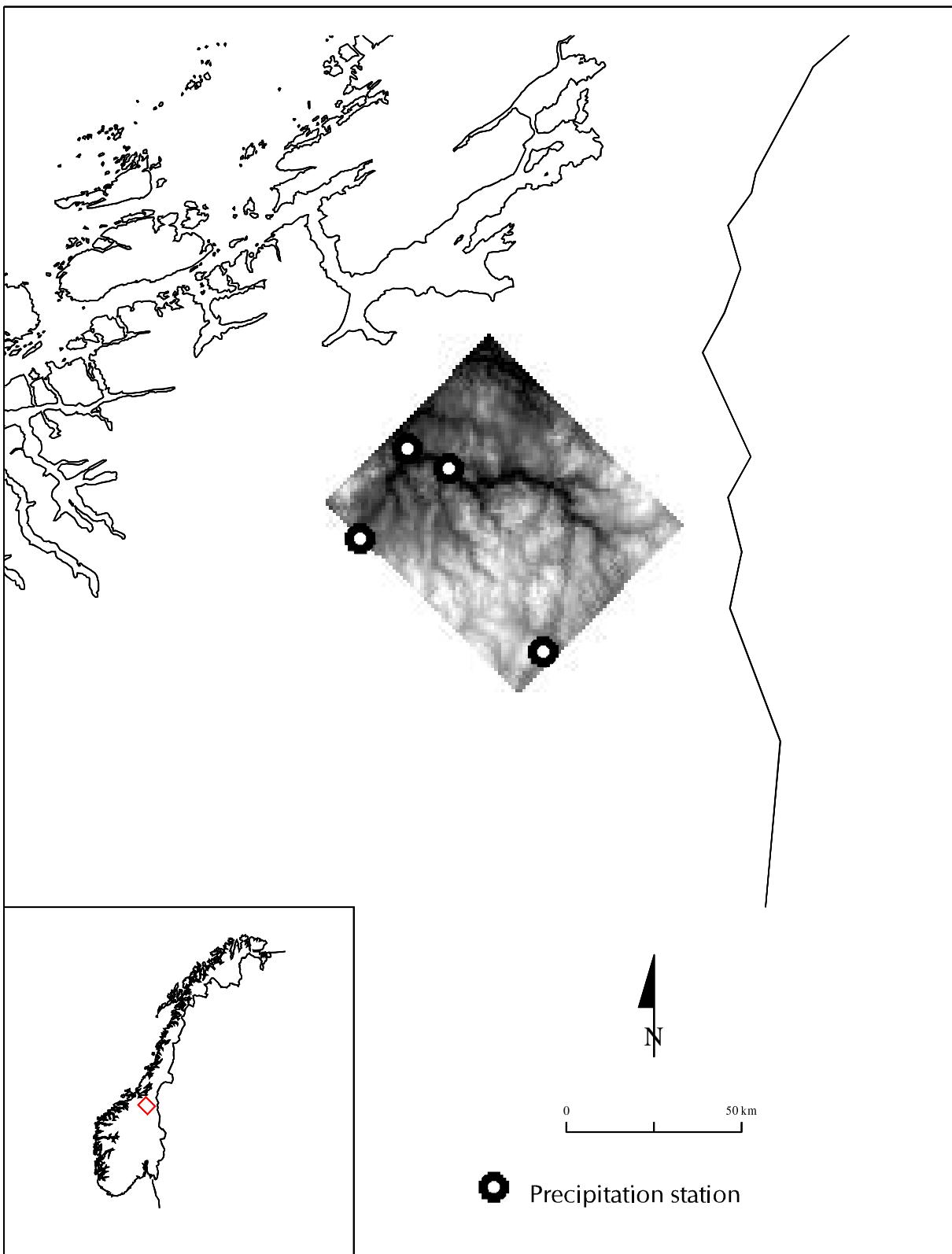
### **3. Interpolation of temperature**

Daily temperature values for the model grid cells were determined by inverse distance interpolation of data from the two closest HIRLAM grid cells. Differences caused by elevation were corrected by fixed temperature lapse rates equal to -0.47 and -0.62 °C per 100 metres for days with and without precipitation, respectively.

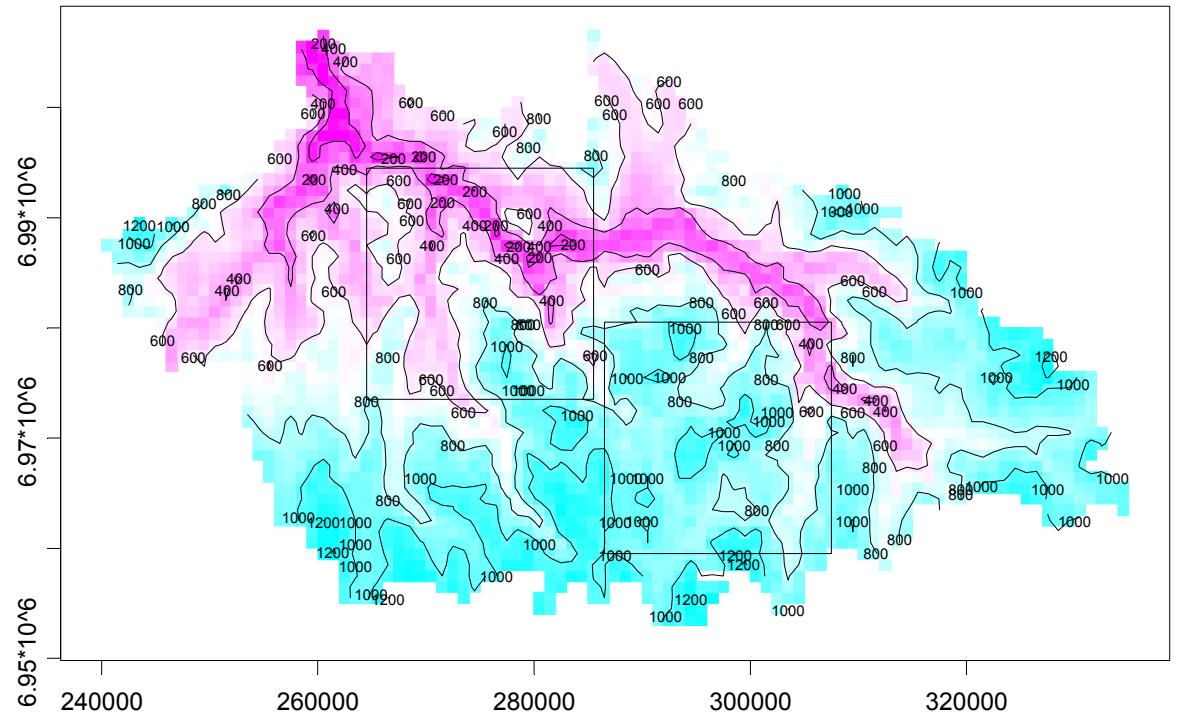
## **Results and discussion**

Time series of daily values of precipitation and temperature, disaggregated and interpolated to  $1 \times 1 \text{ km}^2$ , were estimated from the output of the HIRLAM model for the Gaula catchment for the period 1st October 1999 – 1st September 2002. Figure 2 shows the location of the Gaula catchment ( $3085 \text{ km}^2$ ). Figure 3 shows further the topography of the Gaula catchment with the location of the sub areas, each with size  $484 \text{ km}^2$ . The mean altitude of the Gaula catchment is 729 m.a.s.l whereas the northern sub area has a mean altitude of 585 m.a.s.l and the southern sub area has a mean altitude of 858 m.a.s.l. The GWB model uses a snow routine similar to that of the HBV model and daily values of SWE is calculated for the time period for each grid cell.

Figure 4 shows the cumulative distribution function for SWE (not including zeros) for the three catchments for the start of the accumulation season, the peak of the accumulation season, and at the end of the ablation season. Table 1 shows the spatial mean, spatial standard deviation, coefficient of variation (CV) and the coefficient of skew for the same periods of the season. For the Gaula and southern sub area the distributions are relatively skewed in the beginning of the season. The skew has decreased at the peak of accumulation and increases significantly at the end of the ablation season. For the northern sub area, it appears that the skew has increased steadily throughout the season.

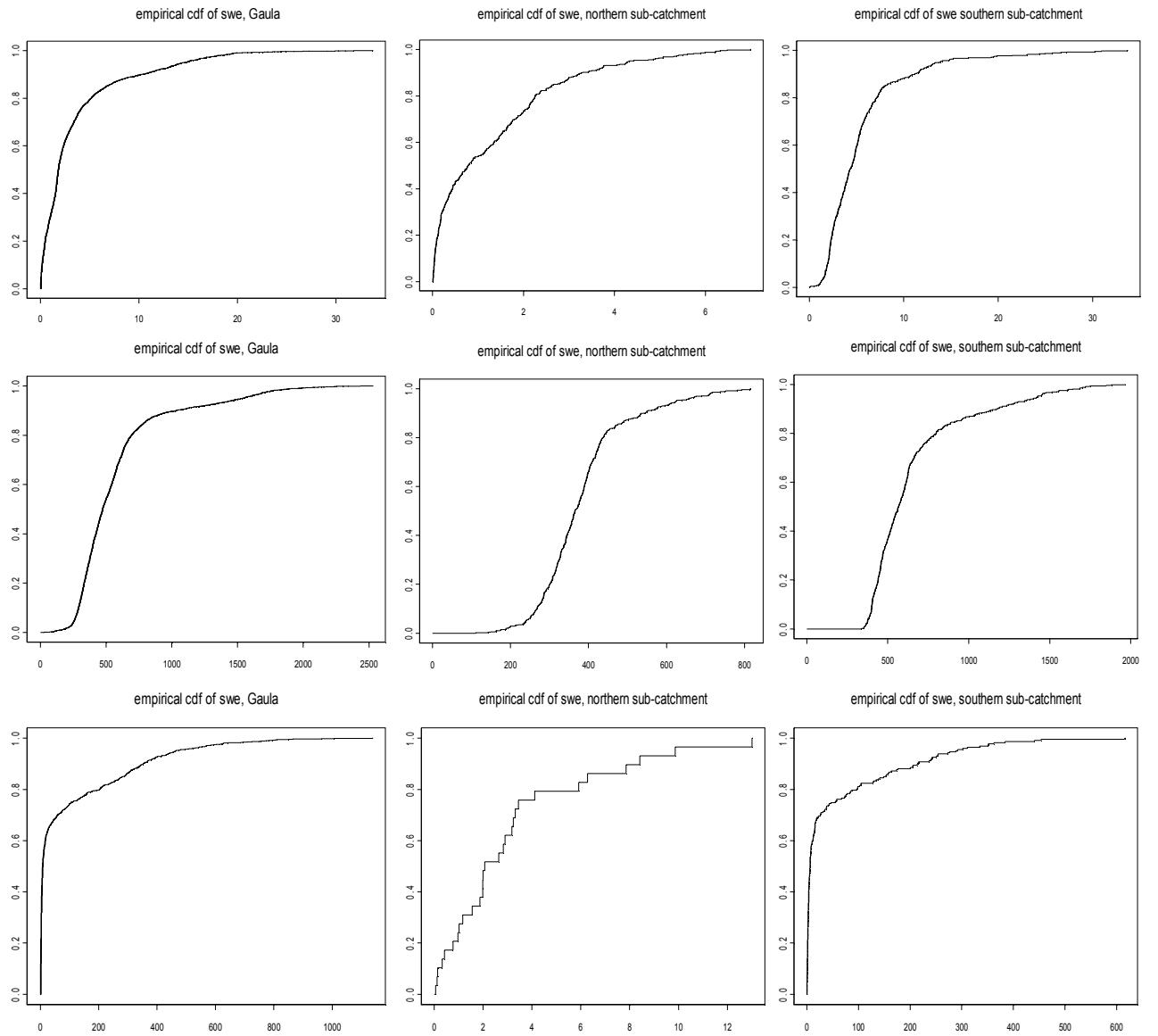


**Figure 2.** Map of the study area in Central Norway. The location of four precipitation gauges is marked.



**Figure 3.** Topography of the Gaula catchment (asl in meters) with the two sub areas (coordinate system UTM, zone 32).

We note that the northern sub area, being situated in the lower region of the Gaula catchment, only received about 60 % of the amount of snow compared to the southern sub area at the peak of the accumulation season.

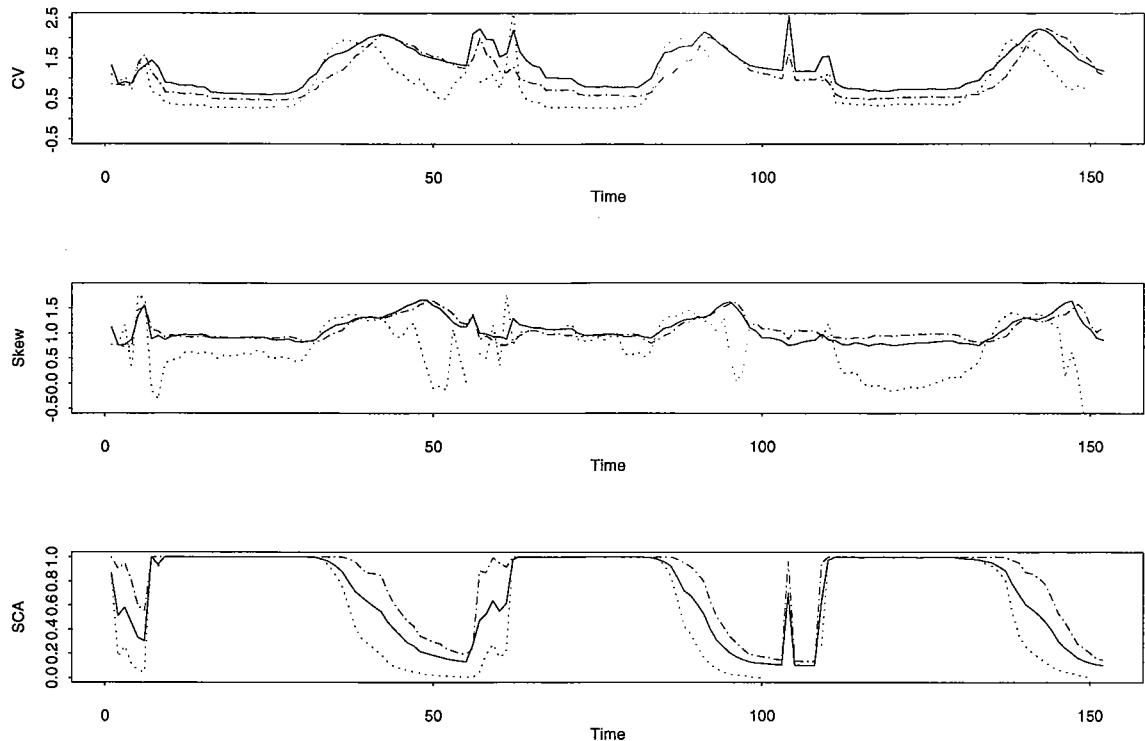


**Figure 4.** Cumulative distribution functions of the spatial distribution of SWE for the Gaula catchment (left column), northern sub area (middle column) and southern sub area (right column) for the start of the accumulation season (top row), peak of accumulation season (middle row) and end of the ablation season (bottom row). Note the different horizontal scales.

**Table 1.** Statistical parameters for the spatial distribution of SWE for the start of the accumulation season (8<sup>th</sup> October 1999) peak of the accumulation season (14<sup>th</sup> April 2000) and end of the ablation season (18<sup>th</sup> August 2000)

Catchment	Date	8th October	14th April	18th August
Gaula	Mean (mm)	3.5	579.5	97.3
	Std.dev. (mm)	4.7	369.9	173.9
	CV	1.34	0.63	1.78
	Skew	1.12	0.88	1.55
Northern Sub-area	Mean (mm)	1.3	383.6	3.22
	Std.dev. (mm)	1.5	114.5	3.2
	CV	1.15	0.30	0.99
	Skew	1.13	0.49	1.09
Southern Sub-area	Mean (mm)	5.5	661.8	55.25
	Std.dev. (mm)	4.7	315.5	102.1
	CV	0.85	0.48	1.85
	Skew	0.76	0.93	1.45

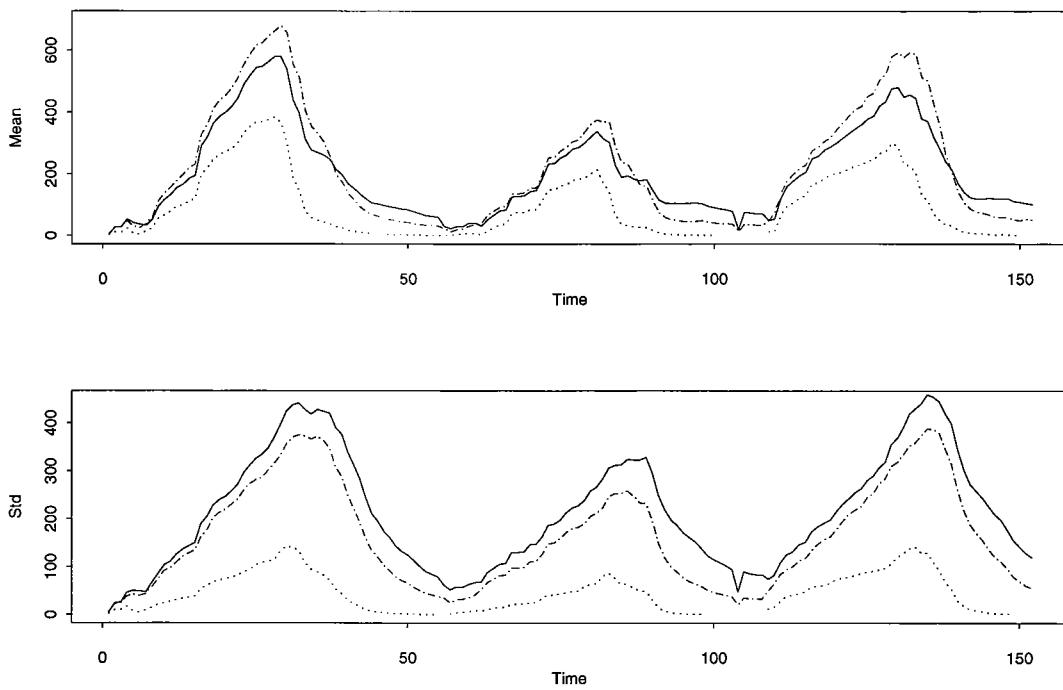
A similar pattern of the temporal variability of the statistical parameters CV and skew can be observed in Figure 5, where the CV, skew and SCA for the three areas are plotted as time series. The coefficient of variation and the skew are obviously correlated, and high fluctuations occur in the beginning of the season, then the parameters decrease during the accumulation season, and increase as the melting season sets in. This development is in accordance with the principle of the central limit theorem (Feller, 1971, p.258) which would give a decrease in skew as the accumulations increase, and the ablation process acts as a reverse providing an increasingly more skewed distribution.



**Figure 5.** Time series of the CV and Skew for the spatial distributions of SWE and SCA, for the Guala catchment (solid line), northern sub area (dotted line) and southern sub area (dashed line). The time index refers to week numbers beginning at the start of the first accumulation season.

An interesting feature of Figure 5 is that the coefficient of skew is consistently much higher for the southern sub area than for the northern sub area. It is a commonly observed feature that when one compares snow courses carried out in forested and alpine areas (above the timber line) one finds that the distribution in the alpine areas tend to be more skewed (Alfnes et al. 2004; Marchand and Killingtveit, 2002). There have been many attempts to link the spatial distribution of SWE to physical parameters like elevation, slope, aspect, net solar radiation and the type and density of vegetation cover (Erxleben et al. 2002, Elder et al. 1989). Large portions of the observed variability in the snow depth remain, however, unexplained. Redistribution due to wind have also been a popular

descriptor, but more difficult to verify. These results may be explained by a mismatch in spatial scales between predictor and descriptors, which, as pointed out by Blöschl (1999), may seriously affect predictions. This important point is valid when there is a mismatch in process scales, but will also play a role when the spacing and extent of data do not capture the variability of the processes involved. In this study, features of the spatial distribution of SWE was not especially linked to such physical parameters as described above, and all features we observe of the spatial distribution of SWE, at least in the accumulation season, stem from the effects of accumulating disaggregated precipitation fields. From Figure 6, which shows the temporal development of the spatial mean and the spatial standard deviation for the three catchments, we observe that for similar values of the spatial mean for the three areas, the spatial variability is considerably less for the northern sub area, thus having the smallest CV during the accumulation season. As we can observe from Figure 5, the coefficient of skew and the CV appear to be correlated, and several popular choices for the spatial distribution of SWE, like the lognormal and gamma distribution (Sælthun and Killingtveit, 1995; Skaugen, 1999; Skaugen et al. 2004), have theoretical expressions of the coefficient of skew being proportional to the CV. It thus appears that the skewed spatial distribution of SWE we find in alpine areas, can partly be explained by the increased spatial variability of precipitation in such areas.



**Figure 6.** Time series of the mean and standard deviation for the spatial distributions of SWE, for the Gaua catchment (solid line), northern sub area (dotted line) and southern sub area (dashed line) The time index refers to week numbers beginning at the start of the first accumulation season.

Only in a very few limited cases can the distribution of accumulated events be described analytically. Spatial independence and independence between events are typical constraints. Also, seen from a spatial point of view, the events should be identically distributed. Both the normal and the gamma distribution have analytical expressions for the distribution of the accumulations. Analytically these models break down when we deal with precipitation fields of varying spatial distributions which are correlated in space and time. Despite of these theoretical problems, Skaugen et al. (2004) proposed a statistical model that takes into account the dynamical behaviour of the spatial distribution of SWE. In Skaugen (1999), the distribution of accumulated snow was

modelled as a summation of independent, identically distributed gamma variables. This modelling framework allows positively skewed gamma distributed single events, whereas the distribution of the accumulated events will also be gamma distributed but with parameters determined by the original gamma distribution and the number of accumulations. The distribution of the accumulated events will converge to a normal with a rate depending on the parameters of the gamma distribution and the number of accumulations. This approach was carried further in Skaugen et al. (2004), where a gamma distributed unit SWE was introduced. An event of accumulation or ablation, which may comprise a number of units will, under an assumption of independence, also be gamma distributed. This modelling framework was implemented within the HBV model, and a routine for modelling the temporal development of the snowcovered area, linking the change in SCA as a response to an accumulation- or ablation event directly to the parameters of the spatial distribution of SWE was developed. The main features of the accumulation-ablation model of Skaugen et al. (2004) are described as follows: Let  $y$  be a SWE equivalent and a gamma distributed random variable with probability density function (PDF):

$$f_{\alpha,\nu}(y) = \frac{1}{\Gamma(\nu)} \alpha^\nu y^{\nu-1} e^{-\alpha y} \quad \alpha, \nu, y > 0 \quad (1)$$

where  $\alpha$  is the scale parameter and  $\nu$  is the shape parameter. The mean equals  $E(y) = \nu / \alpha$  and the variance equals  $Var(y) = \nu / \alpha^2$ . If the variable  $y_i$  is approximated to be an independent and identically distributed gamma variable in time and space, then  $z_n(x) = y_1 + y_2 + \dots + y_n$  is distributed as a gamma variable with parameters  $\alpha$  and  $n\nu$  (Feller, 1971, p.47). The parameters  $\alpha$  and  $\nu$  are catchment specific parameters determined by the local climate, whereas the parameter  $n$  represents the accumulated number of SWE equivalents in the snow reservoir at a certain time of interest. Thus, the spatial distribution of accumulated SWE ( $z$ ) at a given time has mean and variance equal to:

$$E(z) = n\nu / \alpha \quad (2)$$

and

$$Var(z) = n\nu / \alpha^2 \quad (3)$$

We further find that the coefficient of variation  $CV$  and the skew,  $\gamma$ , are dynamical parameters in that they are functions of the parameter  $n$ :

$$CV = 1/\sqrt{n\nu} \quad (4)$$

and the skew is:

$$\gamma = 2/\sqrt{n\nu} \quad (5)$$

It is clear from (4) and (5) that the value of the  $CV$  and  $\gamma$  should decrease during the accumulation season when  $n$  grows, and increase in the ablation season when  $n$  decreases, in a similar manner as have been demonstrated by the simulation exercise.

The procedure of modelling the spatial distribution of SWE as a summation of a gamma-distributed variable is approximate in that the assumption of independence in time and space may be compromised to an unknown degree. Regarding the spatial independence, several authors (Elder et al. 1989, Faanes and Kolberg, 1996; Gottschalk and Jutman, 1979) report low autocorrelation for a range of distances. The studies of the latter two references were carried out for Norwegian and Swedish data respectively and are thus representative for the present study. Studies attempting to link snow depth to terrain features have often showed very weak correlations (Elder et al. 1989; Faanes and Kolberg, 1996; Erxleben et al. 2002), so that an assumption of independence in space can be justified. In order to investigate possible temporal dependencies, Skaugen et al. (2004) reports of data from a snowpillow, Vauldalen (820 m.a.s.l.) located in the central southern Norway, which was tested for autocorrelation. Out of 16 sequences with more than 13 days with snowfall, only 5 sequences showed to have significant autocorrelation for lag 1 day, none of the sequences showed significant autocorrelation for longer time lags. An assumption of temporal independence can thus be justified.

## Conclusions

Time series of the spatial distribution of SWE have been simulated using a distributed hydrological rainfall-runoff model with input data from the HIRLAM atmospheric model. The rainfall input has been disaggregated, and the temperature input has been interpolated to the spatial scale of  $1 \times 1 \text{ km}^2$ . Time series of the spatial distribution of SWE have been constructed for three areas of different mean altitudes. The time series for all three areas

display the same dynamical behaviour of the parameters of the distribution in that the skew decreases during accumulation season and increase during the ablation season.

From the simulation exercise we found that the spatial variability of precipitation could explain a more skewed spatial distribution of SWE in mountainous areas. This may prove an important point in the modelling of snow in rainfall runoff models in that the parameters of the model can be determined from an investigation of the spatial variability of rainfall and not from the calibration process of the rainfall runoff model. This is a point of departure for future work.

Modelling the spatial distribution of SWE as sums of gamma distributed variables, takes into account the dynamic properties of the spatial distribution of SWE in that it allows for a dynamical change in the shape of the distribution in accordance with the simulation exercise, observations and the principles of the central limit theorem.

## References

- Alfnes, E., L. M. Andreassen, R.V. Engeset, T.Skaugen and H-C Udnæs. 2004.  
Temporal variability in snow distribution. *Annals of Glaciology*, in press.
- Beldring, S., Engeland, K., Roald, L.A., Sælthun, N.R., Voksø, A. 2002. Regional  
calibration of a distributed hydrological model. *XXII Nordic Hydrological Conference,  
Røros, Nordic Hydrological Programme, Report No. 47*, 441-450.
- Bergström, S. 1995. The HBV model. In: Singh, V.P. (Ed.), *Computer Models of  
Watershed Hydrology*. Water Resources Publications, Highlands Ranch, 443-476.
- Blöschl, G. 1999. Scaling issues in Snow Hydrology. *Hydrol. Process.* **13**, 2149- 2175.
- Elder, K., J. Dozier and J. Michaelsen. 1989. Spatial and temporal variation of net snow  
accumulation in a small alpine watershed, Emerald Lake basin, Sierra Nevada,  
California, USA, *Annals of Glaciology* **13**: 56-63
- Erxleben, J. K. Elder and R. Davis. 2002. Comparison of spatial interpolation methods for  
estimating snow distribution in the Colorado Rocky Mountains, *Hydrol. Process.* **16**  
3627-3649.
- Faanes, T. and S. Kolberg. 1996. Optimal utnyttelse av snømagasinet. SINTEF report No.

STF22 A96412 (in Norwegian).

- Feller, W. (1971) An introduction to probability theory and its applications. John Wiley & Sons, Ltd., New York, 669 pp.
- Gao, X, and S. Sorooshian, 1994. A stochastic precipitation disaggregation scheme for GCM applications', *J. Clim.*, 7(2), 238-247.
- Gottschalk, L. and T. Jutman. 1979. Statistical analysis of snow survey data, Swedish meteorological and hydrological institute, SMHI, Report No. 20.
- Killingtveit, Å. and N-R. Sælthun. 1995. Hydrology, (Volume No. 7 in Hydropower development). NIT, Trondhjem, Norway.
- Mackay, N.G., R.E. Chandler, C. Onof, and H.S. Wheater. 2001. Disaggregation of spatial rainfall fields for hydrological modelling, *Hydrology and Earth System Sciences*, 5(2), 165-173
- Marchand, W-D. and Å. Killingtveit. 1999. Statistical properties of snow cover in mountainous catchments in Norway, Northern Research Basins, Twelfth International symposium and Workshop, Reykjavik, Iceland: pp.227-239
- Marchand, W-D. and Å. Killingtveit. 2002. Statistical probability distribution of snow on sub-grid cell level. Nordic Hydrological Programme, NHP Report no. 47, pp 461-471.
- Onof, C., N. G. Mackay, L. Oh, and H. S. Weather. 1998. An improved rainfall disaggregation technique for GCMs. *J. Geophys. Res.* 103 (D16), 19577-19586.
- Skaugen, T. 1999. Estimating the mean areal snow water equivalent by integration in time and space. *Hydrol. Process.* 13, 2051- 2066, 1999.
- Skaugen, T. 2002. A spatial disaggregating procedure for precipitation, *Hydrological Sciences Journal*, 47 (6), 943-956.
- Skaugen, T., E. Alfnes, E. G. Langsholt and H.-C. Udnæs. 2004. Time variant snow distribution for use in hydrological models. *Annals of Glaciology*, in press.
- Sælthun, N.R. 1996. The Nordic HBV model. *Norwegian Water Resources and Energy Administration Publication No. 7*, Oslo, 26 pp.

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